Central Force Motion

Central Force Problem

Find the motion of two bodies interacting via a central force.

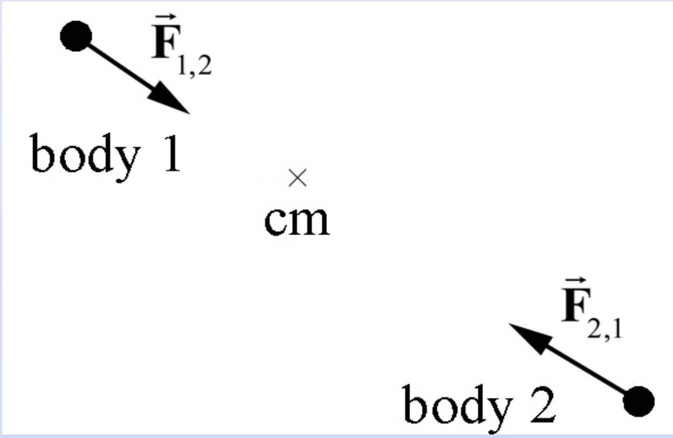
Examples:

Gravitational force (Kepler problem):

$$\vec{\mathbf{F}}_{1,2}(r) = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}$$

Linear restoring force:

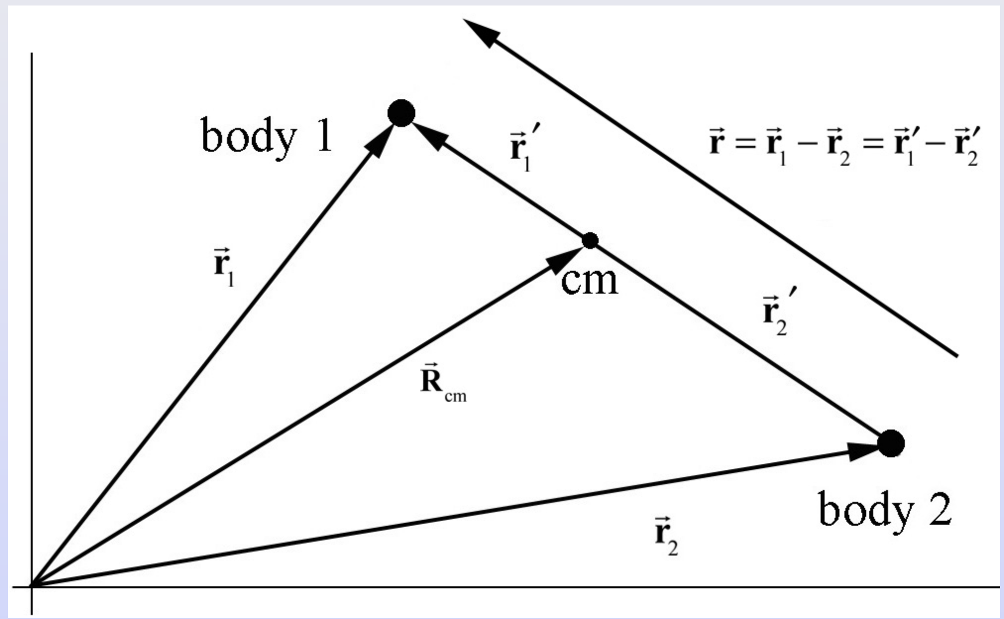
$$\vec{\mathbf{F}}_{1,2}(r) = -kr\,\hat{\mathbf{r}}$$



Two Body Problem: Center of Mass Coordinates

Center of mass

$$\vec{\mathbf{R}}_{cm} = \vec{\mathbf{r}}_1 - \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2}$$



Relative Position Vector

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2 = \vec{\mathbf{r}}_1' - \vec{\mathbf{r}}_2'$$

Reduced Mass

$$\mu = m_1 m_2 \, / \, (m_1 + m_2)$$

Position of each object

$$\vec{\mathbf{r}}_1' = \vec{\mathbf{r}}_1 - \vec{\mathbf{R}}_{cm} = \vec{\mathbf{r}}_1 - \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2} = \frac{m_2\left(\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\right)}{m_1 + m_2} = \frac{\mu}{m_1}\vec{\mathbf{r}}$$

$$\vec{\mathbf{r}}_2' = -\frac{\mu}{m_2}\vec{\mathbf{r}}$$

Reduction of Two Body Problem

Newton's Second Law
$$F_{1,2}\,\hat{\mathbf{r}} = m_1 \frac{d^2\vec{\mathbf{r}}_1}{dt^2} \qquad F_{2,1}\,\hat{\mathbf{r}} = m_2 \frac{d^2\vec{\mathbf{r}}_2}{dt^2}$$

Divide by mass
$$\frac{F_{1,2}}{m_1}\hat{\mathbf{r}} = \frac{d^2\vec{\mathbf{r}}_1}{dt^2} \qquad \frac{F_{2,1}}{m_2}\hat{\mathbf{r}} = \frac{d^2\vec{\mathbf{r}}_2}{dt^2}$$

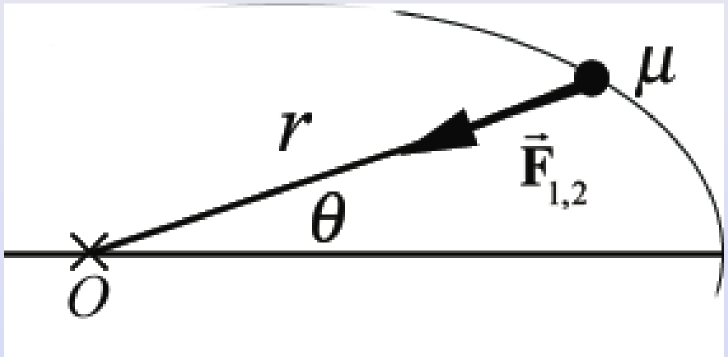
Subtract:
$$\left(\frac{F_{1,2}}{m_1} - \frac{F_{2,1}}{m_2}\right)\hat{\mathbf{r}} = \frac{d^2(\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2)}{dt^2} = \frac{d^2\vec{\mathbf{r}}}{dt^2}$$
$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$$

Use Newton's Third Law (in components) $F_{1,2} = -F_{2,1}$

Summary
$$\left(\frac{1}{m_1} + \frac{1}{m_2}\right) F_{1,2}\,\hat{\mathbf{r}} = \frac{d^2\vec{\mathbf{r}}}{dt^2} \quad \Rightarrow \quad F_{1,2}\,\hat{\mathbf{r}} = \mu \frac{d^2\vec{\mathbf{r}}}{dt^2}$$

Reduction of Two Body Problem

Reduce two body problem to one body of reduced mass μ moving about a central point *O* under the influence of gravity with ***position vector corresponding to the relative position vector*** from object 2 to object 1



Solving the problem means finding the distance from the origin $r(t)$ and angle $\theta(t)$ as functions of time

