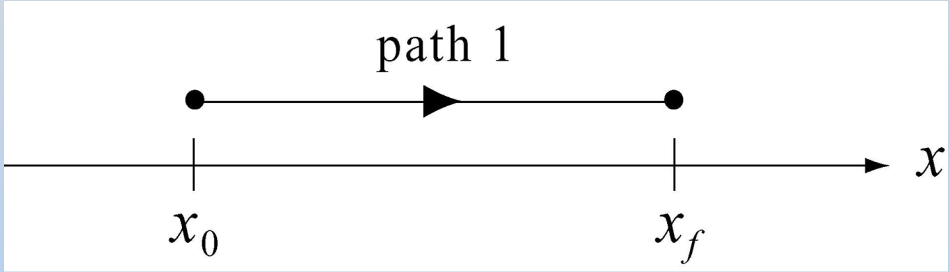
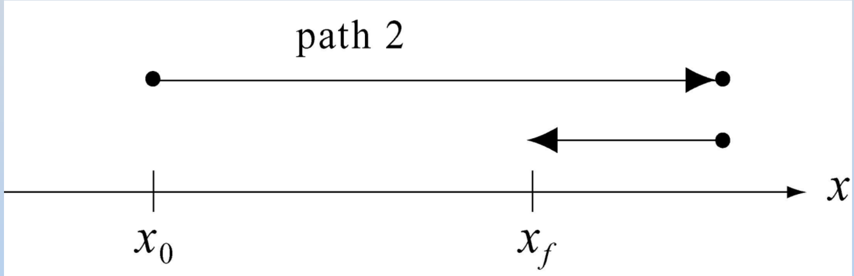
Conservation of Mechanical Energy

8.01

Non-Conservative Forces

Work done on the object by the force depends on the path taken by the object





Example: friction on an object moving on a level surface

$$F_{\text{friction}} = \mu_k N$$

$$W_{\text{friction}} = -F_{\text{friction}} \Delta x = -\mu_k N \Delta x < 0$$

Non-Conservative Forces

Definition: Non-conservative force Whenever the work done by a force in moving an object from an initial point to a final point depends on the path, then the force is called a *non-conservative force*.

Change in Energy for Conservative and Non-conservative Forces

Total force:

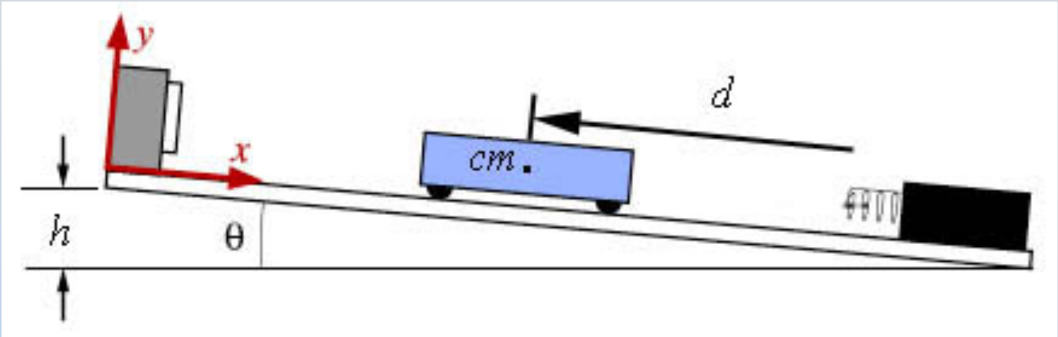
$$\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_c^{\ total} + \vec{\mathbf{F}}_{nc}^{\ \ total}$$

Total work done is change in kinetic energy:

$$W_{total} = \int_A^B \vec{\mathbf{F}}^{total} \cdot d\vec{\mathbf{r}} = \int_A^B \left(\vec{\mathbf{F}}_c^{total} + \vec{\mathbf{F}}_{nc}^{total}\right)\cdot d\vec{\mathbf{r}} = -\Delta U^{total} + W_{nc} = \Delta K$$

