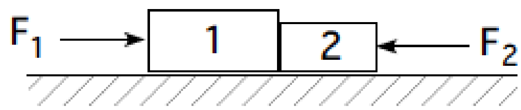
Application of Newton's Laws
Concept Questions

Question 1 Two blocks 1 and 2, on a frictionless table, are pushed from the left by a horizontal force $\vec{F}_1$, and on the right by a horizontal force of magnitude $\vec{F}_2$ as shown above. The magnitudes of the pushing forces satisfy the inequality $|\vec{F}_1| > |\vec{F}_2|$.



Which of the following statements is true about the magnitude N of the contact force between the two blocks?

1. $N > |\vec{F}_1| > |\vec{F}_2|$

2. $|\vec{F}_1| > N > |\vec{F}_2|$

3. $|\vec{F}_1| > N = |\vec{F}_2|$

4. $|\vec{F}_1| = N > |\vec{F}_2|$

5. $|\vec{F}_1| > |\vec{F}_2| > N$

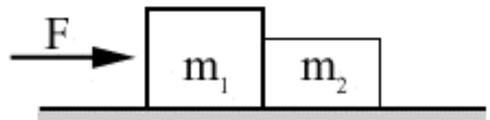
6. Cannot be determined from the information given.

Answer 2

Considered as a whole the blocks are pushed to the right by a force greater than that pushing to the left. Thus both blocks accelerate to the right. Block 1 is pushed to the right by $\vec{F}_1$ and to the left by $\vec{N}$. It accelerates to the right thus $|\vec{F}_1| > N$. Block 2 is pushed to the right by $\vec{N}$ and to the left by $\vec{F}_2$. It accelerates to the right, thus $N > |\vec{F}_2|$.

Question 2: Newton's Laws of Motion: *Force Applied to Two Blocks*

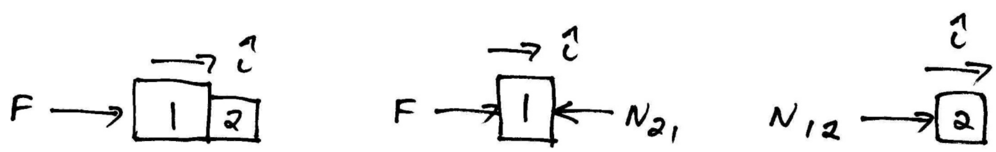
Two blocks sitting on a frictionless table are pushed from the left by a horizontal force $\vec{\mathbf{F}}$, as shown below. The magnitude of the contact force between the two blocks is



a) greater than the magnitude of the pushing force $\vec{\mathbf{F}}$.
b) lesser than the magnitude of the pushing force $\vec{\mathbf{F}}$.
c) equal to the magnitude of the pushing force $\vec{\mathbf{F}}$.
d) Cannot determine from the information given.

Briefly explain your reasoning.

Answer b. Consider the three free-body force diagrams shown below which depict only the horizontal forces acting on the blocks. (There is no friction; the vertical normal forces (between the table and the blocks) cancel the weights, and so vertical forces will be neglected.)



Take the positive direction to be to the right in the figure. There is a contact force of between blocks 1 and 2 with magnitude $N \equiv N_{12} = N_{21}$; this force is directed to the left on the block 1 and to the right on the block 2. From the first free-body force diagram, applying Newton's Second Law in the horizontal to the system of both blocks yields

$$F = (m_1 + m_2)\,a\,. \tag{0.1}$$

Applying the Second Law to just block 1 yields

$$F - N = m_1\, a \tag{0.2}$$

or

$$F = N + m_1\, a \tag{0.3}$$

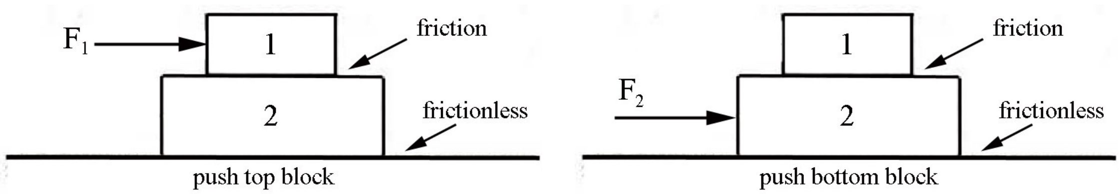
indicating that the magnitude of the pushing force is greater than the magnitude of the contact force between the blocks. As a check Newton's Second Law applied to block 2 yields

$$N = m_2\, a \tag{0.4}$$

We can solve for a from Eq. (0.1) and substitute that into Eq. (0.4) yielding

$$N = \frac{m_2}{m_1 + m_2} F < F\,. \tag{0.5}$$

Question 3 Consider two textbooks that are resting one on top of the other. The bottom book has mass M_2 and is resting on a nearly frictionless surface. The top book has mass M_1, with $M_1 < M_2$. Suppose the coefficient of static friction between the books is μ_s.



Consider the following two experiments. You push the top book horizontally with the maximum force so that the two books move together without slipping? You push the bottom book horizontally with the maximum force so that the two books move together without slipping? Explain which force is greater in magnitude.

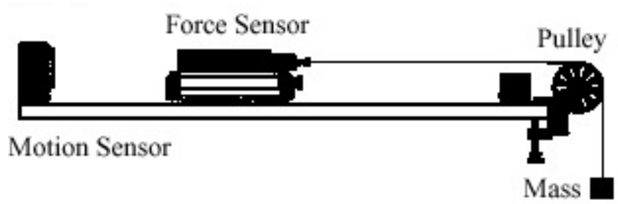
Solution The acceleration of the books is greater when you push the lower book than when you push the upper book (recall that the problem statement specified $M_1 < M_2$).

