

MITOCW | MIT8_01SCF10mod10_01_300k

And our next problem is problem 3.7. 3.7 deals with circular orbits and satellites and orbital periods. It's always a little tricky, always a little tricky. Here we have the earth and the earth has a radius r_{earth} . I will use this symbol for earth. And v_{earth} at a height h above the surface over the earth-- that is here height h . At a radius little r . So h equals little r minus capital R . There's a satellite going around the earth. Here is the satellite and here is its velocity. And let's assume that it's a circular orbit, so the magnitude of v is constant.

Clearly there will be a force pointing exactly towards the center of the earth. Let's call that force F . Let us call the gravitational acceleration at the surface of the earth, let's call that g with this symbol for the earth so that there's never any confusion what I have in mind. Oh, I should have put the symbol here too.

Well, at location r , the gravitational acceleration is less because I'm farther away from the center of the earth. And the gravitational acceleration just like the gravitational force goes as inverse r squared. And so this value equals the value that you would have measured right at the surface of the earth times R at the earth squared divided by r squared. And notice, since r is larger than this capital R , this value is smaller than the gravitational acceleration at earth.

Now the force that I have here, this gravitational pull equals m times this g . But it also equals $m v$ squared over r , which is the centripetal force, which is required for this object with mass m to go around in a circular orbit.

Now I can replace this v by the circumference $2 \pi r$ divided by the time to go around T . This ins now the period in seconds to go around. And I can square that.

Now, the m 's cancel. And if I combine this part of the equation with that part of the equation, then I find that $g r$ is this quantity. Oh, by the way, I forgot a little r here. This r must be here. So $g r$ equals this quantity, but I can replace this $g r$ by this. And so I'm going to get g measured at the surface of the earth times the radius of the earth squared divided by r squared. That now equals to 4π squared. This r sub 1 r here, so I end up with only one r divided by T squared. And out pops the orbital period of this satellite, which equals 2π times the square root of r cubed divided by the radius of the earth squared times the gravitational acceleration as measured at the earth. And what you see here now, perhaps a little bit to your surprise, that this equation doesn't contain the mass of the earth and it doesn't contain the gravitational constant g . I'll get back to that in a minute.

When you look at this equation, there are a few things quite intuitive. Notice that the larger r is, the

larger T is. So if r is larger-- that's my shorthand notation-- T is larger.

Well we all know that the orbital period of the moon around the earth is substantially larger than the orbital period, for instance, of the shuttle. The orbital period of the moon around the earth, if I round that off is about 25 days. And the orbital period of the shuttle is something like 90 minutes. So it's completely pleasing that r is upstairs here and when r goes up that T goes up. Of course, it's by no means obvious that this one is proportional to r to the power $3/2$. That's is a little bit harder to see to say the least. But in any case, qualitatively, this goes exactly along the lines of our intuition.