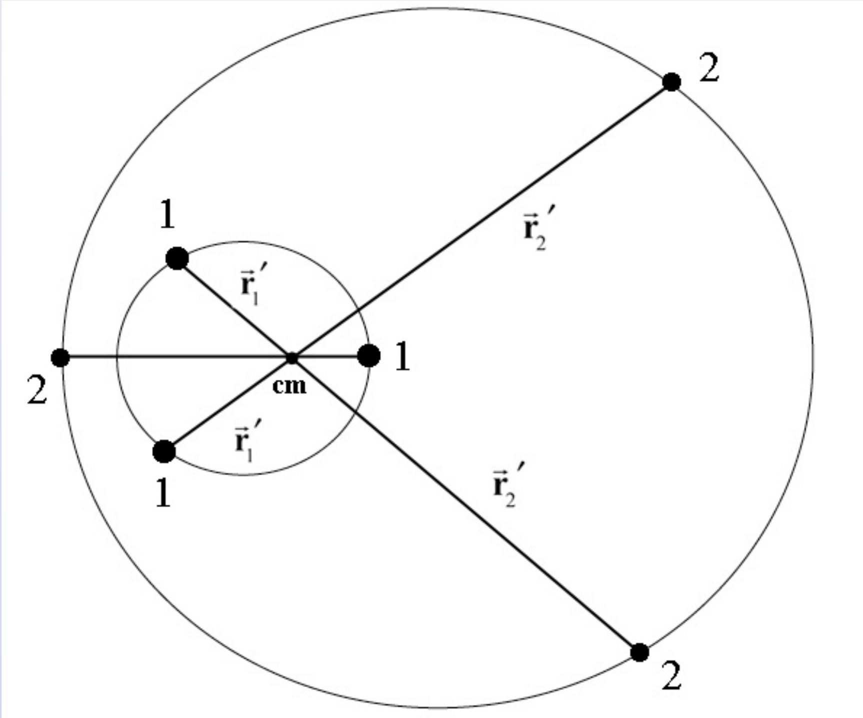
Equivalently, finding $r(\theta)$ as a function of angle θ

$$\vec{\mathbf{F}}_{1,2} = \mu \frac{d^2\vec{\mathbf{r}}}{dt^2}$$

Interpretation of Solution: Motion about Center of Mass

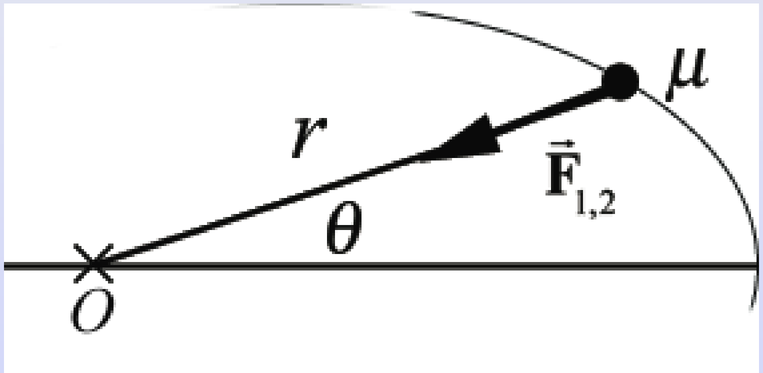
Knowledge of $\vec{\mathbf{r}} = \vec{\mathbf{r}}_1' - \vec{\mathbf{r}}_2'$ determines the motion of each object about center of mass with position.

$$\vec{\mathbf{r}}_1' == \frac{\mu}{m_1}\vec{\mathbf{r}} \qquad \vec{\mathbf{r}}_2' = -\frac{\mu}{m_2}\vec{\mathbf{r}}$$



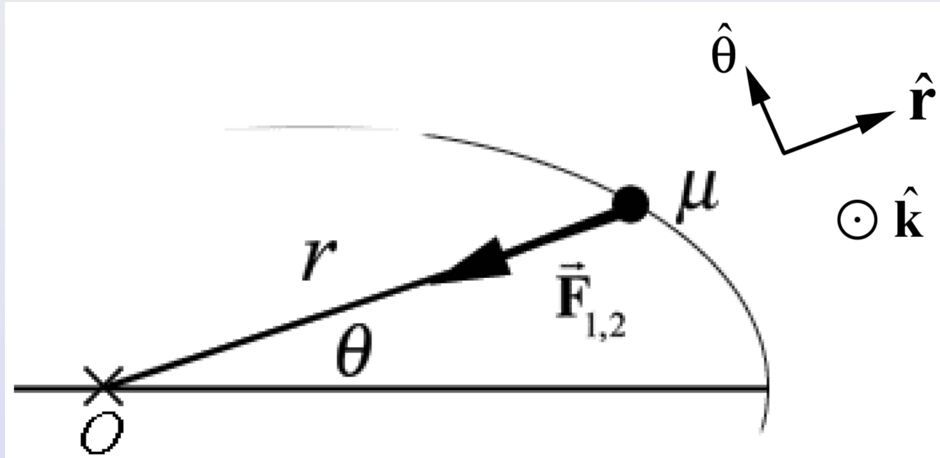
Checkpoint Problem: Angular Momentum

The angular momentum about the point *O* of the “reduced body”



1. is constant.
2. changes throughout the motion because the speed changes.
3. changes throughout the motion because the distance from *O* changes.
4. changes throughout the motion because the angle θ changes.
5. Not enough information to decide.

Angular Momentum about *O*



Torque about *O:*

$$\vec{\boldsymbol{\tau}}_O = \vec{\mathbf{r}}_O \times \vec{\mathbf{F}}_{1,2}(r) = r\hat{\mathbf{r}} \times F_{1,2}(r)\hat{\mathbf{r}} = \vec{\mathbf{0}}$$

Velocity $$\vec{\mathbf{v}} = \frac{dr}{dt}\hat{\mathbf{r}} + r\frac{d\theta}{dt}\hat{\boldsymbol{\theta}}$$

Angular Momentum $$\vec{\mathbf{L}}_O = \vec{\mathbf{r}} \times \mu\vec{\mathbf{v}} = r\hat{\mathbf{r}} \times \mu\left(\frac{dr}{dt}\hat{\mathbf{r}} + r\frac{d\theta}{dt}\hat{\boldsymbol{\theta}}\right)$$

$$\vec{\mathbf{L}}_O = \mu r^2 \frac{d\theta}{dt}\hat{\mathbf{k}}$$

$$L \equiv L_z = \mu r^2 \frac{d\theta}{dt}$$

Useful Relation: $$\frac{L^2}{2\mu} = \frac{1}{2\mu}\left(\mu r^2 \frac{d\theta}{dt}\right)^2 = \frac{1}{2}\mu\left(r\frac{d\theta}{dt}\right)^2$$

Recall: Potential Energy

Find an expression for the potential energy of the system consisting of the two objects interacting through the central forces given by

a) Gravitational force $\vec{\mathbf{F}}_{1,2} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

b) Linear restoring force $\vec{\mathbf{F}}_{1,2} = -kr\hat{\mathbf{r}}$

Gravitation:

$$\Delta U_{grav} = -\int_{r=r_0}^{r=r_f} -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}} \cdot d\vec{\mathbf{r}} = -\int_{r=r_0}^{r=r_f} -\frac{Gm_1m_2}{r^2}dr = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

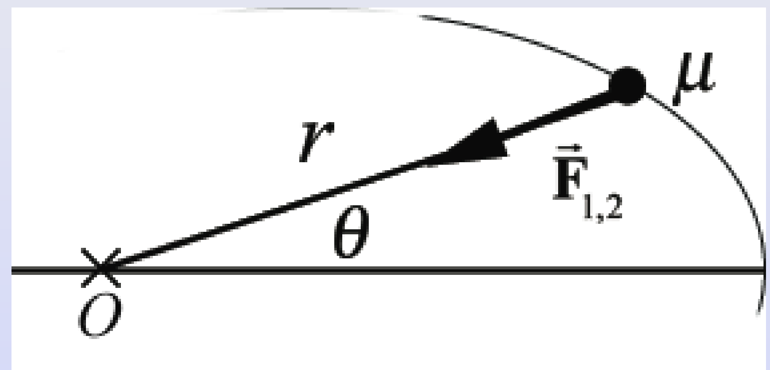
Linear restoring:

$$\Delta U_{spring} = -\int_{r=r_0}^{r=r_f} -kr\hat{\mathbf{r}} \cdot d\vec{\mathbf{r}} = \frac{1}{2}k(r_f^2 - r_0^2)$$

Checkpoint Problem: Energy

The mechanical energy

1. is constant.
2. changes throughout the motion because the speed changes.
3. changes throughout the motion because the distance from *O* changes.
4. is not constant because the orbit is not zero hence the central force does work.
5. Not enough information to decide.



