So, I think we have hit that hard enough. Let's go to-- my goodness-- 4.B.5. It's Monday morning and it's early, believe it or not. The gravitational and electric potential energy. Let's first do gravity and there is no fiction. We have assumed no friction, no air drag of any kind.

Here is the sun. It has a mass M sun. This symbol stands for the sun. Here's an object of mass little m at position 1 at separation r1 from the sun. And here is that same mass at position 2 at separation r2 for the sun. And I now want to evaluate the forces, gravitational forces, pulling from the sun. This is F1. It's regularly pointing towards the sun, and the magnitude is m times the mass of the sun times g divided by r1 squared.

And this force, which is smaller because it's farther away from the sun, F2, equals m times the mass of the sun times the gravitational constant divided by r t squared. And when I go all the way to infinity, F then at infinity, the gravitational pull from the sun equals 0. Because 1 over r squared becomes 0.

Now if I bring the object from position 1 to position 2, I have to overcome the gravitational force. So I move my hand to the right, I pull to the right, and I move it to the right, so I do positive work. And per definition, U2, that is the gravitational potential energy here, minus U1, the gravitational potential energy here. [INAUDIBLE] per definition, the work I do in moving that object from position 1 to position 2. And since here r2 is larger than r1, U2 minus U1 is larger than 0. So I do positive work.

If I bring it from here to here, then r2 is smaller than r1. r2 minus U1 is smaller than 0, and I do-- no. U2 minus U1 is always larger than 0. But if I go from here to here, then I go from a higher potential energy to a lower potential energy, and so I do negative work. So I confused you. r2 is always larger than r1 in this case. U2 minus U1 is always larger than 0. If I go from 1 to 2, I therefore have to do positive work. If I go from 2 to 1, I therefore, have to do negative work. I go from a higher potential to a lower potential.

Now before we continue, I would like to examine with you a somewhat simpler case.

Suppose here I have an object with mass m and I have here a force mg. And here is my force Walter Lewin. I'm bringing it from-- here's the floor. Here's a table top. I call this table top arbitrarily, y equals 0. I could've called this something else. This table top is at location A. This is at location B. And I bring it up to a position h, so y equals h here.

Now to bring this mass from A to B I have to do positive work. And per definition, Ub minus UA is the work that I have to do to bring it from A to B. And it's immediately obvious that that is mgh. This is easy to remember. mgh stands for Massachusetts General Hospital. At least, that's the way I always remember it. So that's the work I do when I bring it from A to B.

Now here, since we are moving it, that distance is very small compared to the size of the earth, the gravitational force is constant all the way when I go from A to B. So my force is constant. So that's why I can do it in this simple way.

However, if we are going back to the problem with the sun, then of course, the situation is somewhat different.

If I go back to the sun, it is obvious that this force is not the same as this force. So now I cannot simply when I do the integration in going from 1 to 2, I cannot assume that the force is constant. So F1 is not the same as F2 because r1 is smaller than r2.

So now what is the work that I do? Well that is the integral r1 to r2 of m M sun times g divided by r squared dr.

I go from here and I move it out and I'm at some random location r. And I move it out over the distance dr. Over this small trajectory I assume that the force remains constant. This is the magnitude of my force when I have to pull it to an increasing value of r. dr is positive here and my force itself is also positive. I have to pull away from the center. You could call this force negative if you wanted.

Well, clearly, I'm dealing here with a dot product between my force, Walter Lewin, and dr. But since the angle is 0 between the two, the cosine angle equals 0. So I can ignore that for now. And so I get that U at r2 minus Ur1, which is the work that I have to do to bring the object from 1 to 2 becomes m M sun times g times 1 over r1 minus 1 over r2. And I have assumed that there is no fiction here.

Now if r2 is larger than r1, which is the case here, then this whole thing is larger than 0. And I do positive work. That's immediately obvious because as I move the object farther away from the sun, clearly I have to overcome the gravitational pull. So it is immediately obvious that I do positive work, and so the potential energy at location 2 is larger than the potential energy at location 1.

What now is the potential energy at position 1? See when you're dealing with very close to earth situations in our laboratory, you can effectively choose the 0 level any way you want it. You can choose it on the floor, you can choose it at the table top, or you can choose it at that location B. Remember, I was moving it from the table top to that location B. It doesn't really matter, the 0 has no consequence. What counts in physics is the difference between potential energy. Difference between potential energy determines the force. The force determines the acceleration. That means the change of velocity. So no matter how you decide, where you decide to choose your 0, gravitational potential energy in here in the laboratory system, the physics is not different.

Here however, it is meaningful and useful to choose that the potential energy of gravity at infinity equals 0. So I prefer that the choice is not yours now.

Now the consequence of that is, if I fill in for r2 infinity, then I get r equals infinity. Which we defined to be 0. Minus U or r. For r1 I put here now a random location r equals m mass of the sun times g times 1/r because this is 0.

This is the scalar and this is 0, so if I rewrite this a little bit as Ur, I get a minus sign here. So we get Ur gravitational potential energy equals minus m M sun times g divided by r. And it is a scalar. It has no direction. Let's put this in a nice red box because it's a very important equation.

