This is 7.5.5.

I have a spring on a table-- this is x equals 0, that's when the spring would be relaxed. I extend the spring-- I have an object here on this table, and the position of this object equals x. In fact, I bring it out as far as I possibly can so that when I let it go, that it doesn't go back-- it's the maximum possible displacement that I can give this object. The question is how far can I move it so that it doesn't start to slide?

There is a static friction coefficient mu s, and there's a kinetic friction coefficient mu k. Let's make a free body diagram-- mg is non-negotiable. Here is the force-- the spring force-- whose magnitude equals kx. It's opposing x, so I can write down in factorial form that it is minus kx, but I'm only interested in the magnitude now. There is normal force from this table.

I said earlier that there is no friction-- that, of course, is not true. The whole idea is that there is friction, as you can see. As this object wants to go in this direction, the only way that this can be prevented is that there is friction here-- the frictional force is exactly opposing this, and if these two cancel each other out, that will be no acceleration. The object, in this case, will stay put.

The requirement is that the frictional force-- F friction, the magnitude of the frictional force-- as long as that is equal to kx, then the sum of all forces in the x direction will be 0. I call this the x direction.

The frictional force is always less or equal to the normal force times the static friction coefficient. It's static because it has to start moving from zero speed. N equals mg, you've seen that [? previously ?] so this equals mg times mu s-- this is correct. So the requirement then is that kx must be less or equal to mg times mu s.

What do you find? That the maximum possible distance x-- I can move this out so it doesn't start to slide-- equals mg times mu. That's a bad mu, but it is really a mu, and divide by k. That's, then, my final result.

Let's look at this result and see whether we can understand this qualitatively. The first thing is that if mu s was 0, notice that x maximum equals 0-- that's immediately obvious. If there were no friction at all, there's no way you could move it to any distance, and it would start to slide immediately. The fact that it is linearly with us is quite pleasing.

Suppose that k is very small-- this is my way of saying it's very small-- then the spring is very sloppy, so this force is small. It's immediately obvious that you can bring it out very far, and still have equilibrium-- so you can move it out very far, the spring is not very strong, and it will still stay put.

It's also clear that if you increase the friction coefficients, if you make the friction coefficient very high, then you can bring it out much further so it stays put and doesn't fly back.

I think we've hit this hard enough, and now I would like to go to the next part. That is that I give this one a teeny little push so that instantaneously I break loose the surface here, and I break loose the contact. My static friction coefficient will all of a sudden become the kinetic friction coefficient. This force, which was mg mu s, goes down-- it becomes now mg mu k. Therefore, this spring force is stronger, and so the object will go to the left.

If I call this the positive direction of x to the right, then I can write down the differential equation: ma is, of course, all in the x direction which would then be minus kx. This was its maximum position, because that's where I started from-- this is the spring force opposing x plus mg times mu k. Notice there is no kinetic friction coefficient, because the friction in this direction is reduced, because I break loose the contact.

This is the equation of motion, if you want to put it that way, and so that also equals minus mg times mu s minus mu k. I just substitute in here for x max the value that we just calculated, and so the absolute magnitude of the acceleration which is in a minus x direction-- I want to remind you of that-- the magnitude equals a, and it cancels, equals mu s times mu k times g. That's the instantaneous acceleration once I break it loose.

When is the speed maximum? That's is a very interesting question-- so when is the speed a maximum? Here's that spring-- let this be the position x equals 0, and this is the position x maximum. It starts to move, and there comes here a position x, and I will call that where F equals 0. Remember when it starts to move, this spring force is larger than the frictional force, because of this mu k being smaller than mu s. But as it moves to the left, the spring force goes down and down and down, because the spring force is proportional to x. There comes a point here that these two forces cancel each other out, the object is no longer being accelerated, and so right here you must have reached you maximum speed.

The maximum speed will be reach them when the spring force-- F of s-- the magnitude of the spring force equals mu k times m times g. We know that the spring force, the magnitude of the spring force, equals kx, so you will find easily that that will happen when x equals mu k times mg divided by the spring constant k. Earlier, we had x maximum-- we had mu s here, now we an x, and we have mu of k here. You can really see that this must be smaller than x maximum. It's unusually obvious because this mu of k is smaller than mu of s. It's immediately obvious that this point where that will happen, where the speed reaches a maximum, is on the left side of the maximum displacement where the object is still standing put.

Now I want to ask you something extra, and that is: what is its maximum speed? This is not being asked, so you don't have to answer that question, but I thought it would be useful-- what is the maximum speed? Now, we're going to use the concept of energy. Keep in mind that when I release the object at its maximum displacement x max, there is potential energy in the spring, which is 1/2 k x squared. When it reaches the point where the force is 0, that is only this much left-- F equals 0-- so the potential energy was large, the potential energy is now smaller, and the difference, however, is not 1/2 any squared of the object.

The reason is that there is friction, and the friction sucks energy out, and I must take that into account. The loss of energy must be taken into account before I put this equal to 1/2 half mv squared, which is the kinetic energy of that object. I must now take out the energy that the friction takes out that goes out into the form of heat. You must subtract the frictional force times the distance that the frictional force moves-- x of F equals 0. You know what I mean by that-- we just calculated that value.