Mechanical Energy and Effective Potential Energy

There are no non-conservative forces acting so the mechanical energy is constant.

Kinetic Energy

$$K = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}\mu\left(r\frac{d\theta}{dt}\right)^2 = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2\mu}$$

Mechanical Energy

$$E = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}\frac{L^2}{\mu r^2} + U(r) \equiv K_{effective} + U_{effective}$$

Effective Potential Energy

$$U_{effective} = \frac{L^2}{2\mu r^2} + U(r)$$

Effective Kinetic Energy

$$K_{effective} = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2$$

Force and Potential Energy

Effective Potential Energy $$U_{eff} = \frac{L^2}{2\mu r^2} + U(r)$$

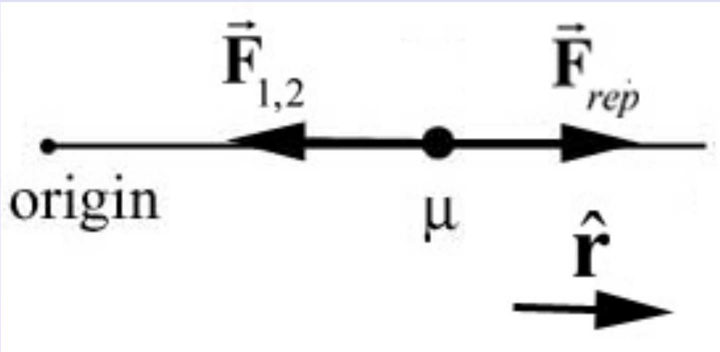
Repulsive Force $$F_{rep} = -\frac{d}{dr}\frac{L^2}{2\mu r^2} = \frac{L^2}{\mu r^3}$$

Central Force $$F_r = -\frac{dU(r)}{dr}$$

Effective Force $$\vec{\mathbf{F}}_{eff} = -\frac{dU_{eff}(r)}{dr}\hat{\mathbf{r}} = \left(\frac{L^2}{\mu r^3} - \frac{dU(r)}{dr}\right)\hat{\mathbf{r}}$$

Reduction to One Dimensional Motion

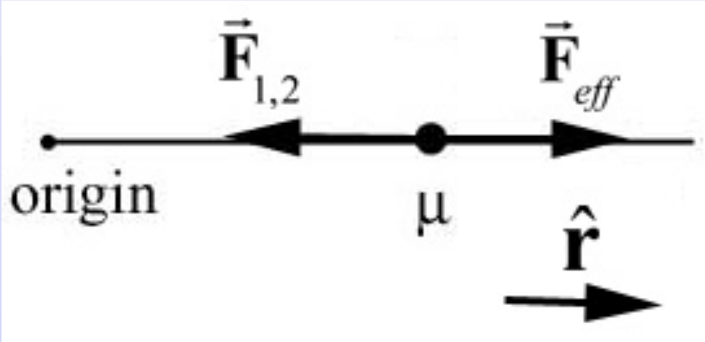
Reduce the one body problem in two dimensions to a one body problem moving only in the radial direction but under the action of two forces: a repulsive force and the central force



$$\vec{\mathbf{F}}_{eff} = -\frac{dU_{eff}(r)}{dr}\hat{\mathbf{r}} = \left(\frac{L^2}{\mu r^3} - \frac{dU(r)}{dr}\right)\hat{\mathbf{r}} = \mu\frac{d^2\vec{\mathbf{r}}}{dt^2}$$

Reduction to One Dimensional Motion

Reduce the one body problem in two dimensions to a one body problem moving only in the radial direction but under the action of two forces: a repulsive force and the central force



$$\vec{\mathbf{F}}_{eff} = -\frac{dU_{eff}(r)}{dr}\hat{\mathbf{r}} = \left(\frac{L^2}{\mu r^3} - \frac{Gm_1m_2}{r^2}\right)\hat{\mathbf{r}}$$

$$\mu\frac{d^2r}{dt^2} = \frac{L^2}{\mu r^3} - \frac{Gm_1m_2}{r^2}$$

Central Force Motion: constants of the motion

Total mechanical energy *E* is conserved because the force is radial and depends only on *r* and not on θ

Angular momentum *L* is constant because the torque about origin is zero

The force and the velocity vectors determine the plane of motion

Linear Restoring Force

Central Force

$$\vec{\mathbf{F}}_{1,2} = -kr\hat{\mathbf{r}}$$

Energy

$$E = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}\frac{L^2}{\mu r^2} + \frac{1}{2}kr^2 = K_{effective} + U_{effective}$$

Angular Momentum

$$L = \mu\, r^2 \frac{d\theta}{dt}$$

Kinetic Energy

$$K_{effective} = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2$$

Effective Potential Energy

$$U_{effective} = \frac{L^2}{2\mu r^2} + \frac{1}{2}kr^2$$

Repulsive Force

$$F_{repulsive} = -\frac{d}{dr}\left(\frac{L^2}{2\mu r^2}\right) = \frac{L^2}{\mu r^3}$$

Linear Restoring Force

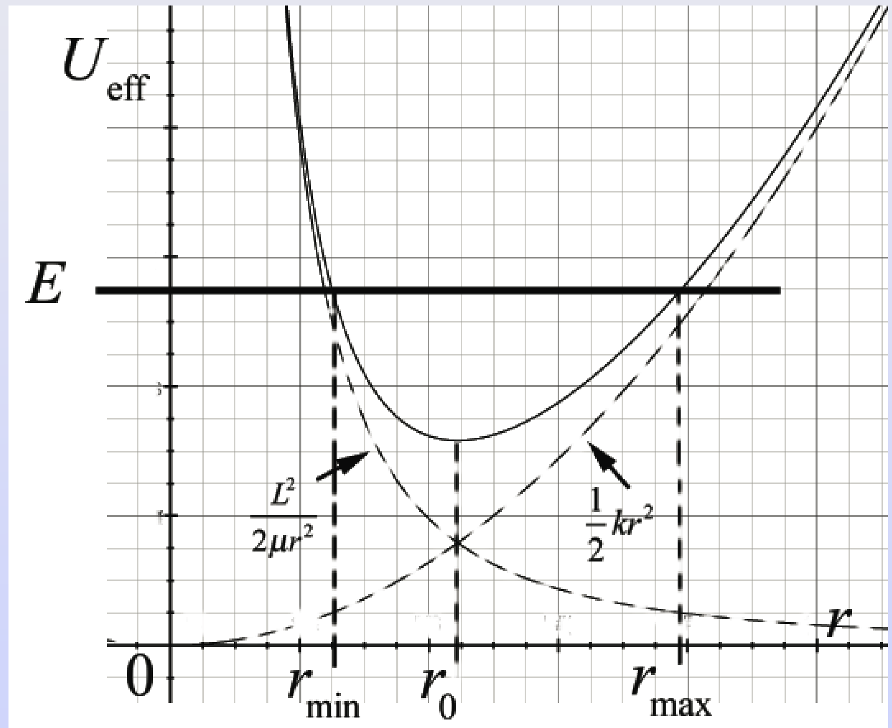
$$F_{spring} = -\frac{dU_{spring}}{dr} = -kr$$