This negative sign is the price that we have to pay for the fact that we call U at infinity equals 0. Put in r equals infinity and you see that U indeed becomes 0.

If we wanted to draw a graph of the potential energy as a function of r, here is r. Here is U potential. These values would all be positive. This will be 0. And these values would all be negative. Then, the gravitational potential energy is 1 over r. It's a hyperbola. So it would go like this. Goes to 0 at infinity. But of course, this is proportional by the way, to 1/r. But when you get inside the sun, then of course, if this is r sun, then this has to be reevaluated. This certainly does not continue as a hyperbola.

And as r decreases, if r decreases, U decreases. For B, to move it further in, I have to do negative work. As r increases, U increases. Notice if it is negative here, but it's still increasing. And for me to move it further out, I have to do positive work.

Now if you know how U depends on x, y, or z, or r, for that matter, you can easily find the vector components of the forces that are responsible for the potential energy.

F of x-- that is the x component of that force-- equals minus dU dx. What does that mean?

I move the object only in the x direction. I keep y and z constant. And then, I watch how the potential energy is changing. And this equation then gives you the x component of that force. Equally, Fy equals minus dU dy and Fz equals minus dU dz. And if you have something that only depends on the radius, like the case of the sun, Fr equals minus dU dr.

Let's take the situation of the table. On the table we had U as a function of y. y was increasing in this way. Let's call it mgy. I then arbitrarily choose-- at y equals 0 I choose U equals 0. Feel free to do this, but that makes it in this case, rather easy.

Well, if this is a scalar, if now I calculate the y component of the gravitational force, I get minus dU dy, so I get minus mg. And this minus sign is simply telling you that it is in the opposite direction as increasing y. So this force is in downwards direction, which is exactly what it should be. Fx equals 0 and Fz equals 0 because there's no dependents of the potential energy on either x or z for that matter.

Now the minus sign is also easy to see. In the case of the sun where we have only a radial dependence, the gravitational force would then be minus dU dr. I have to do positive work to increase U. If dU is positive, it means that dr is positive. I go outwards. And we all know that the gravitational force is directed inwards. that's the meaning now of this minus sign. Just in a similar way as this minus sign was telling you that it is in the direction opposite to an increasing value of y. This is telling you that this minus sign is pointing in an opposite direction than in the direction of increasing r. So the force of Walter Lewin, if I bring it out, would be plus dU dr. Because my force is exactly in the opposite direction than the force from gravity.

So U in the case of gravity, it was the sun equals m M sun times g divided by r. Notice that when r increases that U increases. Because of this minus sign.

Now dU dr equals plus m M sun times g divided by r squared. So F in the radial direction of gravity is minus dU dr. So F in the radial direction equals minus m M sun g divided by r squared. And the minus sign then indicates that at this point in the opposite direction as increasing r I prefer a different notation.

If here is the sun and here is r, I would like to introduce-- but I think Professor Guth is not doing that. I would like to introduce a unit vector r roof. It has lengths 1 and it is radially pointing out. And I would like to put here an r roof and here a vector. Then there is simply no confusion whatsoever. You see immediately that the gravitational force is opposing the unit vector r roof, which is pointing outwards to increasing value of r But this is a matter of taste, and I'll leave it up to you what you prefer.

Now we turn to electric potential energy. Now we have a charge plus q and plus capital Q. And unlike gravity, these repel. So already you feel in your stomach that everything is going to be very, very similar, except that minus signs may change to plus signs and plus signs may change to minus signs. Yeah, that's true. But remember, in electricity the charges can also have opposite signs. And if that's the case, they attract each other. So then, you would expect to see something extremely similar to what we have seen with gravity. Because with gravity, masses cannot be negative and positive. So with gravity we always have attracting forces.

All right, here we have plus q and here we have plus capital Q. They are at a distance r and they repel, so the electric force is in this direction outwards. And the electric force here equaling strength in the opposite direction. And the magnitude of this electric force equals q times Q divided by 4 pi epsilon 0. Something you will see later in 802. It's a constant. Divided by r square, inverse r square.

If I increase the distance between these two objects, I have to do negative work. If I have to bring them closer together, I have to overcome this repelling force. Then I have to do positive work. And if again, I assume that the potential energy at infinity is 0, I now get that the electrostatic potential energy-- not gravitational, but electrostatic equals plus q times capital Q divided by 4 pi epsilon 0 r. This is nearly identical to gravity except that with gravity we had a minus and here we have a plus.

Now, the nice thing about this equation is that if you change the signs of q and capital Q-- little q and capital Q-- if you make them both negative, the plus sign still remains plus. Because minus times minus is still plus. So this still holds.

If however, they have opposite signs, either this plus and this minus or this plus and this minus, then the upstairs here becomes negative and you get a situation, which is completely analogous to gravity. Which is exactly what you predict because now you have attracting forces. And when you have attracting forces, the upstairs here, once you make this multiplication, the upstairs becomes negative. So it's very nice and very cute that you see here the beautiful symmetry with gravity in case that the electric charges attract each other.