Whether you push the top or the bottom, the pushing force is equal to the total mass times the acceleration, $F = (M_1 + M_2)a$. In order to determine which book, top or bottom, you can push with a greater force before the books slip, we must decide in which case the maximum value of the acceleration is greater.

The maximum value of static friction between the books has the same magnitude whether you push the top or bottom book, $f_{s,\max} = \mu_s M_1 g$. When you push the top book, the static friction is the force that accelerates the lower book. So the maximum acceleration is given by $a_{1.\max} = f_{s,\max} / M_2$. When you push the lower book, the static friction is the force that accelerates the upper book with maximum acceleration is given by $a_{2.\max} = f_{s,\max} / M_1$. The resulting acceleration $a_{2.\max} > a_{1.\max}$ will be greater than the acceleration when you push the lower book, because $M_1 < M_2$.

Therefore you can push the lower book with a greater maximum force $F_{2,\max} = (M_1 + M_2)a_{2,\max}$ just before the books slip than when you push the upper book with a maximum force $F_{1,\max} = (M_1 + M_2)a_{1,\max}$ just before the books slip.

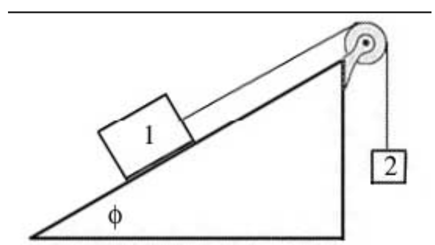
Question 4: A force sensor on a cart is attached via a string to a hanging weight. The cart is initially held. When the cart is allowed to move does the tension in the string



1. increase?
2. stay the same?
3. decrease?
4. cannot determine. Need more information about friction acting on the system.

Answer 3: The tension must decrease. Before the cart is released, the magnitude of the tension is equal to the magnitude of the gravitational force $T = mg$. Once the object is released, using Newton's Second Law the tension is now $mg - T = ma$. So $T = m(g - a)$ and hence has decreased. This is analogous to an elevator accelerating downward, where the normal force is less than the gravitational force (since $mg - N = ma$).

Question 5 A block of mass m_1, constrained to move along a plane inclined at angle φ to the horizontal, is connected via a massless inextensible rope that passes over a massless pulley to a bucket to which sand is slowly added. The coefficient of static friction is μ_s. Assume the gravitational constant is g. What happens to the tension in the string just after the block begins to slip upward?



1. Increases
2. Decreases
3. Stays the same
4. Oscillates
5. I don't know
6. None of the above

Answer 2. Before bucket starts to accelerate, the tension in the string satisfies $mg - T = 0$ so $T = mg$, where m is the combined mass of the bucket and sand. Once the bucket starts accelerating downward , $mg - T = ma$ or $T = m(g - a)$. Thus the tension in the string is decreases.

Question 6 Weightlifting

A weightlifter and a barbell are both at rest on a large scale. The weightlifter begins to lift the barbell, ultimately holding it motionless above her head. Does the scale reading ever differ from the combined weight of the two bodies at any time during the lift? Explain.

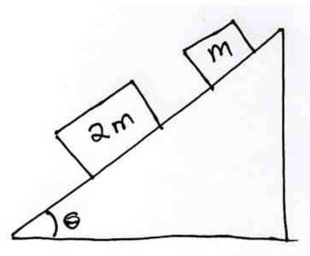
Solution: Choose the positive direction upwards. The motion of the barbell is as follows:

Stage 1: The barbell is being accelerated upwards as weightlifter jerks it from the floor. In this stage, $N - m^{\text{total}} g = m^{\text{total}} a_y$, where a_y is the acceleration of the center of mass of the lifter-barbell system and N is the magnitude of the normal force that the scale exerts on the lifter and hence the scale reading . The acceleration a_y is positive upwards, and hence $N = m^{\text{total}} g + m^{\text{total}} a_y > m^{\text{total}} g$; the scale reading shows an increase.

Stage 2: The barbell moves at constant velocity. (In practice, this would happen only instantaneously.) The acceleration is zero, $N = m^{\text{total}} g$, and so the scale reading returns to its original (pre-lift) value.

Stage 3: The barbell is being decelerated as the weight lifter brings them to rest above her head. The acceleration during this stage is downward (negative upwards), and so $N = m^{\text{total}} g + m^{\text{total}} a_y < m^{\text{total}} g$; the scale reading is less than its lift or pre-lift value.

Question 7 You place a single and a double brick on a level board and then slowly raise one end of the board. The double brick has twice the mass and twice the surface contact area as the single brick. The coefficient of static friction between each of the bricks and the surface is μ_s. The coefficient of kinetic friction between each of the bricks and the surface is μ_k. Which of the following statements is true?



1. The single brick slides before the double brick.
2. Both slide at the same time.
3. The single brick has a greater acceleration than the double brick.
4. The acceleration is the same for both.
5. None of the above.
6. Two of the above.

Answer 3 and 6.

The static friction force and the gravitational force are both proportional to mass. When you raise one end and the blocks have not slipped, the magnitude of the static friction force is equal to the component of the gravitational force along the incline plane, $f_s = mg\sin\theta$. As you continue to raise one end, the static friction reaches it maximum possible value $f_{s,\max} = \mu_s mg\cos\theta_{\max}$ at the angle $\theta_{\max}$. The forces still balance hence $\mu_s mg\cos\theta_{\max} = mg\sin\theta_{\max}$, and we can solve $\tan\theta_{\max} = \mu_s$ which is independent of the mass so both blocks will slip at the same angle. When the blocks are sliding, $mg\sin\theta_{\max} - \mu_k mg\cos\theta = ma$ and hence the acceleration is independent of mass. Therefore both blocks have the same acceleration.