

Let us now examine the difference between the angular velocity and the angular frequency, which unfortunately-- both often have the same symbol, ω .

Let this be a pendulum. This is the equilibrium position. I look at the situation a little later in time-- this angle is θ with length l , and this angle is $d\theta$. This object at this moment here has a velocity-- let's say it has a velocity in this direction, tangential-- I call it V_θ . In the time that this angle $d\theta$ takes place, there is a little arc-- I call the arc ds . That little arc ds is $d\theta$ in radians times the length l .

It is velocity V_θ , which is ds/dt , that becomes $d\theta/dt$ times l . This $d\theta/dt$, which is the angular velocity, is often called ω , and so this is ωl . This ω has nothing to do with the ω that is defined as angular frequency, which is 2π divided by P . This angular velocity is 0 when the object stands still, when the angle of θ has reached the maximum value of $+\theta_{\text{maximum}}$ or $-\theta_{\text{maximum}}$, because the velocity is then 0-- it stands still, so the $d\theta/dt$ better be 0. It is a maximum when it goes through equilibrium either in this direction, or when it goes in this direction.

The angular velocity, this ω , changes in time. Now, the angular frequency: the angular frequency shows up in the equation x equals A times cosine ωt plus α , or if you want to write it down in θ , you can do that as well-- it's the same thing. This ω is called angular frequency, and this ω determines the period of the oscillation.

For a given pendulum, and for a given length, this ω is a constant-- it's not changing in time. In fact, we will show that through a good approximation, the ω doesn't even change if you change the amplitude of the oscillation. It's independent of A and independent of α -- it's a constant for a given simple harmonic oscillation, and angular velocity is not.

It is extremely unfortunate that angular velocity, which changes in time for a pendulum, is called ω , and that angular frequency, which is a constant for a given simple harmonic oscillation and does not change in time, that they both have the same symbol of ω . It's very confusing, and that's why I spent some time on it. If you keep them completely separate at all moments, and if both occur in the same problem, you may even decide to give one of the two ω s a different symbol, or maybe an ω' . Keep them apart-- that is an absolute must.