

And I now want to go to problem number 6.3, which is a real classic.

A girl, eskimo girl is sliding off an igloo. It's like a sphere with a radius r . She starts here and her speed is 0. She starts to slide in this direction. And at point P the angle is θ . She has here a certain tangential velocity. I call that v . And I want to put all the forces on her that I can think off.

First of all, there is here if her mass is m , mg . I can decompose that in two directions. I can decompose that into tangential direction, which would be $mg \sin \theta$ and into a radial direction, which is $mg \cos \theta$. Now as long as this girl is sitting on the igloo and feels a push from the igloo upwards, there has to be a normal force from the igloo on her seat. And since there is no friction, there cannot be any tangential force on her. So, she can only experience from the igloo a force upwards. And this is due to gravity.

Now, in the radial direction, and the reason why there can only be a normal force from the igloo is there's no friction here. Now in the radial direction, this girl must have the centripetal acceleration, which is a requirement. Otherwise, she couldn't go around this circle with radius r with that speed v . And therefore, $mg \cos \theta - N$ because that's in the other direction, must be $m v^2 / r$ at that location P divided by r . That's a key equation. That's equation number one. But that's not all.

She starts to slide here with velocity v . And if mechanical energy is conserved, which is the case here because there's no friction, the sum of potential energy and kinetic energy here must be the sum of potential and kinetic energy here. So what matters is really how much potential energy difference there is between these two points.

This height here h equals $R \sin \theta$. And therefore, if I write down the conservation of mechanical energy between this point and that point, then I get that mgh , which is the potential energy of this point above this point, equals $mg r \sin \theta$. And that now must all have been converted to kinetic energy at point P, which is $\frac{1}{2} m v^2$. What I have effectively done, I have effectively called u here 0. But it's irrelevant where you call u zero because it's only the difference that shows up. And this is equation number two.

Well, I can combine these two equations. Notice I have $m v^2$ here and I can substitute that in here. So I get $mg \sin \theta - N = m \frac{v^2}{r}$. And now I'm going to write down for $m v^2$

squared, I'm going to write down twice this term. So I get 2 . Oh, I already had the m . So I'm going to write down for $m v$ squared, I'm going to write on $m g r$, 1 minus cosine theta times 2 . So I get 2 times $m g$ times r times 1 minus cosine theta. And then I have here an r downstairs, which is this r . Is that correct? Yep, that's this r , $m v$ squared over r . r cancels. And if I work this out, then I find N divided by $m g$ equals 3 cosine theta minus 2 . And so what I have derived here is by combining these two key equations, I have derived here an equation, which tells me what the normal force is. The force that the girls feels pushing vertically, radially outwards when she's sitting on the igloo as a function of theta.

Let's stick in this equation theta equals 0 . 3 minus 2 is 1 , so you find N equals $m g$. Well, that's rather obvious. When she is sitting right on top of the igloo, then her mass attracted by the earth $m g$ and there is no velocity-- she is sitting still. So the normal force that she experiences from the igloo better be exactly equal to $m g$. So that is very pleasing.

Now notice, as theta increases, the cosine of theta goes down. And if the cosine of theta goes down, N must goes down. So as she slides down, she will experience that this force pushing radially outward, which she feels from her seat pushing her radially outwards, which the bathroom scale would indicate if she were sitting on the bathroom scale. That force slowly goes down and down and down with increasing value of theta. And then there comes a time that N becomes 0 . And when that's the case, she's literally, floating on top of her seat. You could call her weightless. She's in free fall. She has lost contact with her seat. The seat doesn't have to push on her any more.

And when does that happen? When N is 0 . So when cosine of theta equals $2/3$. So that is when theta is approximately 48 degrees.

And what will happen then of course, she will be in free fall. So if I have my original figure here again, she will let go somewhere. She will slide down. Let go here and then, there will be a parabolic trajectory crushing down here on the ground. And that's a very interesting idea. That sliding down that you will let go. This actually leads to a very classic problem, which some of you may have seen.

If I have a ruler and I support the ruler-- let me make a drawing for you first. I support the ruler. A piece of wood or whatever. Here's the ruler vertically up, and this is a frictionless floor. And I let the ruler fall starting with 0 speed. There comes a certain angle theta when this ruler let go here. As it is falling, the ruler is pushing against this block and the block is pushing back. But there comes a time that that force goes to 0 and that the ruler will let go and start sliding in this direction. And you may all have seen that.

And it just so happens that that angle is exactly the same. That is also about 48 degrees. The cosine of that angle is also $2/3$. I will show it to you first from above. This is my timer that I use to time my lectures. And then I will show it to you from in front.

So this is the ruler that I have. The ruler is as vertical as I can get it. Of course, this is not exactly frictionless. That's why it will come to a stop. If this were completely frictionless, and the angle were 48 degrees, and then it would continue to go. But of course, it will stop because of friction.

First look at the phenomenon. Did you see it go? It's no longer in contact. It slips away. There it goes. Now let's now do it in a way that you can, perhaps see the angle.

Boy, this is also smoother. It comes closer to being frictionless. I can just barely see you by the way.

So now I have this ruler and I'm going to let it fall. And so it will do this and then it will start to slide. And when this angle here from here to here is roughly 48 degrees, that's when it will let go. You ready? There we go. You see? it slides.

That's a classic problem. You will see it often at exams. Now maybe not at 801. Although, I think by the end of the course you should, in principle, be able to calculate it. But it's a little bit more tricky than the igloo. Very well. Let's do a time check. Looks good. Only one minute behind, but that's no problem.