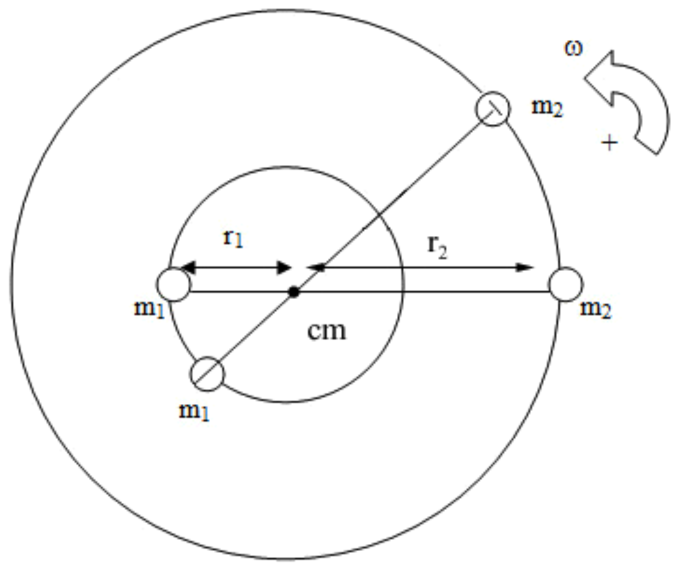
Problem Solving Circular Motion Dynamics
Challenge Problems

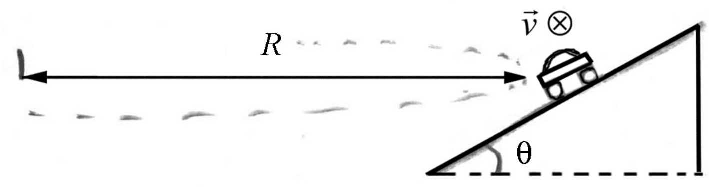
Problem 1: Double Star System

Consider a double star system under the influence of gravitational force between the stars. Star 1 has mass m_1 and star 2 has mass m_2. Assume that each star undergoes uniform circular motion about the center of mass of the system. If the stars are always a fixed distance s apart, what is the period of the orbit?



Problem 2: ***Circular motion: banked turn***

A car of mass m is going around a circular turn of radius R, that is banked at an angle θ with respect to the ground. The coefficient of static friction between the tires and the road is μ. Let g be the magnitude of the gravitational acceleration. You may neglect kinetic friction (that is, the car's tires do not slip).



a) At what speed v_0 should the car enter the banked turn if the road is very slippery (i.e. $\mu \to 0$) in order not to slide up or down the banked turn?

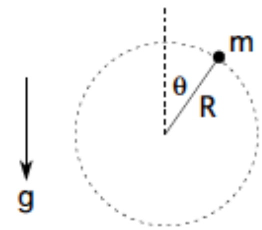
b) Suppose $\mu \tan\theta < 1$. What is the maximum speed v_{max} with which the car can enter the banked turn so that it does not slide up the banked turn?

c) Suppose $\mu \tan\theta < 1$. What is the minimum speed v_{min} with which the car can enter the banked turn so that it does not slide down the banked turn?

d) Suppose the car enters the turn with a speed v such that $v_{max} > v > v_0$. Find an expression for the magnitude of the friction force.

Problem 3:

Sally swings a ball of mass m in a circle of radius R in a vertical plane by means of a massless string. The speed of the ball is constant and it makes one revolution every t_0 seconds.



a) Find an expression for the radial component of the tension in the string $T(\theta)$ as a function of the angle θ the ball makes with the vertical[1]. Express your answer in terms of some combination of the parameters m, R, t_0 and the gravitational constant g.

b) Is there a range of values of t_0 for which this type of circular motion can not be maintained? If so, what is that range?