Energy Diagram: Graph of Effective Potential Energy vs. Relative Separation

For $E > 0$, the relative separation oscillates varies between

$$r_{\text{min}} \le r \le r_{\text{max}}$$

The effective potential has a minimum at r_0



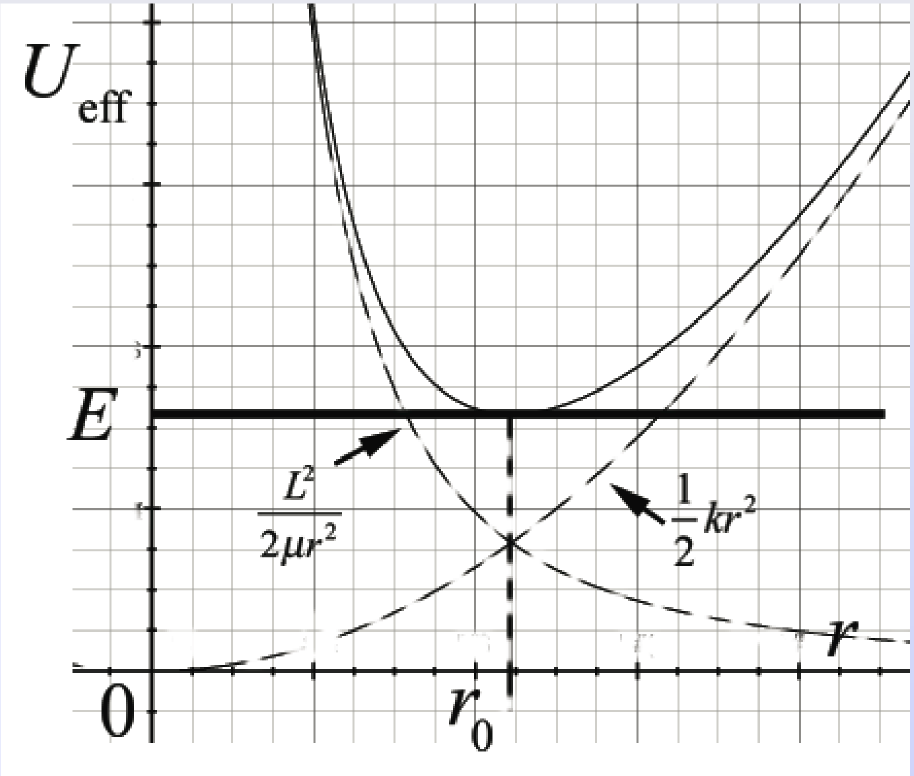
$$U_{effective} = \frac{L^2}{2\mu r^2} + \frac{1}{2}kr^2$$

Checkpoint Problem: Lowest Energy Solution

The effective potential energy is

$$U_{effective} = \frac{L^2}{2\mu r^2} + \frac{1}{2}kr^2$$

Find the radius and the energy for the lowest energy orbit. What type of motion is this orbit?



Orbit Equation: Isotropic Harmonic Oscillator

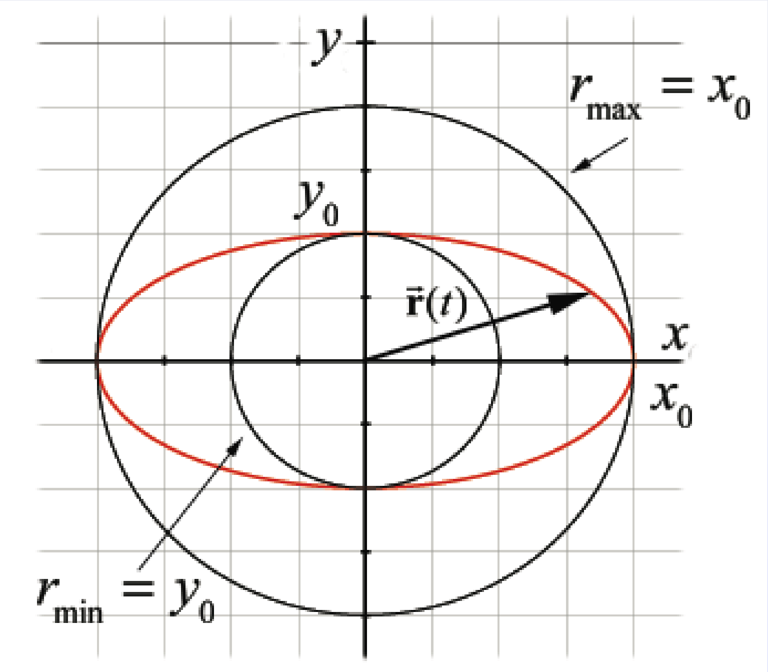
A special solution of the equation of motion for a linear restoring force

$$\mu \frac{d^2\vec{\mathbf{r}}}{dt^2} = -kr\,\hat{\mathbf{r}}$$

is given by $\vec{\mathbf{r}}(t) = x(t)\,\hat{\mathbf{i}} + y(t)\,\hat{\mathbf{j}}$

with $x(t) = x_0 \sin(\omega t)$

$$y(t) = y_0 \cos(\omega t)$$



where for the case shown in the figure with

$y_0 < x_0$ $r_{\min} = y_0$ $r_{\max} = x_0$

The solution for $\vec{\mathbf{r}}(t)$ is an ellipse centered at the origin

Summary: Kepler Problem

- Reduced mass $$\mu = \frac{m_1 m_2}{m_1 + m_2}$$
- Angular Momentum $$L = \mu r^2 \frac{d\theta}{dt}$$
- Kinetic Energy $$K = \frac{1}{2}\mu v^2 = \frac{1}{2}\mu\left(\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\theta}{dt}\right)^2\right) = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}\frac{L^2}{\mu r^2}$$
- Potential energy $$U(r) = -\frac{Gm_1 m_2}{r}$$
- Energy $$E = K + U(r) = \frac{1}{2}\mu\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}\frac{L^2}{\mu r^2} - \frac{Gm_1 m_2}{r} = K_{eff} + U_{eff}(r)$$
- Effective Kinetic Energy $$K_{eff} = \frac{1}{2}\mu\left(dr/dt\right)^2$$
- Effective Potential Energy $$U_{eff} = \frac{L^2}{2\mu r^2} - \frac{Gm_1 m_2}{r}$$
- Effective Repulsive Force $$F_{rep} = -\frac{dU_{rep}}{dr} = \frac{L}{\mu r^3}$$
- Gravitational Force $$F_{grav} = -\frac{dU_{grav}}{dr} = -\frac{Gm_1 m_2}{r^2}$$

