And we now go to a problem, which has a slingshot. 3.9.

I twirl a stone in a vertical plane and I release it and it makes [INAUDIBLE] trajectory, and it hits the ground.

Now, let this be the vertical plane, so the gravitational acceleration g is in this direction. And let's assume the object at this moment in time is here. Let the radius of this circle, this critical plane be r, and let the velocity be v.

Now, there must be a net force on this object, which is nonnegotiable and that net force must be pointing exactly towards the center of my circle. I call it F net. And F net, the magnitude of F net must be the centripetal force, which is m v squared divided by r. It's the magnitude of this speed in orbit, which we have assumed somewhat artificially in this problem to be constant.

But remember, this object m has a mass, and so there is a force mg here. And in order for the net force to be this force, there must be another force acting on this object in this direction. And I call that the tension in the string. It's just an accident that this angle looks about 90 degrees. That's purely accidental.

But what I want you to appreciate is that F net must be mg, which is a vector, plus this tension T. And the tension can only come from the string. So what is this telling you if the object goes around like this?

When the object is going upwards-- I call this upwards and I call this downwards-- it must be clear that the angle between v and T, this angle must be less than 90 degrees if the object is going upwards. Now you should be able to convince yourself that if the object is going downwards that the angle between the velocity v and T must be larger than 90 degrees when the object is going downwards. And therefore, it is very hard to envision what I should do when I hold that string in my hands and I rotate it so that I will get this vertical circle. It's very hard to see what I should do with my hand, the exact motion of my hand, so that here the angle is 90 degrees-- less than 90 degrees. But when I'm on this side on the way down, that the angle is larger than 90 degrees. I find it hard to envision and frankly, I believe that it is nearly impossible to actually achieve this. It's a little bit artificial, and I don't think you can actually make the speed in this circular orbit to be constant everywhere. And the reason for that is of course, this gravity.

If the gravity were so small that you could ignore it, which we will do in the remainder of the problem, then of course, the situation becomes relatively easy. In any case, before we do, we all of a sudden cut the line. We cut the line and the object will then start a trajectory. And that brings me back to the first problem that we have that was problem 3.2. The object goes like so, and it hits the ground. And I assume that the radius of the circle can be ignored compared to these dimensions. So I have an angle theta, I have here a velocity v. We can decompose the velocity in the x direction and the y direction, and during this flight, during this trajectory-- everywhere and always there will be this force mg and nothing else. No longer a tension. It's now in free fall. And it hits the ground here at location x max.

Perhaps you remember that the maximum range would be when theta equals 45 degrees. You can clearly see that from problem 3.2 because x max equals v0 times cosine of theta times 2 times v0 times the sine of theta divided by g at earth. This is the gravitational acceleration at earth.

I have put the [INAUDIBLE] on here. I just will write for it g for the same reason will I not write down here v0, but I will simply write down here this velocity v because that is the velocity with which the object was released when I cut the string in my vertical motion. So I will simply write for that here v squared times the sine of 2 theta divided by g. So this is now the velocity with which it is released when I cut the string. And 2 sine theta cosine theta is sine 2 theta. And this g is the same as this g.

And what do you see? You see that x max is proportional to that v squared. That initial velocity that I have when the object is released. x max is proportional to v squared.

Well if I make the assumption that T equals much, much larger than mg, then the angle between the velocity vector at the moment that I let it go and T is effectively 90 degrees. So this problem that I discussed earlier will completely go away, and the tension itself through a very good approximation is then mv squared divided by r. So effectively, I forget gravity. So I get now that the net force, the magnitude of the net force is very close to the magnitude of the tension. And that now is approximately mv squared divided by R. v being velocity, the speed at the moment that I cut the rope and that the object will start its trajectory. So this equals m divided by R. And I write down now for this v squared, which I have here, I write down g x max divided by the sine of 2 theta. So g times x max divided by the sine of 2 theta. Now this here is v squared.

I'll take the case that the object reaches the farthest distance possible. That means the sine of 2 theta must be 1. That means theta is 45 degrees. So I will assume that this here equals 1. And what I find then, that's one of the results that we're asked to calculate. That then the tension a very good approximation. This [INAUDIBLE] equals mg times x maximum. I think in your problem they call this x divided by capital R. And I think they call these l. But that of course, is just a matter of semantics. So this is now the net force at the moment of release when the angle of release is 45 degrees with the horizontal.

Now if the maximum value of T is set by the strength of the string. In other words, I couldn't make T any larger because if I'm make T any larger, then the string would break. Then I can apply this equation and see what happens if I change certain parameters. But of course, I can then not change T. T is then a given. It's the maximum value that is possible.

So what you see now is that since g is not changing and since-- yeah, since g is not changing, you find now that x max is therefore, proportional to R over m. And you could change R and you could change m. But remember, T is now a given, a maximum value that you cannot exceed. And g of course, is not changing. And therefore, if you double r, if r goes up by a factor of 2, x max will go up by a factor of 2.

And if you double m, notice that also x max will go-- not also that x max will go down by a factor of 2.

So, increasing r. Larger x max, larger m. Smaller x max. And if you keep r over m constant, for instance, you make this one double and you make this one double, that will have no impact on x max.

Well, if you want to intuitively digest this, be my guest. Not so easy. Not so easy to have a good feeling for that. But by all means, try it. I think Professor Gooch calls that scrutinizing the solution. Some cases it's easy. This case it's not so easy. But give it a shot anyhow.