Perhaps the easiest example of a simple harmonic oscillation is a spring with an object at the end. Let's suppose we have such a spring here-- this will be the spring in a relaxed position. I call this x equals 0, I arbitrarily call this the plus direction, and this the minus direction. This is the relaxed spring, and I extend the spring with an object mass m here, and this position equals x.

The spring, in my approximation, is massless. There are no mass less springs, but what it means is that the mass of the spring can be neglected compared to the mass of this object. Also, there is no friction of any kind: this could be on a horizontal table, the friction is stable, and there is no air drag. It is an idealized situation.

The spring has a spring constant k, and when I bring this object out over distance x, there is a restoring force-- the spring force that is driving it back-- F, to equilibrium. F-- this is that force, according to Newton's Law-- equals ma, and a is the second derivative of x versus time. I can write m times d 2 x d t squared, but I prefer to write for the second derivative an x with a double dot, and for the first derivative, an x was dot, so I get m x double dot.

The magnitude of x equals kx, if it is an ideal spring-- if Hooke's law holds. But F is a vector: it has a magnitude and a direction. Since this is a one dimensional problem, I can simply remove the vectorial notation, and simply write F equals minus kx. Notice that when x is positive, automatically F in this direction will be negative. When x is negative-- so, in this direction-- notice that automatically the force in this direction will be positive.

This minus sign takes into account that F is a vector. The minus sign indicates that there's a restoring force: if x is positive, it wants to bring it back to equilibrium. If F is negative, it wants to bring it back to equilibrium. I can now rewrite-- forgive me for my cold-- F equals m x double dot simply as minus kx equals m x double dot. I get mx double dot plus kx equals 0, I divide by m, and I get the famous relation x double dot plus k divided by m times x equals 0. That is famous in that this means immediately when you see this that this is a simple harmonic oscillation.

k over m is a given for a particular system. Let's write it in more general terms for its c-- we write for the c for now, so I could have, then, a differential equation-- plus c x is 0. That would be a simple harmonic oscillation in x, and if you ever encounter something like this, the c's could be different, of course. Then it would be a simple harmonic oscillation in y, and if you ever encounter something like this, then it would be a simple harmonic oscillation in terms of an angle, like when we had the pendulum-- the angle theta.

The solution to any one of these equations: the solution is that A-- which would be x equals A times the cosine of omega t plus alpha, and you may prefer a sign. If I choose that, then x dot, which is the velocity, would become minus A times omega times the sine-- omega t plus alpha-- and that means x double dot would become minus A omega squared times the cosine omega t plus alpha. That is the same as minus omega squared times x, because x A cosine omega t plus alpha here, and so this is x again.

I will substitute this result now in this equation-- I'll call this equation number one. I have to substitute x double dot, and then I have plus c x. What is x double dot? That is minus omega squared x plus cx equals 0. I don't think we need this equation anymore-- it's clear what I have done-- x double dot is this value, and cx is this value. I will leave this on the floor for now, and this is 0.

You'll find immediately that this is true, and only true, if omega squared equals c, and so we have omega equals the square root of c. So in the case of our spring system, omega-- the angular frequency-- would be the square root of k over m in radians per second. The period of the oscillation in seconds, which is 2 pi divided by omega, would be 2 pi times the square root of m over k, and this will then be in seconds. Omega, which is the angular frequency, equals 2 pi divided by P.

Notice that both omega and P are independent of A and independent of alpha. A and alpha depend on the initial conditions. For instance, and this is only one example, if you chose the time t equals 0-- and you are always free to choose what you call time t equals 0-- if you choose that at the moment that the object x is at plus A, and that the velocity at that moment equals 0, then you'll find that alpha equals 0 when you have this cosine term. A, then, is the amplitude of the oscillation: it's the maximum displacement in one direction, and then it is the maximum displacement in the other direction.

Alpha is only 0 if you define it in this matter-- if you define the times 0 at other moments of time, then alpha will not be 0. There's not that much physics in alpha-- really, all the physics is in the omega, and is in the period.