I now want to discuss the multiplication of vectors.

In this course, we will encounter two kinds of multiplications-- dot products and vector products with very, very different characteristics.

If I have a vector A dot vector B [INAUDIBLE] bold dot. We call this a dot product. As you will see, the dot product is a scalar, so this is often also called the scalar product.

And then we have a totally different way of multiplying vectors with a cross here. And we call it A cross B. And we call this the vector product. And no surprise of course, this is a vector.

Let's first discuss the dot product.

The dot product is defined as follows: A dot B. If the x component of A multiplied by the B component-- by the x component of B plus the y component of A multiplied by the y component of B. Plus the z component of A multiplied by the z component of B. These are three numbers, and those numbers are going to be added. Some of them may be positive, some of them may be negative. So out of this comes one number, characteristic for the scalar.

Suppose we have A equals 3 x roof minus 2 y roof plus 4 z roof. And B equals minus x roof plus 3 y roof plus 2 z roof. So A has a component in the x direction of 3 in the plus x direction, in the y direction minus 2, in z direction plus 4.

What now is the dot product of A with B? It is the x component of A times the x component of B, which is 3 times minus 1, is minus 3. Plus the y component of A times the y component of B, which is minus 6. Plus the z component of A times the z component of B, which is plus 8. And so the outcome equals minus 1. So this dot product is a scalar if the outcome, which is minus 1.

I'd now like to look at the vectors A and B in their own plane. And I'll make that plane of course, the plane of the paper.

So let this be vector A and let this be vector B. We know already that A dot B is the dot product, equals A of x B of x plus A of y B of y plus A of z B of z. I'm now telling you, without proof, that it also equals the magnitude of A times the magnitude of B times the cosine of theta, if theta is the angle between A and B. This is completely identical. If you want to prove it, give it a shot.

Now, in our case, the example that we discussed, the dot product was minus 1. It's a scalar. And so minus 1 then equals the magnitude of A, which was the square root of 29. The magnitude of B is the square root or 14. And then we get cosine theta here. And so I find that cosine theta equals minus 0.0496. That gives me two values of theta. You always get two values. 92.8 degrees and I find 267.2 degrees. It shouldn't surprise you that you get two angles. One angle is this one and the other angle is this one. Notice that the sum of the two is 360 degrees.

When the smallest angle of these two is larger than 90 degrees, which is the case here, then the dot product is always negative. You see that here. So I would like to write that down in its more general form.

So the dot product is positive when the smallest angle of the two is smaller than 90 degrees. It's 0 when the smallest angle of the two equals 90 degrees. The other angle then, of course, is 270 degrees. The sum always being 360. And the dot product is negative when the smallest angle of the two is larger than 90 degrees. So what it comes down to, in a nutshell, if we have here B and we have here A, if this angle is smaller than 90 degrees the dot product is positive. If the angle between A and B as you see here is larger than 90 degrees, then the dot product is negative. And if the angle between A and B were exactly 90 degrees, then the dot product equals 0.