This is 9B.4. We're going to dispose of nuclear waste, and we have assumed that we have an orbit-- we have a rocket in orbit with mass m. The rocket is in an Earth-like orbit with radius R from the sun, and this sun has mass M. The speed of this rocket in an assumed circular orbit equals v, the same as the Earth.

The first question is what is that speed? There is gravitational attraction between the Earth and the rocket, and that gravitational attraction equals m M g divided by R squared. That must be exactly equal to centripetal force to hold this rocket with nuclear waste in orbit, so to speak, and that equals m speed of the Earth divided by R. I lose my m, I lose one R, and so you can immediately calculate that the speed of the Earth, and therefore, also of this rocket around the sun is approximately 30 kilometers per second.

What would be the minimum speed required for this object to escape the gravitational field of the Sun? We just assume that the Earth is nowhere nearby. If I increased the speed and give it a speed v escape, and I want it to go to infinity, I now use the conservation of mechanical energy. That tells me that the kinetic energy plus the potential energy here is the same as the kinetic energy and the potential energy at infinity. The kinetic energy plus potential energy is a constant.

Let's start at infinity-- I did such a careful speed, that as it reaches infinity, that its speed is 0 at infinity. That's the minimum energy I have to add-- that's the minimum increase of speed. Why should I waste any energy, and why should I have a speed which is not 0 here? The kinetic energy is 0 at infinity and the potential energy is 0, by definition, at infinity. The kinetic energy here equals 1/2 m v escape squared, and then the potential energy here, which follows from the assumption that the potential energy at infinity is 0, becomes minus m M g over R-- this is this R here.

You have one equation with one unknown, so you can calculate v escape, and v escape is the square root of the one that we calculated earlier-- the square root of 2. That is about 12 kilometers per second. That is 42 kilometers per second-- I'm getting ahead of myself-- and therefore, I have to increase the speed by about 12 kilometers per second. That increase will be needed for this object to escape to infinity.

Now comes the hard part. I want to slow the rocket down that it falls into the Sun and just grazes the

Sun as it falls in. I started this at point P, and I have fired the rockets so that they slow down to a substantially smaller speed than they had before, which is v at P. The Sun is here-- highly exaggerated in scale, and this is absolutely not to scale-- this is capital R, which is 150 million kilometers, and this is the radius of the Sun. Let the center of the Sun the s, and let the point of closest approach be Q.

This object will graze the Sun, and obviously will be eaten up by the Sun. The velocity here at Q will be enormously larger than the velocity at P-- I call it v Q. Only at the closest approach, and at the point where it's farthest away, will the angle between the position vector and the velocity be 90 degrees. Keep that in mind, because that's going to be important very shortly.

I first apply the conservation of mechanical energy. Mechanical energy here must be the same as mechanical energy there. Therefore, I have minus m M g over R, and I start with point P, plus 1/2 m v at P squared-- that's the total energy here, the total mechanical energy-- equals minus m M g divided by R Sun, which is this radius, and that's the distance s Q, plus 1/2 m v Q squared. This is one equation with two unknowns, so you need one more equation.

The other equation that we have is the conservation of momentum, but the conservation of momentum of the rocket around the Sun only holds with respect to point s-- only with respect to that center of the Sun, and it doesn't hold relative to any other point. Angular momentum relative to s is constant-- there is no torque relative to s, and the reason for that is that when the object is here, the force goes through s. When the you object is here, the force also goes through s. The tau relative to s is always 0, so the angular momentum cannot change. The angular momentum is the position vector relative to s crossed with the momentum itself, which is not this capital P, of course.

That gives me the equation, and I will write it down up here m times v p times R-- because of this angle being 90 degrees, I can forget the cross-- equals m v Q times R Sun, and because of the 90 degrees here, I can forget the cross here. I lose all these m's. Remember that R divided by R Sun is approximately 1,500 divided by 7, which is also v Q divided by v P.

I now have two equations-- one here, and one here, with two unknowns, v P and v Q. I can solve for both. If you're clever, however, you will see immediately that this part can completely be ignored relative to this part, because it's about 40,000 smaller because of this. This part can be ignored relative to this part, because it is about 200 times smaller, so all you have to use if you're clever is this equation set equal to 0. You can immediately calculate v Q, and once you know v Q, you can immediately calculate v

P.

When I calculated v P, I found v P equals 2.9 kilometers per second. I started off with 30, and I have to go down to 2.9, so I have to decrease the speed by 27 kilometers per second. This is way less efficient than what we did earlier when we had to increase the [UNINTELLIGIBLE]. Clearly, getting rid of it to infinity is a much better way than trying to hit the Sun.