Energy Diagram: Graph of Effective Potential Energy vs. Relative Distance

Case 1: Hyberbolic Orbit

$E > 0$

Case 2: Parabolic Orbit

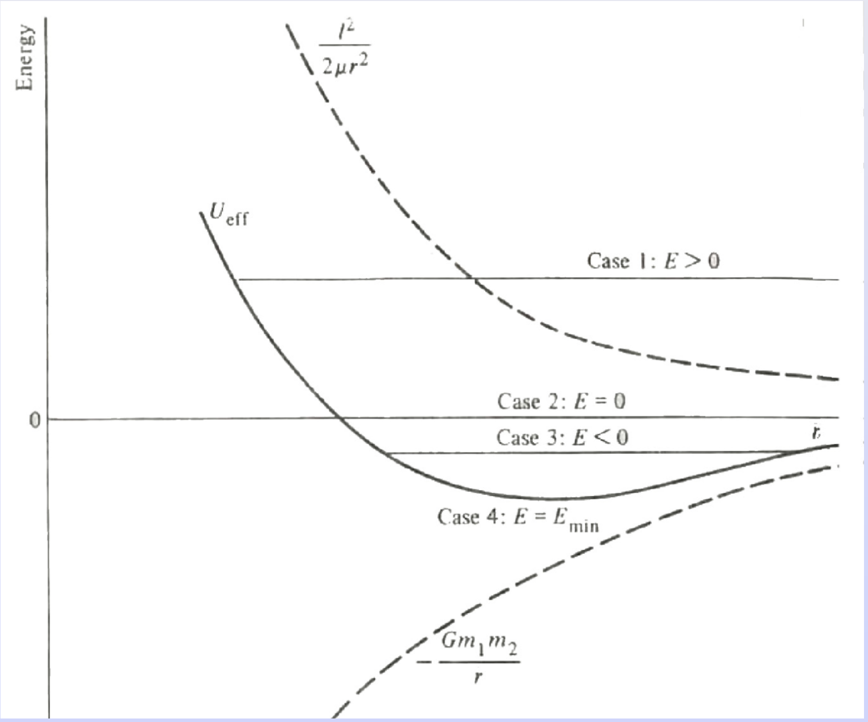
$E = 0$

Case 3: Elliptic Orbit

$E_{\min} < E < 0$

Case 4: Circular Orbit

$E = E_{\min}$



$$U_{effective} = \frac{L^2}{2\mu r^2} - \frac{Gm_1m_2}{r}$$

Checkpoint Problem: Lowest Energy Orbit

The effective potential energy is

$$U_{effective} = \frac{L^2}{2\mu r^2} - \frac{Gm_1 m_2}{r}$$

Make a graph of the effective potential energy as a function of the relative separation. Find the radius and the energy for the lowest energy orbit. What type of motion is this orbit?

Checkpoint Problem: Circular Orbital Mechanics

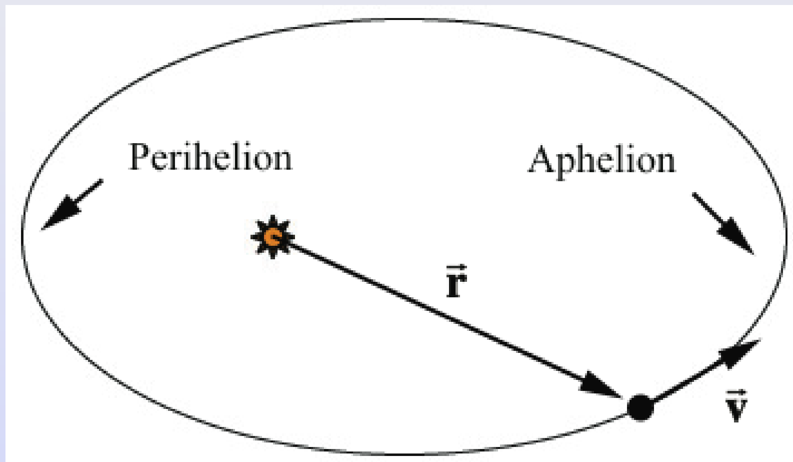
A double star system consisting of one star of mass m_1 and a second star of mass m_2 are orbiting each other such that the relative separation remains constant $r = R$.

a. Find the ratio of kinetic to potential energy

b. Suppose the orbits remain circular but the relative separation R increases. Do the following quantities increase, remain the same, or decrease: angular momentum, velocity, kinetic energy, potential energy, energy, and eccentricty?

Kepler's Laws

1. The orbits of planets are ellipses; and the center of sun is at one focus



2. The position vector sweeps out equal areas in equal time

3. The period *T* is proportional to the length of the major axis *A* to the 3/2 power $T \quad A^{3/2}$

Equal Area Law and Conservation of Angular Momentum

Change in area

$$\Delta A = (1/2) r v_\theta \Delta t$$

per time

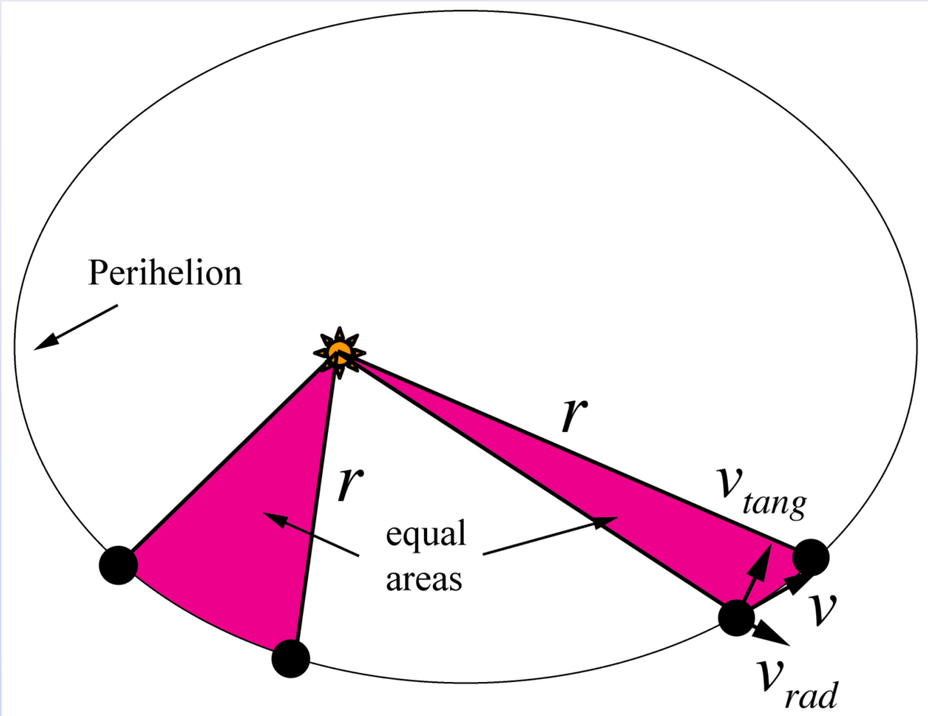
$$\frac{\Delta A}{\Delta t} = \frac{1}{2} v_\theta r$$

