

We have problem 4.3.3.

This is kind of a classic-- I have a pendulum here which has length  $l$ , and the pendulum here has an angle  $\theta_0$ . If the pendulum could swing freely, then it would go back and forth like this. However, there is a pin here-- this distance here is capital  $L$ -- and so this remaining part is little  $l$  minus capital  $L$ . When this string hits the pin, it goes around, this angle becomes  $\alpha$ , and it comes to a halt here-- let's call this point C, let's call this point B, and let's call this point A.

The question now is what is this angle  $\alpha$ ? Let's assume we release this object at first at velocity 0. We will also assume later that we give it a certain velocity  $v_0$  tangentially here in this direction, or it will give it a tangential velocity  $v_0$  in this direction.

Let's first agree on where we define the potential energy to be 0. It's completely arbitrary, but I would say that the gravitational potential energy-- I would call that at point B 0. There's nothing sacred about it-- I just think that's kind of nice in this case. If there is no loss of mechanical energy, there's no air drag, and there's no friction, then I can write down the law of the conservation of energy, which then states that the total energy in A is the total energy in B is the total energy in C. That means the total potential energy in U, which in this case, is all gravity at point A plus the kinetic energy at point A, is the total potential energy in gravity at point B plus the kinetic energy at point B and is a total gravitation of potential energy at point C, plus the kinetic energy at point C.

When we release it, if we release it, at speed 0, then of course this one would be 0. When it comes to a halt at points C, this one is certainly 0, and so you can immediately see that the potential energy at points C must be exactly the same as the potential energy as point A if there is no kinetic energy at A and no kinetic energy at C-- this is addressed here, and I release it at 0.

The fact that these two are the same means, then, that this point C is at the same height above me as point A. This distance here-- if you want to call it  $h$ , be my guest-- must be the same at this distance here. That allows you now to calculate that angle  $\alpha$ , because how far is A above the-- well, that's this distance. Am I doing that right-- point A is the distance above.

This is  $L$ -- capital  $L$ -- and this is also capital  $L$ . This piece here-- which I will indicate in blue-- is  $L \cos \theta_0$ . This point here, which is the difference between A and B in height, equals  $l \sin \theta_0$  while one minus  $\cos \theta_0$ -- that's this height.

How high is point C above B? Now, we have to look at this triangle here. Here we have  $l \sin \alpha$ , and so part here is  $l \sin \alpha$  times the cosine of  $\alpha$ . What remains here is  $l \sin \alpha$  minus  $l \sin \alpha$  times cosine  $\alpha$ . The two have to be the same, so I have  $l \sin \alpha$  minus  $l \sin \alpha$  times cosine  $\alpha$ , and I have one equation here, and  $\alpha$ , which is  $\alpha$ , so I should be able to solve for this.

The situation is different when there is a velocity here-- then the problem is a little bit more complicated. I would like to find my original equation, because now this value is not 0. This value would be  $\frac{1}{2} m v^2$  at that location--  $\frac{1}{2} m v^2$ . This would still be 0, so now you have to calculate the angle  $\alpha$ . This recalculated potential energy here is  $mgh$ , so it is this value times  $mg$ .

The potential energy in C that we also know that is this value times  $mg$ . You have to add to this one  $\frac{1}{2} m v^2$ , so you now again have one equation with one unknown-- namely, you have  $\alpha$  as unknown, so you can calculate a new value for  $\alpha$ .

Since this value is larger than 0 for sure, you will see the value for  $\alpha$  is going to be higher. That's completely intuitive, because if you give this either the speed in this direction, or you give it a push in this direction, it will obviously swing higher here. There is extra kinetic energy, so when it comes to a halt, the potential energy at point C will be higher than the potential energy at point B, and so it will swing higher.

I don't think this was too difficult-- what do you think? Let's now go to the next problem, and we're still one minute ahead of time. I don't know I managed to do that.