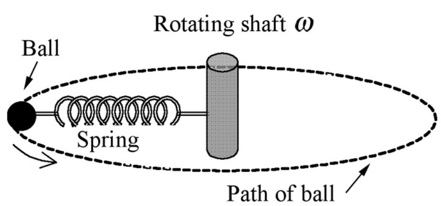
[1]Note added after the fact: The ball moves in a circle, but Sally's hand cannot remain at the center of the circle if a constant speed is to be maintained.

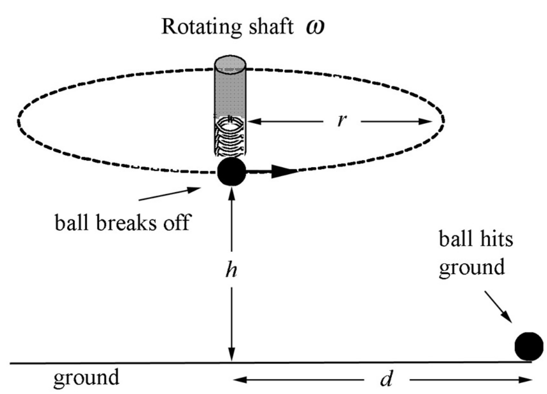
Problem 4:

A ball of mass m is connected by a spring to shaft that is rotating with constant angular velocity ω. A student looking down on the apparatus sees the ball moving counterclockwise in a circular path of radius r. When the spring is unstretched, the distance from the mass to the axis of the shaft is r_0. The orbital plane of the ball is a height h above the ground. Suddenly the ball breaks loose from the spring, flies through the air, and hits the ground an unknown horizontal distance d from the point the ball breaks free from the spring. Let g be the magnitude of the acceleration due to gravity. You may ignore air resistance and the size of the ball.

a) What is the magnitude of the spring constant k? Show all your work. Answers without work will not receive credit.

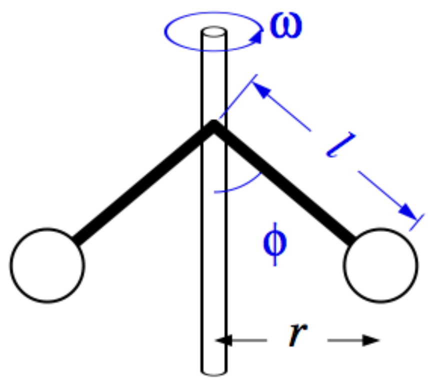
b) Find an expression for the horizontal distance d the ball traveled from the point the ball breaks free from the spring until it hits the ground.





Problem 5: Circular Motion

A governor to control the rotational speed of a steam engine was invented by James Watt. Two spheres were attached to a rotating shaft by rigid arms that were free to rotate up and down about a pivot where they attached to the shaft, as in the diagram above. As the arms pivoted up and down they actuated a mechanism to control the throttle of the steam engine. Assume the rigid arms have length l and no mass. All of the mass is then concentrated in the two spheres at the end of the arms, each having mass m.



a) Describe the acceleration of the spheres. Explaining and quantifying your knowledge of the acceleration will help you model the problem. Show all relevant free body diagrams.

b) Show that there is a minimum angular velocity $\omega_{\min}$ below which the governor will not function as intended.

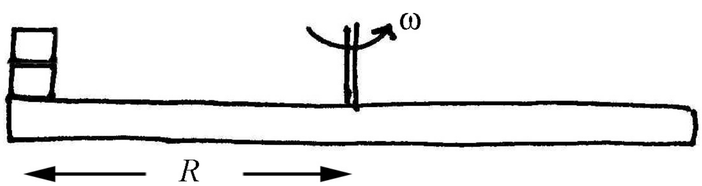
c) Derive an expression for the radius r of the circular path followed by the spheres. Express your answer only in terms of as few of the quantities m, ω, l, and g (the acceleration due to gravity) as you can. (Do not use the angle ϕ in your answer).