Energy Change: $\Delta K + \Delta U^{total} = W_{nc}$

Checkpoint Problem: Cart-Spring on an Inclined Plane



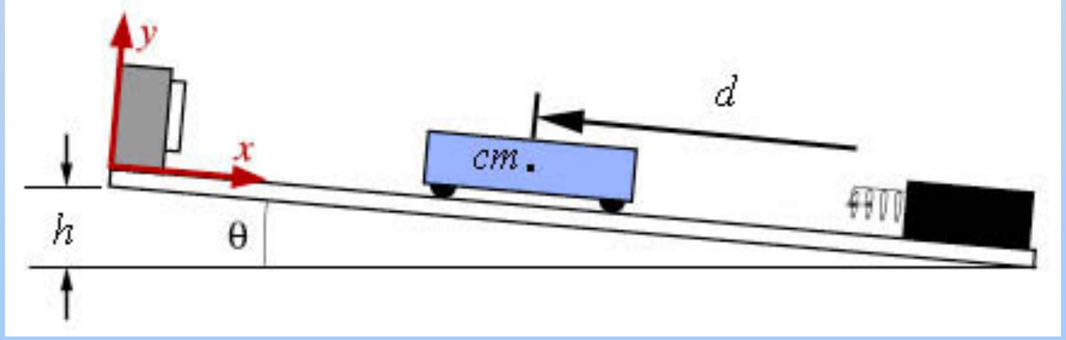
An object of mass *m* slides down a plane that is inclined at an angle θ from the horizontal. The object starts out at rest. The center of mass of the cart is a distance *d* from an unstretched spring with spring constant *k* that lies at the bottom of the plane.

a) Assume the inclined plane to be frictionless. How far will the spring compress when the mass first comes to rest?

b) Now assume that the inclined plane has a coefficient of kinetic friction μ. How far will the spring compress when the mass first comes to rest? How much energy has been transformed into heat due to friction?

Reading Quiz : Cart-Spring on an Inclined Plane

An object of mass *m* slides down a plane that is inclined at an angle *θ* from the horizontal. The object starts out at rest. The center of mass of the cart is an unknown distance *d* from an unstretched spring with spring constant *k* that lies at the bottom of the plane. Assume the inclined plane to be frictionless. The spring compress a distance *x* when the mass first comes to rest? Find an expression for the distance d.



1. $$d = \frac{1}{2mg\sin\theta} kx^2 - x$$

2. $$d = \frac{1}{2mg\sin\theta} kx^2$$

3. $$d = \frac{1}{2mg} kx^2 - x\sin\theta$$

4. $$d = \frac{1}{2mg} kx^2$$

5. $$d = \frac{1}{2mg\sin\theta} kx^2 + x$$

6. $$d = \frac{1}{2mg} kx^2 + x\sin\theta$$

Strategy: Using Multiple Ideas

Force and Energy

Need second law in radial direction

Summary: Change in Mechanical Energy

Total force:

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}_{\text{c}}^{\text{ total}} + \vec{\mathbf{F}}_{\text{nc}}^{\text{ total}}$$

Total work:

$$W^{\text{total}} = \int_{\text{initial}}^{\text{final}} \vec{\mathbf{F}}^{\text{total}} \cdot d\vec{\mathbf{r}} = \int_{\text{initial}}^{\text{final}} \left(\vec{\mathbf{F}}_{\text{c}}^{\text{total}} + \vec{\mathbf{F}}_{\text{nc}}^{\text{total}}\right) \cdot d\vec{\mathbf{r}}$$

Change in potential energy:

$$\Delta U^{\text{total}} = -\int_{\text{initial}}^{\text{final}} \vec{\mathbf{F}}_{\text{c}}^{\text{total}} \cdot d\vec{\mathbf{r}}$$

Total work done is change in kinetic energy:

$$W^{\text{total}} = -\Delta U^{\text{total}} + W_{\text{nc}} = \Delta K$$

Mechanical Energy Change:

$$\Delta E^{\text{mechanical}} \equiv \Delta K + \Delta U^{\text{total}}$$

Conclusion:

$$W_{\text{nc}} = \Delta K + \Delta U^{\text{total}}$$

Modeling the Motion using Force and Energy Concepts

Force and Newton's Second Law:

- Draw all relevant free body force diagrams
- Identify non-conservative forces.
- Calculate non-conservative work $W_{\text{nc}} = \int_{\text{initial}}^{\text{final}} \vec{\mathbf{F}}_{\text{nc}} \cdot d\vec{\mathbf{r}}.$

Change in Mechanical Energy:

- Choose initial and final states and draw energy diagrams.
- Choose zero point *P* for potential energy for each interaction in which potential energy difference is well-defined.
- Identify initial and final mechanical energy.
- Apply Energy Law. $W_{\text{nc}} = \Delta K + \Delta U^{\text{total}}$

Mechanical Energy Accounting

Initial state:

- Total initial kinetic energy

$$K_{\text{initial}} = K_{1,\text{initial}} + K_{2,\text{initial}} + \cdots$$

- Total initial potential energy

$$U_{\text{initial}} = U_{1,\text{initial}} + U_{2,\text{initial}} + \cdots$$

- Total initial mechanical energy

$$E_{\text{initial}}^{\text{mechanical}} = K_{\text{initial}} + U_{\text{initial}}$$

Final state:

- Total final kinetic energy

$$K_{\text{final}} = K_{1,\text{final}} + K_{2,\text{final}} + \cdots$$

- Total final potential energy

$$U_{\text{final}} = U_{1,\text{final}} + U_{2,\text{final}} + \cdots$$

- Total final mechanical energy

$$E_{\text{final}}^{\text{mechanical}} = K_{\text{final}} + U_{\text{final}}$$

- Apply Energy Law:

$$W_{\text{nc}} = E_{\text{final}}^{\text{mechanical}} - E_{\text{initial}}^{\text{mechanical}}$$

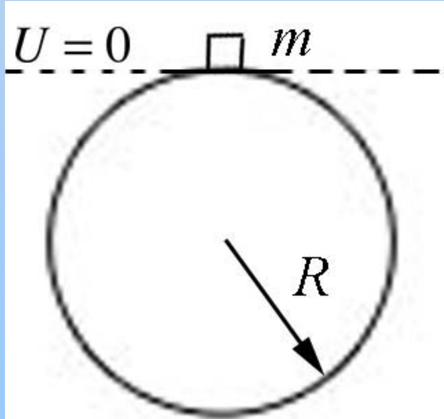
Worked Example: Block Sliding off Hemisphere

A small point like object of mass m rests on top of a sphere of radius R. The object is released from the top of the sphere with a negligible speed and it slowly starts to slide. Find an expression for the angle θ with respect to the vertical at which the object just loses contact with the sphere.

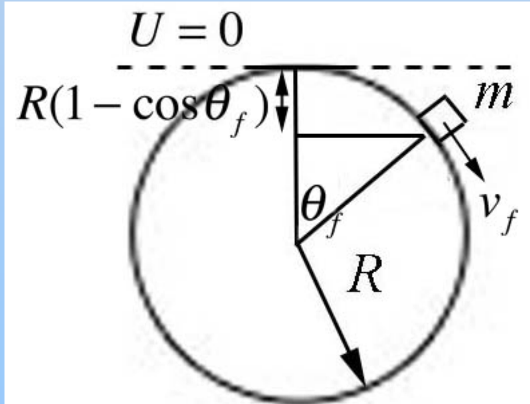
Example: Energy Changes

A small point like object of mass *m* rests on top of a sphere of radius *R*. The object is released from the top of the sphere with a negligible speed and it slowly starts to slide. Find an expression for the angle θ with respect to the vertical at which the object just loses contact with the sphere.

Energy Flow diagrams



initial state



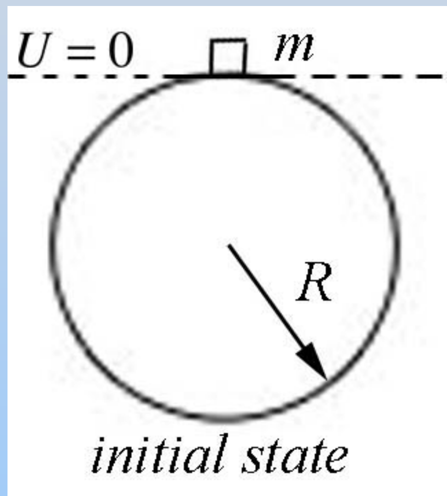
final state

Example: Energy Changes

A small point like object of mass m rests on top of a sphere of radius R. The object is released from the top of the sphere with a negligible speed and it slowly starts to slide. Find an expression for the angle θ with respect to the vertical at which the object just loses contact with the sphere.

