PROFESSOR: All right, if you're ready, I'm ready. Problem number 1.6.1. Oh, this pen is a little bit too thick for that.

Problem 6.1. A rollercoaster. You are on a rollercoaster at location A. You are a dare devil and you go down here from point A. Here you are in your rollercoaster. You're sitting there innocently at least at this moment in time, not being aware of the terrible things that are going to happen to you. You start with speed 0 and there you go [INAUDIBLE]. And you go inside this loop upside down. Let us call arbitrarily the gravitational potential energy at the ground level 0. So u equals 0. And let this height of A above the ground level be h. The radius of this circle, let that be r. And this is point C. And let this be point B, the highest point of the circle. And this separation here, the distance of point B above the ground, would be 2r.

Now, let's assume there's no friction. So mechanical energy is completely conserved. You have no kinetic energy here. You have only potential energy and you pick up speed, and you reach point C with a velocity, which I call v of C. You're climbing up against the gravitational fields again, so you'll lose kinetic energy and you finally reach point B. Let us assume with speed vB tangentially. No fiction. Mechanical energy is conserved. Gravitational potential energy plus kinetic energy here is gravitational potential energy plus kinetic energy here is gravitational potential energy plus kinetic energy here.

All right, we have already agreed that we have arbitrarily chosen to be the gravitational energy 0 here. So that makes life a little easy. Let's call the mass of your rollercoaster capital m and u was a modest of little m. So there we go.

First, point A. No kinetic energy, only gravitational potential energy. So we have M plus little m times g plus h. That is the gravitational potential energy relative to this level. Then we arrive at C where that is no gravitational potential energy because I've set this 0, so we only have kinetic energy. Which is 1/2 M plus little m times v at location C squared. And now we arrive at point B, so we have again, picked up gravitational potential energy over this height. So we have this equals M plus little m times g tomes that height, which is 2r plus the kinetic energy that we have at the point B 1/2 M [INAUDIBLE] at location vB squared. This is a very important equation. This is the conservation of mechanical energy.

And notice that all these M plus m's cancel. And what you see immediately, that if you make h higher, that vC is higher. That's immediately intuitive. If you start at a higher altitude, than obviously, when you reach the ground level, you would have a higher speed. Well you see that in front of you. Again, no friction. And also, you notice that vB will be higher if h is higher. So all of that is intuitively, extremely pleasing.

Now this picture is a little bit more complicated than you may think from this equation. Because what makes you think that you ever make it to point B? When you reach point B, by the way, we'll be hanging inside this loop upside down. It's a very awkward feeling I would imagine. But apart from that, what makes you think that you will actually reach that point B? So lets first evaluate that in some more detail.

Let this be that circle. So this is point C. And this is point B. And you have here this velocity, v of B.

Now what is required for this object to stay in orbit to make this curvature with this speed vB, this tangential velocity vB? What is required that there is a centripetal acceleration, which is downwards. And that centripetal acceleration equals vB squared divided by r. And let us now assume that this is exactly equal to g. So that gravity provides precisely the necessary centripetal acceleration to the center of this point.

The rollercoaster and you will now be floating when you reach this point B. You will experience no weight. I will get back to that. Just the same as we discussed in an earlier help session, that these astronauts in orbit, they have a tangential speed in orbit. Let's call it v. There is an centripetal acceleration pointed towards the earth, which is v squared divided by r. And that is exactly equal to their local g. And therefore, if you can recall my arguments were, therefore they are weightless.

For the same reason, if this were the case here, then the rollercoaster would be weightless and you will be weightless. Well, the way you could test that, and that's the example that I mentioned earlier with the astronaut. Put a bathroom scale between your bum, so to speak, and the seat. You will see if this is the case, that the bathroom scale will read 0. Even though you're hanging upside down, you're not falling at all, but you have no weight.

All right, let's now-- by the way, first, I want you to demonstrate that if you want this situation to occur, I want you to show that that will happen and only then when the height from which you release it is 5/2-- 2 and 1/2 times r. In which case, you reach at that point B, a velocity, a speed, which I call the critical velocity. And so this is the critical height.

If for some reason the h from which you start your rollercoaster is less than 2 and 1/2 r, you'll never make it to point B. But what happens then the following: you come down the slope [INAUDIBLE], you go inside here, and you fall out, and you crash. So this will give you a situation where you do not reach point B.

So what now is necessary for you to reach that point B apart from this special case that v squared over r is exactly g?

If you want to play it a little bit more comfortable, then it's clear that v squared B over r should be larger than g. And therefore, h should be larger then 2 and 1/2 r. And therefore, the speed, the tangential speed that you reach at point B will be larger than that critical speed, which I just calculated.

If that happens, the track will push down on the rollercoaster. If this is the track and here are the wheels of the track, everything is now upside down. And you are sitting there. The track will now push down on the rollercoaster. The velocity, the speed, the tangential speed, is now v, which is larger than the critical value. So let us only draw the forces on U.

You have a mass little m, so that is mg. But now there has to be a push from your seat onto you in this direction, which I will call N. And the reason why this has to be there is that we now have mvB squared divided by r must now be N plus mg. And remember, when this wasn't there, then we had the critical velocity at B. But we know this value is larger than the critical velocity. So there must be an extra force helping gravity to be sure that you have the necessary centripetal acceleration in the downwards direction. So what do you experience? You're hanging upside down, and you feel a push from you seat in down direction. Well I'm sitting straight up now. You better believe me. And so, I feel a push from the seat up. So what do I think? That gravity is down.

Now imagine that you were here. You feel a push from the seat down. So you effectively think that gravity is up. So you perceive is in this direction. So whether you notice that there's no worry for you to fall. You don't even have any fear because you really feel gravity being up. This is a crucial equation. And this now, according to my earlier definition of weight, is really your weight. Now if you want to call it perceived weight, be my guest.

If you put a bathroom scale between you and your seat, that N, in Newtons, or if calibrate in kilograms of pounds, that's fine. That would indicate your weight. The bathroom scale would indicate N and nothing else. And in the case that I mentioned earlier, that the v squared divided by r is exactly g. That bathroom scale would indicate 0. So if you read the bathroom scale, it would really read this number. And I therefore, call that your weight. That's my definition of weight.

Now you're being given that-- oh, not mg. You're being given that you read on the bathroom scale a weight 1/2 mg. That's a given. And so I can rewrite this equation, so I got m vB squared divided by r now equals 3/2 mg. That's a given. m cancels. And so, if you know what r is, and you know what g is, and I assume you do. Out pops that velocity vB. And that vB better be larger than the critical value. Otherwise, you would have made a mistake.

And if you substitute this vB in our equation number one, which I may have saved somewhere. Where is my equation number one? Here it is. If you substitute vB in here, you can solve for h. And you better find that h is larger than 2 and 1/2 r. If not, you would have made a mistake. So that vB substituted in equation one must give you an h, which is larger than 2 and 1/2 r.