

MITOCW | MIT8_01SCF10mod09_02_300k

I have discussed several times the difference between angular velocity and angular frequency. And I would like to expand on that a little bit. And it's always a confusing thing because in physics we both give them the symbol ω .

Let's start again with a simple pendulum. A pendulum, which has length l and the object has mass m . And we are swinging it around. This is the maximum possible angle. I'll call this θ_{max} . On the other side of course, there's also a θ_{max} . I assume there's no damping. And let this angle here be θ .

The angular velocity for which we write ω in physics, I will in this exercise put an ω there, so that you always know which ω I'm talking about. Angular velocity per definition is $d\theta/dt$. And this is in rads per second. The magnitude of this ω has a maximum, so it is the magnitude that I'm talking about now. The magnitude has a maximum when $\theta = 0$. When the object goes through equilibrium, then the angle changes the most per time unit Δt . The value equals 0 when $\theta = \theta_{\text{max}}$. Because then the object here and here stand still. So $d\theta/dt$ is 0.

The angular velocity can be positive if $d\theta/dt$ is increasing. So if it goes in this direction. And it can be negative when it goes in this direction. So the angular velocity can be larger than 0, can be equal to 0, and can be smaller than 0. The bottom line is, it is changing all the time. It can change sign and it changes magnitude.

Right here at this point the linear velocity is either in this direction or in this direction. I call that the linear velocity. And right here the linear velocity is either in this direction or it is in this direction depending upon whether it swings up or whether it swings down. You can do the same here. These linear velocities are $l \cdot d\theta/dt$. And so they are l times the angular velocity. And you can immediately see if the angular velocity is 0, mainly here, that the linear velocity is 0. You can also see when ω changes sign that you can have a different sign for the linear velocity. This, for instance, could be called the plus and this would be a minus. This is a plus and this is a minus. And the magnitude can also greatly vary because when the angular velocity reaches a maximum where the object goes through equilibrium, then of course, ω -- this ω , this angular velocity has reached a maximum value. $d\theta/dt$ is maximum, so the linear velocity is a maximum. So much for the angular velocity.

Let us now think about the simple harmonic oscillation, which is to a good approximation the motion of this pendulum. In terms of θ , I can write down it's an approximate simple harmonic oscillation. θ

equals $\theta_{\max} \cos(\omega t + \alpha)$. You may prefer the sine of ωt plus some phase angle α . This ω has nothing to do with the ω of the pendulum. This is the angular frequency. This one is a constant. Angular frequency is a constant in time. It never changes. But the other one changes.

The period of one oscillation of this simple harmonic oscillator, for which you can either write a T or you can write a P . I always prefer a P when I deal with strings because I want to avoid confusion with tension. That period for one oscillation, for a complete oscillation equals 2π divided by this angular frequency.

If you look at the θ , at the position of θ at time t_1 , at a random time t_1 , and you look again at time $t_1 + 2\pi/\omega$, then the cosine function will repeat itself verbatim. That's why we call this the period. After so many seconds, the whole thing will repeat itself. So there's a huge difference between angular frequency and the angular velocity. Angular velocity changes in time. Angular frequency does not change in time.

Now I want to take a rotating disk. A disk that rotates uniformly, whereby ω is a constant. The angular velocity is a constant. Unlike in the case of the pendulum, whereby the angular velocity was not constant. It could also be a satellite going around the earth at a constant radius r . In other words, a circular motion.

Well, I could call this angle increased θ . I would have here a circumferential linear velocity v . It's tangential to the circle. Here the linear velocity and magnitude would be the same. But of course, not in direction. And this linear velocity-- I will write down linear-- equals $r \frac{d\theta}{dt}$, which is r times the angular velocity. And you can see that the magnitude is constant everywhere because the angular velocity $\frac{d\theta}{dt}$ is constant. Because this object rotates around with a constant angular velocity.

The time for one complete rotation-- just to remind you, the angular velocity equals $\frac{d\theta}{dt}$. The time for one complete rotation and you may call that period P or you call that T , whatever you prefer, equals 2π divided by the angular velocity. This is immediately obvious. One rotation is 2π radians. The object moves with the angular velocity, which is in radians per second. So to go 2π radians and you go with an angular velocity of ω radians per second, that would take so many seconds. So it's clear that this is the time that it takes for one complete rotation.

So this time is the time for a rotation. But remember, the time for one complete oscillation when we

were dealing with a simple harmonic oscillator, that time for which we also wrote the letter capital T, that time T or in case of a pendulum you may prefer P, that time was 2π divided by the angular frequency. And the angular frequency comes in through the cosine term of the simple harmonic oscillation.

Now we talk about the frequency of a rotating disk, and that frequency then means rotations per second. So the word frequency, which now doesn't say angular frequency, simply frequency, is in the case of the disk, the number of rotations per second. We express that in terms of hertz. 400 hertz is 400 rotations per second. And that frequency is 1 divided by T. WE often call that F. And that would be the angular velocity divided by 2π . And this is in hertz.

But we also talk about the frequency of a simple harmonic oscillator. In which case, the frequency of a simple harmonic oscillator-- notice I simply use the word frequency-- is now how many oscillation per second it made? How many cycles per second, also in hertz? And that frequency 1 over T or if you prefer to write for that 1 over P, we also call that F. And that is the angular frequency divided by 2π . Look here. It was for a simple harmonic oscillator. In other words, you could argue though, this is perhaps a matter of semantics. That in the case of a uniformly rotating disk, or of a uniformly rotating satellite around the earth in a circular motion, that in that case the angular velocity is the same as the angular frequency. But as I said, this perhaps is only a matter of semantics. However, in the case of a pendulum, the angular velocity is very, very, very different from the angular frequency.