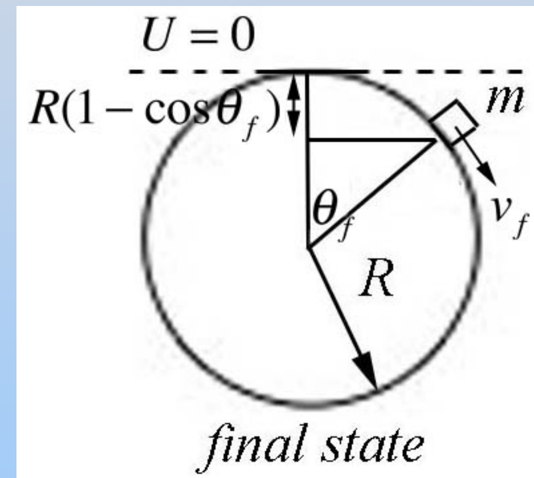
$$K_{\text{initial}} \quad 0$$

$$U_{\text{initial}} = 0$$



$$K_{\text{final}} = \frac{1}{2} m v_f^2$$

$$U_{\text{final}} = -mgR(1-\cos\theta_f)$$



$$E_{\text{initial}}^{\text{mechanical}} \quad 0$$

$$E_{\text{final}}^{\text{mechanical}} = \frac{1}{2} m v_f^2 - mgR(1-\cos\theta_f)$$

$$W_{\text{nc}} = 0 = E_{\text{final}}^{\text{mechanical}} - E_{\text{initial}}^{\text{mechanical}} \Rightarrow$$

$$0 = 0 - \left(\frac{1}{2} m v_f^2 - mgR(1-\cos\theta_f)\right) \Rightarrow \qquad \frac{1}{2} m v_f^2 = mgR(1-\cos\theta_f)$$

Recall Modeling the Motion: Newton ‘s Second Law

- Define system, choose coordinate system.
- Draw force diagram.
- Newton’s Second Law for each direction.
- Example: x-direction
- Example: Circular motion

$$\hat{\mathbf{i}}: \quad F_x^{\text{total}} = m\frac{d^2x}{dt^2}.$$

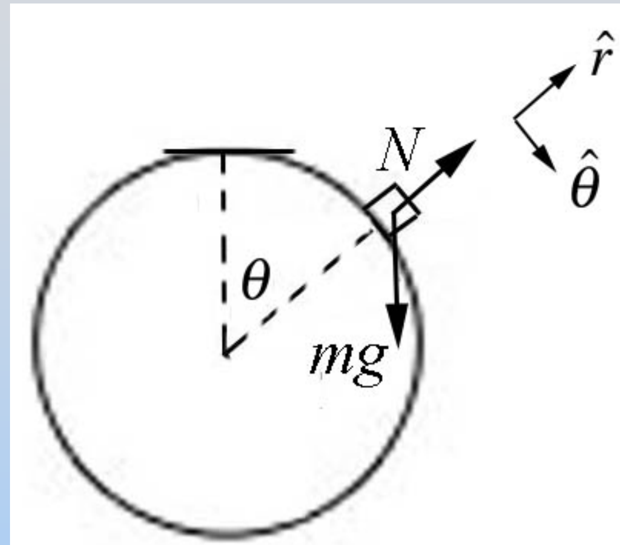
$$\hat{\mathbf{r}}: \quad F_r^{\text{total}} = -m\frac{v^2}{R}.$$

Example (con't): Free Body Force Diagram

Newton's Second Law

$$\hat{\mathbf{r}}:\ N - mg\cos\theta = -m\frac{v^2}{R}$$

$$\hat{\mathbf{e}}:\ \ mg\sin\theta = mR\frac{d^2\theta}{dt^2}$$



Constraint condition: $N = 0$ at $\theta = \theta_f$

Radial Equation becomes

$$mg\cos\theta_f = m\frac{v_f^2}{R} \Rightarrow \frac{1}{2}mv_f^2 = \frac{R}{2}mg\cos\theta_f$$

Energy Condition:

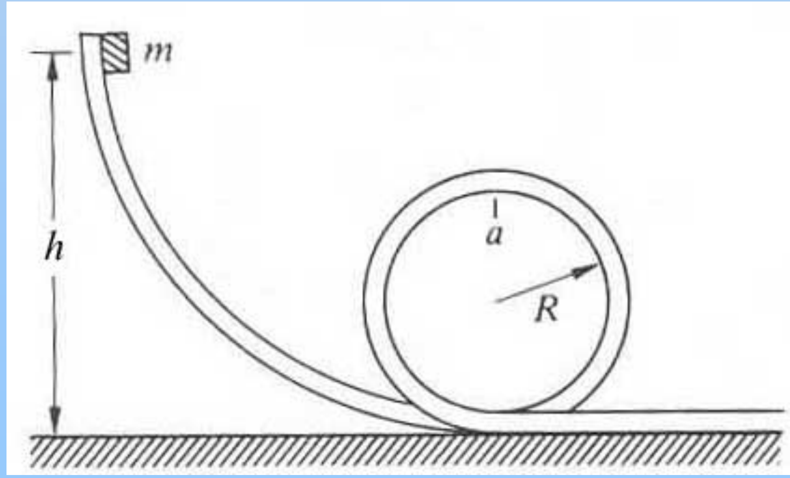
$$\frac{1}{2}mv_f^2 = mgR(1-\cos\theta_f)$$