Problem 6:

Two identical coins each of mass m are stacked on top of each other exactly at the rim of a turntable, a distance R from the center. The turntable turns at constant angular speed ω and the coins rides without slipping. Suppose the coefficient of static friction between the turntable and the coin is given by μ_1 and the coefficient of static friction between the the coins is given by μ_2 with $\mu_2 < \mu_1$. Let g be the gravitational constant.

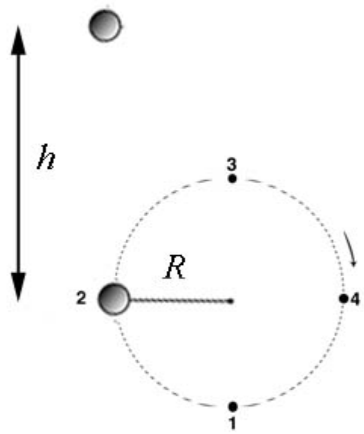
a) What is the magnitude of the radial force (friction force) exerted by the turntable on the bottom coin?

b) As the angular speed increases which coin slips first or do they both slip at the same instant? What is the maximum angular speed ω_{max} such that no slipping occurs?



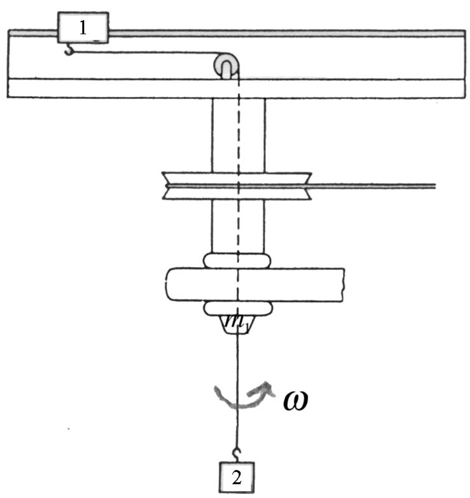
Problem 7: Whirling Stone

A stone (or a ball in the demo), attached to a wheel and held in place by a string, is whirled in circular orbit of radius R in a vertical plane. Suppose the string is cut when the stone is at position 2 in the figure, and the stone then rises to a height h above the point at position 2. What was the angular velocity of the stone when the string was cut? Give your answer in terms of R, h and g.



Problem 8: Uniform Circular Motion: ***Rotating device***

In the device shown below, a horizontal rod rotates with an angular velocity ω about a vertical axis. In the diagram, the horizontal string extending to the right at the middle of the apparatus represents the driving torque that maintains constant angular velocity ω. An object 1 with mass m_1 is constrained to slide along the horizontal rod. A massless inextensible string of length s is attached to one end of object 1, passes over a massless pulley, and attaches to a suspended object 2 of mass m_2. Object 2 hangs along the central vertical axis of the device. Assume the coefficient of static friction between the object 1 and the rod is μ_s, and use g as the gravitational constant. Object 1 moves in a circle of radius r.



a) With what angular velocity can the device spin such that the static friction force is zero?

b) What is the minimum angular velocity with which the device can spin so that object 1 does not move radially inward?

c) What is the maximum angular velocity with which the device can spin so that the object 1 does not move radially outward?

Problem 9: Universal Law of Gravitation and Orbital Uniform Circular Motion

A person, abandoned on a small spherical asteroid of mass m_1 and radius R, sees a satellite orbiting the asteroid in a circular orbit of period T.

a) What is the radius r_{sat} of the satellite's orbit?
b) What is the magnitude of the velocity of the satellite?
c) If the asteroid rotates with a period T_a, at what radius must the satellite orbit the asteroid so that the satellite appears stationary to a person on the asteroid?

Problem 10: Uniform Circular Motion: ***Two strings***

An object with mass m is connected to a vertical revolving axle by two massless inextensible strings of length l, each making an angle of 45° with the axle. Both the axle and the mass are revolving with angular velocity ω. Gravity is directed downwards.

a) Draw a clear force diagram for the object.

b) Find the tensions T_{upper} in the upper string and T_{lower} in the lower string,.