

You ready? I am. Our first problem.

Our first problem is problem 4.1.2. There's a projectile, which is thrown up at an angle θ . It reaches a certain height h . Let's call this the positive y direction. Let's call this point 0. Both in x and in the y direction. The object has a mass m . Its velocity at time t equals 0 when we shoot it up here is v_0 . So its velocity in the x direction, which never changes because there's no acceleration in the x direction. It's $v_0 \cos \alpha$. And its velocity in the y direction, which does change because there's an acceleration in the y direction. It is $v_0 \sin \theta$. My goodness. This must be $v_0 \sin \theta$. And this will also be the y direction comes to a halt when it is at its highest point, and then reverses. Let's call this point A. Let's call this point B. And let this point where it hits the ground C.

Clearly at its highest point, the y component has completely disappeared. But the x component is unchanged, so that it's still the $v_0 \cos \theta$. And when it hits the ground here, the x component has not changed and the y component has changed only in the sense it has the same magnitude, but it has flipped over. So it hits the ground again at the velocity v_0 . Whereby this angle is again θ because of the symmetry of the parabola. And this angle here is θ .

And now the first question is, what is the kinetic energy at point A? Well, that obviously is $\frac{1}{2} m v_0^2 \sin^2 \theta$.

What is the potential energy at point B? The only velocity at point B is the horizontal component, so it is $\frac{1}{2} m v_0^2 \cos^2 \theta$.

And what is the kinetic energy at point C? It must be the same as the kinetic energy at point A because the velocity is the same here. Now I should not say the velocity. The speed is the same here as the speed is there. But the velocity is in a different direction. But kinetic energy is a scalar. This kinetic energy is not a vector, so it is the speed that matters.

We can also look at this from an energy conservation point of view. I could say that the potential energy in this case due to gravity at point A plus the kinetic energy at point A would be the potential energy at point B plus the kinetic energy at point B, and that must be the potential energy at point C plus the kinetic energy at point C. This would be true if there is no loss of energy, if there is no friction, if there is no air drag. Nothing. No energy is converted to heat.

Well, it's immediately obvious that U_A is the same as U_C because they're at the same level. And so if you take this part of the equation equal this part of the equation, you would see immediately that the kinetic energy at point A is the same as the kinetic energy at point C.

If you take the left part of the equation, you can write down that U_B minus U_A would be the kinetic energy at point A minus kinetic energy at point B. U_B has of course, a higher potential energy than U_A , which is mg times h if this height is h . You know the kinetic energy at point A and so you should be able to calculate the kinetic energy at point B. And this would be a very nice way for you to check whether indeed the speed at that point A is indeed, this values. Now of course, you don't have to do that. You're not asked to do that. But it always pays off to do a problem in more than one way. It gives you a little bit more insight. And if you have the time, I certainly would advise you to do that in this case.