This is lost-- this goes out in terms of heat. Note the frictional forces in this direction-- the object moves in this direction, and that's why energy is being sucked out of the system. The dot product is negative-- you see this minus sign here, and this now would be equal to 1/2 m times v maximum squared. It would be the kinetic energy right at the location x where the force is 0. If you massage that a little further, because you know what x max is, and you know what x is at F equals 0, you will find that the maximum speed that you reach at that location equals g-- I hope I didn't make a mistake-- mu s minus mu k divided by the square root of m over k.

Let's see whether we understand these results-- it's always nice to do a few simple checks. Suppose mu s equals mu k: it's clear that if that's the case that the object will never start moving, are you find that v max equals 0. If you have a static friction coefficient which was the same as the kinetic friction coefficient, if I just tapped that object standing still-- just standing still, hanging in there-- and if I tap it, there is no change in frictional force. There is still equilibrium between the spring force and the frictional force, so it's not going anywhere, and doesn't pick up any speed. It is very pleasing that if the two are the same that the maximum speed that you get is indeed 0.

Let's now ask ourselves the question: where does the object come to a stop? I'm going to make the drawing again-- this was the on the relaxed spring-- and then here we have at this point x where F equals 0. This is where the velocity as we just calculated reaches a maximum value, so I'll remind you of that v max. Further on here is the location x max where we could put it without the object moving-- the object would stay put there. When the object is here, there is a frictional force which is maximum.

There is always a maximum, because the object is moving, and there is the spring force-- the magnitude of spring force is kx and x is at this location where F equals 0. From this moment in, it has a certain speed. This force becomes smaller and smaller and smaller, but this one stays the same so the object is going to decelerate.

Let's now ask the question, will it reach point x equals 0-- that is, this the position where the spring is relaxed-- and what speed will it have? Let's assume that it will reach that point, and let us assume that the velocity that is will have there equals v, and x equals 0. Now, follow me very closely-- forget this point here, because I don't need that. I'm going to start here-- right here, in terms of energy, I have 1/2 m max squared, and that is at that location F equals 0. That's kinetic energy.

I also have potential energy in the spring, which is 1/2 k x squared at that location F equals 0. When it reaches this point 0-- it's not even clear that it will-- this is not equivalent to 1/2 m v squared here, because the frictional force takes energy out as we just saw before. I must subtract the work done-- the negative work done-- by the frictional force, which is now from x at F equals zero to x equals zero, assuming that it makes it to that point. x f equals 0-- that is this distance. This, then, would be 1/2 m v x equals 0 squared.

The whole question is if this is positive, then it will make it, and it will have a speed. If it is 0, it will come exactly to a halt there, and if it is less than 0, it won't even make it here, but it will come here to a position which I will call x stop, which will be on the right side of x equals 0.

We've calculated v max, we have calculated this location x, so we can substitute that into this equation, and we have in our problem that mu s equals 0.35 and mu k equals 0.25. You substitute that in here, and what do you find? I found that this was negative, and therefore it will not make it to x equals 0-- it will stop short.

We'll go a little further-- this is the kind of problem that could easily be asked on an exam. Where will it stop? I now want to know precisely the position where it stops-- I know it's on the right side of x equals 0. It will stop here-- what is that location? Again, energy considerations: 1/2 m v maximum squared at F equals 0 was the kinetic energy when it was here.

The potential energy, 1/2 k x squared at F equals 0 is the potential energy here. When it comes to a halt here, there is still potential energy left, because the spring is still stretched. How much potential energy is there left? There's 1/2 k x at that position stop squared-- oh, there's a minus sign, I'm sorry. This is how much potential energy we have, and this is how much there is left, so the difference is what is available in terms of energy, so this must be a minus sign. Now I have to also subtract the work done by this frictional force, so mu k times mg-- and now, it travels over this distance. The distance is x at F equals 0 minus x stop. On this now is exactly the potential energy at the moment that it stops, and that is 0, so when all this energy has been consumed, so to speak, the object will come to a halt.

If you look very closely, you know all these things. The only thing you don't know is x stop. We have one equation with one unknown as x stop, and you can solve for x stop.

What will be the situation when it comes to a halt? Here is x equal 0, here is x stop-- which we know is on the right side of x equals 0. At this moment, there will still be a spring force, and this force will be k times x stop.

What now will be the situation with the frictional force? The frictional force as it was coming in and it was decelerating this object, the frictional force was larger than the spring force. The moment, however, that it comes to a stop, instantaneously the frictional force will be exactly the spring force. It is a sudden drop into frictional force, and the frictional force becomes exactly k x stop. As it comes in, the frictional force is larger than this one-- it remains constant. All of a sudden, when it comes to a halt, the frictional force goes bloop, adjusts itself so that it exactly cancels the k x stop.

Friction is always a tricky thing. Friction can be lower than the maximum possible, but only when it is moving is the frictional force the maximum possible. It's not moving now-- it's stopped, so it [UNINTELLIGIBLE] adjusts, and it stays put. Just imagine that the frictional force remains larger then kx, but then the object would go back-- have you ever seen that?

Have you ever seen an object that comes to a halt because of friction, and that it all by itself goes back? Here I have a little purse for my coins: if I put that on here, and I move it, it's friction that brings it to a halt. Have you ever seen that it would come back all by itself? Of course not-- so I move it like this, and it comes to a halt, and when it comes to a halt in this particular case, the frictional force becomes [UNINTELLIGIBLE] 0. In this particular case, the frictional force has to balance out this spring force.