For those of you who are very good in math, I would like to derive the period of a simple pendulum using the conservation of mechanical energy. If you're not very good in math, just forgot about this-- I'll derive the period of a pendulum in another way. Now, I will do it using mechanical energy, and it's little bit more complicated, but it's cute and nice. It shows you that physics works no matter how you approach it.

Here's a pendulum, and the pendulum is viewed when the angle is theta. This is the equilibrium position of the pendulum, and the pendulum will ultimately swing all the way to an angle theta max. There's the length l, and the velocity at this point here-- let's call this point v-- is v of theta. We have here point A-- equilibrium-- point B, and let's call this point D.

If there is no friction of any kind, then the sum of potential energy and kinetic energy must be conserved here, here, and here. It must be the same as a constant. I can always arbitrarily choose the level of gravitational potential energy, and I called at u equals 0 at A. At point B, I would have to evaluate this distance h-- this equals l cosine theta, so h equals l minus l cosine theta equals l times 1 minus cosine theta.

At that point B, mgh-- remember, Massachusetts General Hospital, that's the difference between potential energy, between here and here-- equals mg times l times 1 minus cosine theta. That is u at point B, and the kinetic energy at that point B where the angle is theta, equals 1/2 m v theta squared. This all holds for point B. At point A, u equals 0, and all the energy is in kinetic energy. In point D, the kinetic energy is 0, and all the energy is in gravitational potential energy.

Now I want to write down at point B, u plus kinetic energy is a constant-- mgl times 1 minus cosine theta plus 1/2 m v theta squared equals a constant. I call that equation number one.

What is v theta? v theta is the same as d theta dt, which is the angular velocity times l. This is the angular velocity, for which very unfortunately sometimes people write omega, not to be confused with angular frequency, which is sometimes also called omega. I'd like to write down this as theta dot times l. This d theta dt will change all the time-- d theta dt 0 here, and has a maximum value here.

Now I want to take the derivative of equation number one, recognizing that v theta is theta dot times l. Let's first write down this equation with a substitution: mgl 1 minus cosine theta plus 1/2 m times l squared times d theta dt squared equals a constant. I want to take the derivative, and this part you may find difficult. This is the u part, and this is the kinetic energy part-- this is a constant, so this is going to be 0 if I take the time derivative.

Now, mgl-- the 1 has no effect. The cosine becomes a minus sign-- I have a minus here-- so I got minus minuses plus sign theta, and then, of course, I have theta dot. I have to use chain rule: plus 1/2 m l squared, and I have to take the derivatives of theta dot squared. That gives me a 2 times theta dot times theta double dot-- chain rule equals 0.

This is du dt, and this is d kinetic energy dt. m cancels, 1 l goes, the 2 goes against the 2, and here I have a theta dot, and here I have a theta dot. The whole situation becomes extremely simple.

What do I end up with? I end up was g sine theta plus l theta double dot equals 0. For small angle approximation, sine theta is about the same as theta if theta is in radians. I find now that theta double dot plus g over l times theta equals 0, and I say yippee! I hope I spelled that yippee correctly-- yippee, this must be a simple harmonic oscillator, because I recognize immediately say theta double dot plus a constant times theta is 0. In fact, I can immediately write down that the angular frequency is the square root of g over l, and the period of oscillation, which is 2 pi divided by the angular frequency, equals 2 pi times the square root of l over g.

This is, of course, no surprise-- this is a very familiar result. You see that you can also derive the period of a simple harmonic oscillation in the case of the pendulum. You can also do that using the conservation of mechanical energy-- whichever method you prefer is up to you.

It's clear that if the object is a simple harmonic oscillation in x, which it was-- we called the equilibrium position x. This is plus x, and this is minus x. If it is a simple harmonic oscillation in x, then it must also be a simple harmonic oscillation in theta, because the sine of theta was x over l. For small angle approximation, the sign of theta is theta. Anything that is a simple harmonic oscillation in x will also be a simple harmonic oscillation in theta.