Angular momentum

$$L = r \mu v_\theta$$

Equal area law

$$\frac{\Delta A}{\Delta t} = \frac{L}{2\mu}$$



Orbit Equation Solution

Orbit Equation

$$\mu \frac{d^2 r}{dt^2} = \frac{L^2}{\mu r^3} - \frac{Gm_1 m_2}{r^2}$$

Change of Variables:

$$u \equiv \frac{1}{r} \Rightarrow \qquad \frac{dr}{d\theta} = \frac{dr}{du}\frac{du}{d\theta} = -\frac{1}{u^2}\frac{du}{d\theta}$$

Angular momentum condition:

$$\frac{d\theta}{dt} = \frac{L}{\mu r^2} = \frac{L}{\mu} u^2$$

Chain rule:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \Rightarrow \frac{dr}{dt} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{L}{\mu} u^2$$

Second derivative:

$$\frac{d^2 r}{dt^2} = -\frac{d^2 u}{d\theta^2}\frac{d\theta}{dt}\frac{L}{\mu} = -\frac{d^2 u}{d\theta^2}\frac{L^2}{\mu^2} u^2$$

One dimensional force equation

$$\mu \frac{d^2 r}{dt^2} = \frac{L^2}{\mu r^3} - \frac{Gm_1 m_2}{r^2} = \frac{L^2}{\mu} u^3 - Gm_1 m_2 u^2$$

Result:

$$-\frac{d^2 u}{d\theta^2}\frac{L^2}{\mu^2} u^2 = \frac{L^2}{\mu^2 r^3} - \frac{Gm_1 m_2}{\mu r^2} \Rightarrow \frac{d^2 u}{d\theta^2} = -u + \frac{\mu G m_1 m_2}{L^2}$$

Orbit Equation

Inhomogenous harmonic oscillator equation $$\frac{d^2u}{d\theta^2} + u = \frac{\mu G m_1 m_2}{L^2}$$

with angle independent solution $$u_0 = \frac{\mu G m_1 m_2}{L^2}$$

Solution:

$$u - u_0 = A\cos(\theta - \theta_0) \Rightarrow u = u_0\left(1 + \frac{A\cos(\theta - \theta_0)}{u_0}\right)$$

Change variables back with: $$r = \frac{1}{u} \qquad \frac{1}{u_0} \equiv r_0 = \frac{L^2}{\mu G m_1 m_2}$$

$$u - u_0 = A\cos(\theta - \theta_0) \Rightarrow \frac{1}{r} = \frac{1}{r_0}\left(1 + r_0 A\cos(\theta - \theta_0)\right)$$

Constants (ε, θ_0) fixed by conditions: choose $\theta_0 = \pi \qquad A = \varepsilon / r_0$

Conclusion:

$$r = \frac{r_0}{1 - r_0 A\cos(\theta)} = \frac{r_0}{1 - \varepsilon\cos(\theta)}$$

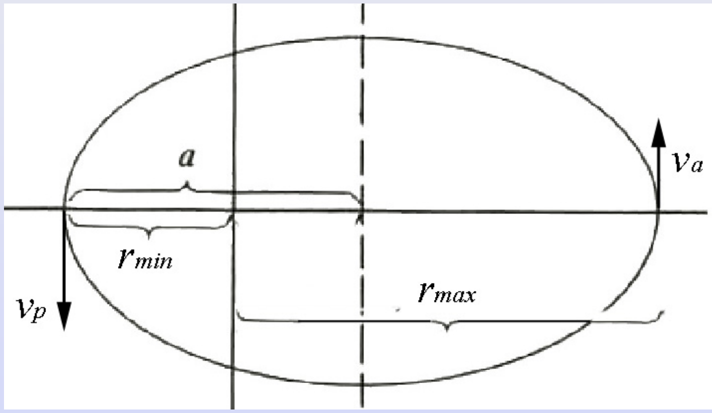
Eccentricity

Orbit Equation $$r = \frac{r_0}{1-\varepsilon\cos(\theta)}$$

Nearest Approach (θ=π): $$r_{min} = \frac{r_0}{(1+\varepsilon)}$$

Furthest Approach (θ=0): $$r_{\max} = \frac{r_0}{(1-\varepsilon)}$$

Semi-major axis: $$a = \frac{1}{2}\left(r_{min} + r_{max}\right) = \frac{r_0}{(1-\varepsilon^2)}$$



Energy at nearest approach:

$$E = U_{eff}(r_{min}) = \frac{L^2\left(1+\varepsilon\right)^2}{2\mu r_0^{\,2}} - \frac{Gm_1m_2\left(1+\varepsilon\right)}{r_0} \Rightarrow E = \left(\varepsilon^2-1\right)\frac{\mu\left(Gm_1m_2\right)^2}{2L^2} = -\frac{Gm_1m_2}{2a}$$

Eccentricity: $$\varepsilon = \left(1+\frac{2L^2E}{\mu\left(Gm_1m_2\right)^2\mu}\right)^{1/2} \Rightarrow \varepsilon = \left(1-\frac{E}{E_0}\right)^{1/2} \qquad E_0 = \frac{\mu\left(Gm_1m_2\right)^2}{2L^2}$$

Constants of the Motion: Energy and Angular Momentum

Angular momentum $$L = \left(r_0 \mu G m_1 m_2\right)^{1/2}$$

where r_0 is the radius of the circular orbit

Energy: $$E = E_{\min}(1-\varepsilon^2) = -\frac{\mu(Gm_1m_2)^2}{2L^2}(1-\varepsilon^2)$$

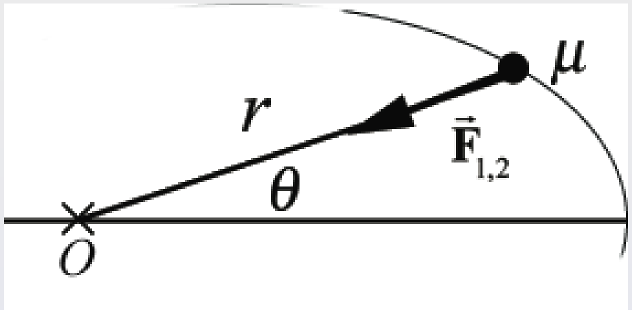
where E_{min} is the energy of the circular orbit

$$E_{\min} = \left(U_{effective}\right)\Big|_{r=r_0} = \frac{1}{2}U_{grav}\Big|_{r=r_0} = -\frac{Gm_1m_2}{2r_0} = -\frac{\mu\left(Gm_1m_2\right)^2}{2L^2}$$

and ε is the eccentricity

$$\varepsilon = \left(1 + \frac{2EL^2}{\mu\left(Gm_1m_2\right)^2}\right)^{1/2} = \left(1 - \frac{E}{E_{\min}}\right)^{1/2}$$

Orbit Equation



Solution:

$$r = \frac{r_0}{1 - \varepsilon \cos\theta}$$

Radius of circular orbit

$$r_0 = \frac{L^2}{\mu G m_1 m_2}$$

Energy of circular orbit

$$E_{\min} = -\frac{1}{2}\frac{\mu (G m_1 m_2)^2}{L^2}$$

Eccentricity

$$\varepsilon = \left(1 + \frac{2EL^2}{\mu\left(G m_1 m_2\right)^2}\right)^{1/2} = \left(1 - \frac{E}{E_{\min}}\right)^{1/2}$$

Orbit Classification:

Case 1: Hyberbolic Orbit ε > 1

$E > 0$

Case 2: Parabolic Orbit ε = 1

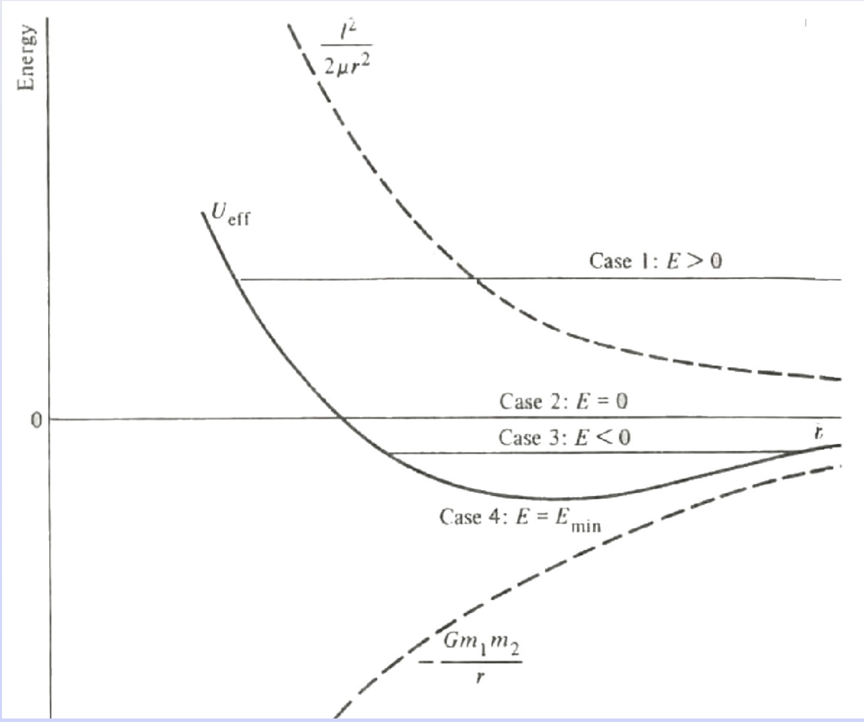
$E = 0$

Case 3: Elliptic Orbit 0 < ε < 1

$E_{\min} < E < 0$

Case 4: Circular Orbit ε = 0

$E = E_{\min}$



$$U_{effective} = \frac{L^2}{2\mu r^2} - \frac{Gm_1 m_2}{r}$$

Properties of Ellipse

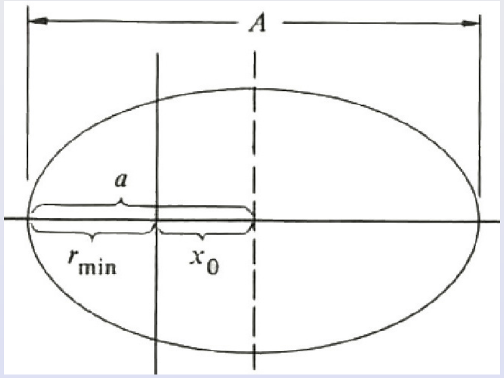
Eccentricity $$\varepsilon = \left(1 + 2EL^2 / \mu\left(Gm_1m_2\right)^2\right)^{1/2}$$

Semi-Major axis $$a = -\frac{Gm_1m_2}{2E}$$

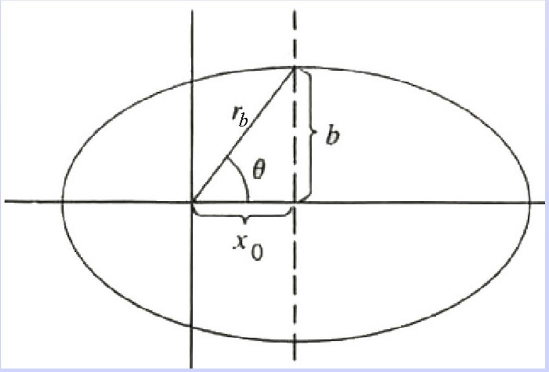
Semi-Minor axis $$b = a\sqrt{1-\varepsilon^2}$$

Area $$A = \pi ab = \pi a^2\sqrt{1-\varepsilon^2}$$

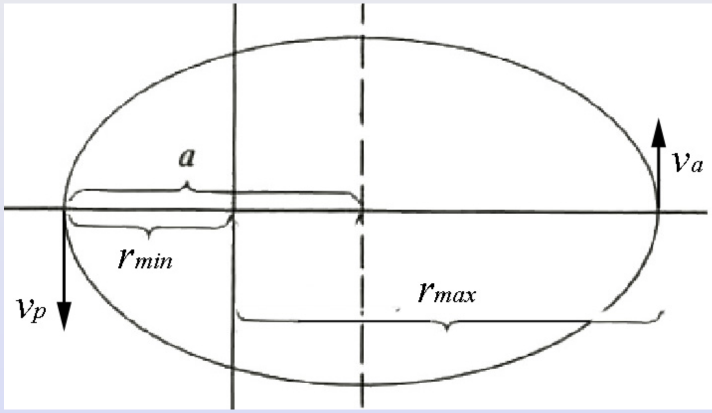
Location of the center of the ellipse $$x_0 = \varepsilon a$$



$$r = \frac{r_0}{1-\varepsilon\cos\theta}$$



Properties of an Elliptic Orbit



Energy $$E = -\frac{Gm_1 m_2}{2a}$$

Angular Momentum $$L = \sqrt{\mu G m_1 m_2 a(1-\varepsilon^2)}$$

Nearest Approach (θ=π): $$r_{min} = a(1-\varepsilon)$$

Speed $$v_p = \frac{L}{\mu r_{\min}} = \frac{L}{\mu a(1-\varepsilon)}$$

Furthest Approach (θ=0): $$r_{\max} = a(1+\varepsilon)$$

Speed $$v_a = \frac{L}{\mu r_{max}} = \frac{L}{\mu a(1+\varepsilon)}$$

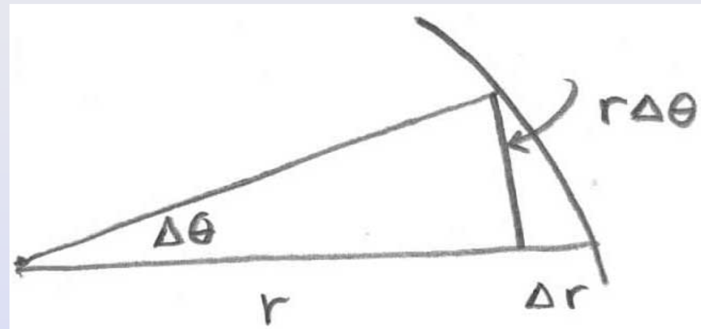
$$r = \frac{r_0}{1-\varepsilon\cos(\theta)}$$

$$r_0 = \frac{L^2}{\mu G m_1 m_2}$$

$$\varepsilon = \left(1 + 2EL^2 / \mu\left(Gm_1 m_2\right)^2\right)^{1/2}$$

Kepler's Laws: Equal Area

Area swept out in time Δt



$$\frac{\Delta A}{\Delta t} = \frac{1}{2}\left(r\frac{\Delta\theta}{\Delta t}\right)r + \frac{(r\Delta\theta)}{2}\frac{\Delta r}{\Delta t}$$

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} \qquad \frac{d\theta}{dt} = \frac{L}{\mu r^2}$$