Conclusion:

$$mgR(1-\cos\theta_f) = \frac{R}{2}mg\cos\theta_f \ \Rightarrow\ \cos\theta_f = \frac{2}{3} \ \Rightarrow\ \theta_f = \cos^{-1}\left(\frac{2}{3}\right)$$

Checkpoint Problem: Loop-the-Loop

An object of mass *m* is released from rest at a height *h* above the surface of a table. The object slides along the inside of the loop-the-loop track consisting of a ramp and a circular loop of radius *R* shown in the figure. Assume that the track is frictionless. When the object is at the top of the track (point a) it pushes against the track with a force equal to three times it's weight. What height was the object dropped from?



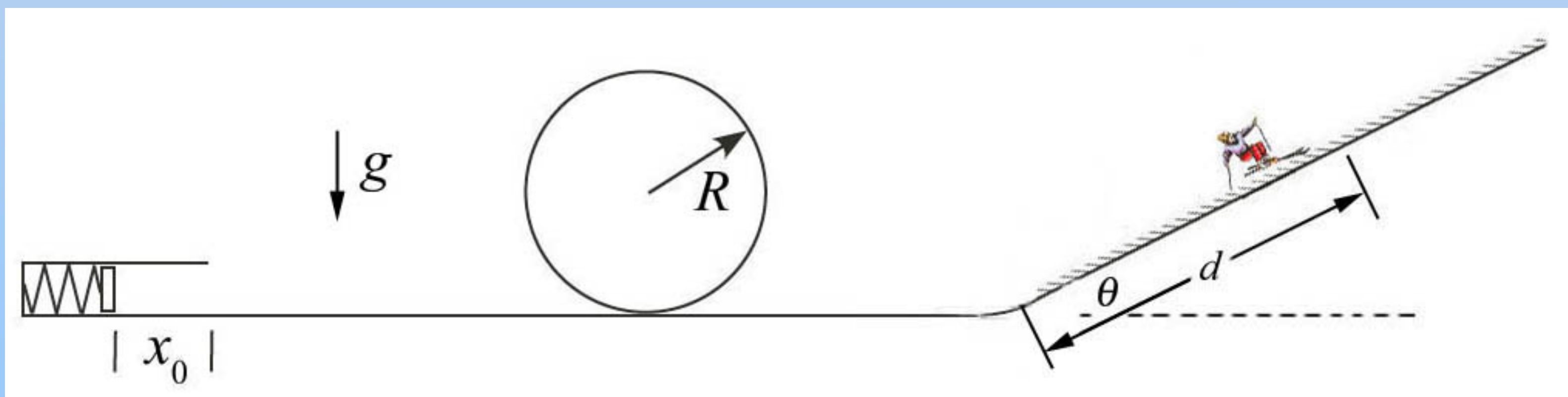
Demo slide: Loop-the-Loop B95

http://scripts.mit.edu/~tsg/www/index.php?page=demo.php?letnum=B 95&show=0

A ball rolls down an inclined track and around a vertical circle. This demonstration offers opportunity for the discussion of dynamic equilibrium and the minimum speed for safe passage of the top point of the circle.

Checkpoint Problem: Extreme Skier

An extreme skier is accelerated from rest by a spring-action cannon, skis once around the inside of a vertically oriented circular loop, then comes to a stop on a carpeted up-facing slope. Assume the cannon has a spring constant k and a cocked displacement x_0 , the loop has a radius R , and the slope makes an angle θ to the horizontal. The only surface with friction is the carpet, represented by a friction constant μ. Gravity acts downward, with acceleration g, as shown. What is the linear distance d the skier travels on the carpet before coming to rest?



Chcckpoint Problem: Block-Spring System with Friction

A block of mass m slides along a horizontal surface with speed v_0. At $t=0$ it hits a spring with spring constant k and begins to experience a friction force. The coefficient of friction is variable and is given by $\mu_k = bx$ where b is a constant. Find how far the spring has compressed when the block has first come momentarily to rest.