Now we come to problem 10.3. We have a human skater, and we'll assume that the human person is a cylinder, which is perhaps not very friendly-- here are her legs. It rotates about the center, which I called the center z, with angular velocity omega 1 in the beginning situation. The mass of the person is m, the person has arms and legs L, and there is at each end a mass m. The moment of inertia about the axis z in position 1 equals 1/2 mr squared, which is the moment of inertia of the cylinder, plus 2ml squared, and this is obviously the additional moment of inertia due to these two arms.

The person pulls her arms in-- let's assume that she's not cheating, and that there are no external torques here at the ice, which I'm not so sure about at all-- and so I'm going to get an I z 2, which is 1/2 mr squared. When these arms have been brought in, and these masses at the distance capital R, then I would have plus 2m R squared. That's the moment of inertia, which it is right now, and this moment of inertia is less then I z 1. If the person pulls her arms in, and is not cheating here, then I z 1 omega 1 must be I z 2 omega 2. In other words, angular momentum is conserved, if there are no external torques. Therefore, omega 2 must be larger than omega 1.

If you take a modest case, whereby the person's weight is 60 kilograms-- no insult implied-- if you take l equals 1 meter, you take m equals 5 kilograms, and you take R equals 0.2 meters, then you will find that this portion of the moment of inertia is only 1.2, and that this portion, which is substantially larger, is 10. In the new position, this is 1.2, but in the new position, this is only 0.4, so the moment of inertia has enormously decreased. The moment of inertia has decreased by a factor of 7. So, omega 2, which is the omega after the person pulls her arms in, is about 7 times omega 1, so she rotates about 7 times faster. This is by no means trivial.

If you look at the situation of kinetic energy, you will find to your surprise that the kinetic energy has increased. To me, it would be obvious, because the kinetic energy of rotation equals 1/2 I about the z axis omega squared-- I omega is constant. If you take the kinetic energy in situation two, when the arms are in, divided by the situation when the arms are not in, you will find immediately omega 2 divided by omega 1, which is approximately 7 in our special case, because I omega is constant.

Where does this come from? As this person is holding her arms stretched-- let me do it this way-- she will have to pull on her arm to hold for that mass in her hands and make it go around in a circle. We call that force to centripetal force-- see her fingers, there is that mass, and there is a centripetal force necessary to hold it in. The distance over which she moves it, s, is also in-- this is provided by her muscles. It's clear that she has to work as she moves this force closer to her body, and it's exactly that work that shows up in terms of kinetic energy.