Equal Area Law:

$$\frac{dA}{dt} = \frac{L}{2\mu} = \frac{1}{2}\sqrt{G(m_1+m_2)a(1-\varepsilon^2)} = \text{constant}$$

Kepler's Laws: Period

Area

$$A = \pi ab = \pi a^2 \sqrt{1 - \varepsilon^2}$$

Integral of Equal Area Law

$$\int_{orbit} \frac{2\mu}{L} dA = \int_0^T dt$$

Period

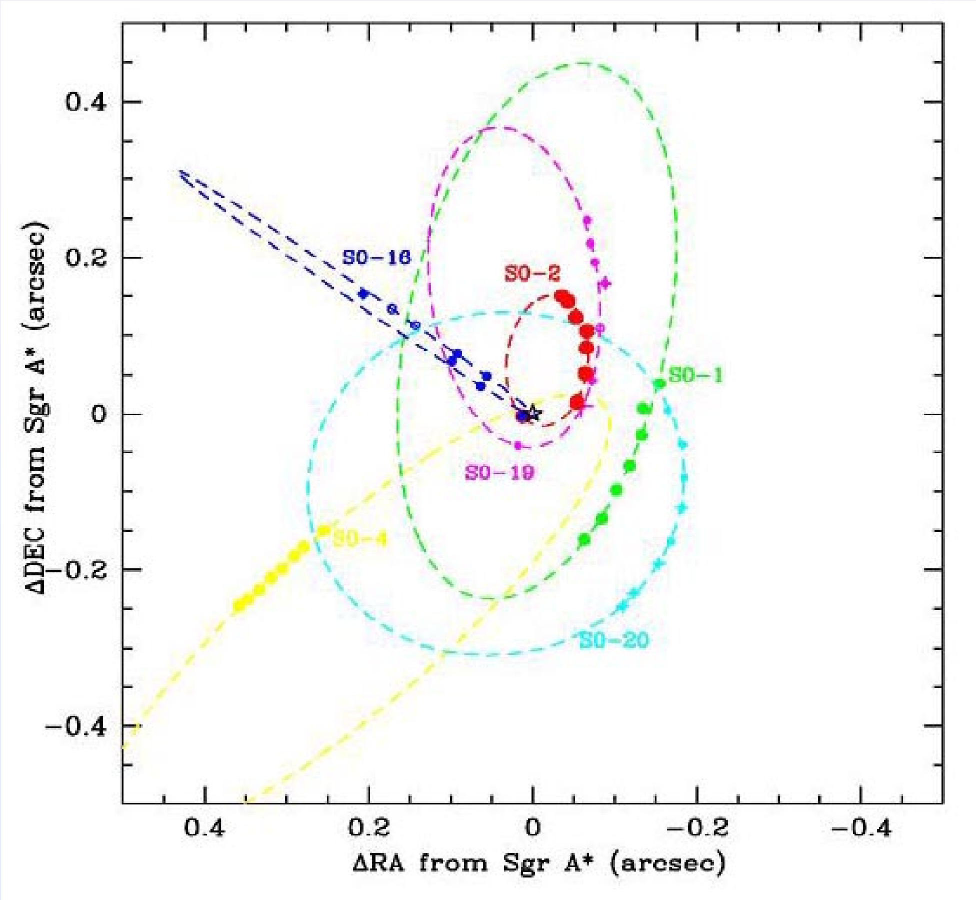
$$T = \frac{2\mu}{L} A = \frac{2\mu\pi a^2 \sqrt{1 - \varepsilon^2}}{L}$$

Period squared proportional to cube of the major axis but depends on both masses

$$T^2 = \frac{4\mu^2}{L^2} \pi a^4 (1 - \varepsilon^2) = \frac{4\pi^2 \mu^2 a^4 (1 - \varepsilon^2)}{\mu G m_1 m_2 a (1 - \varepsilon^2)} = \frac{4\pi^2 a^3}{G\left(m_1 + m_2\right)}$$

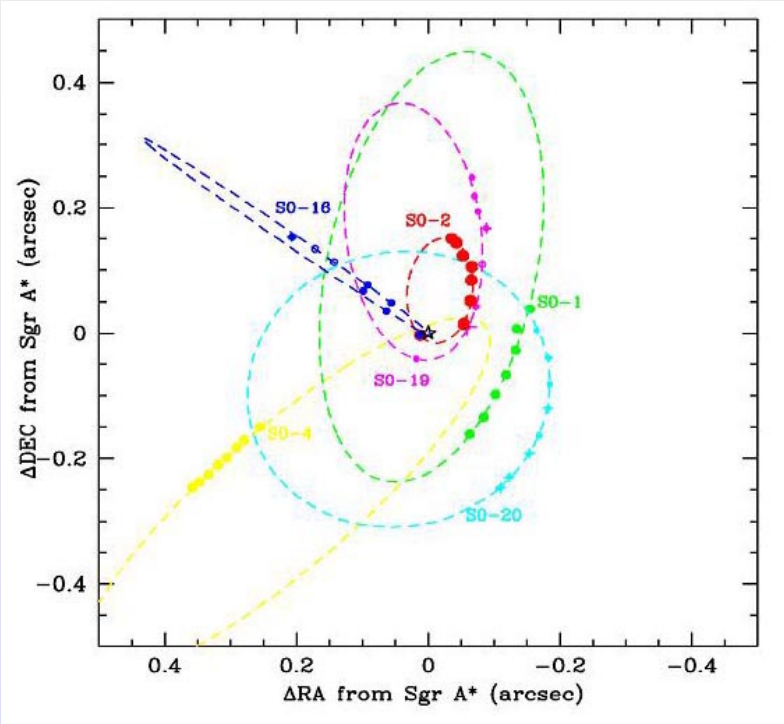
Stars Nearby Galactic Center

The UCLA Galactic Center Group, headed by Dr. Andrea Ghez, developed an animation of the orbitsof eight stars about the galactic center http://www.astro.ucla.edu/~jlu/gc/images/2004orbit_animfull_sm.gif



Astronomical Data

- Observation data is given in terms of the semi-major axis a and eccentricity ε
- Example: orbit of stars around center of galaxy



$$m = \frac{4\pi^2 a^3}{GT^2} \qquad 1AU = 1.50 \times 10^{11} m$$

Star	Period (yrs)	Eccentricity	Semi-major axis (10^{-3} arc sec)	Periapse (AU)	Apoapse (AU)
S0-2	15.2 (0.68/0.76)	0.8763 (0.0063)	120.7 (4.5)	119.5 (3.9)	1812 (73)
S0-16	29.9 (6.8/13)	0.943 (0.019)	191 (24)	87 (17)	2970 (560)
S0-19	71 (35/11000)	0.889 (0.065)	340 (220)	301 (41)	5100 (3600)

Numbers in parentheses are the errors on the given quantities.

Checkpoint Problem: Black Hole Mass

1. Find the mass of the black hole at the center of Milky Way Galaxy using Kepler's 3rd law.
2. What is the ratio of the mass of the black hole to one solar mass?
3. What is the ratio of the mass of the sun to the mass of the earth?
4. How do these ratios compare?

Mass of earth: 6×10^{24} kg
Mass of sun: 2×10^{30} kg
I AU = 1.5×10^{11} m
G= 6.7×10^{-11} N m^2 kg^{-2}

$$m = \frac{4\pi^2 a^3}{GT^2}$$