

$\mathbf{A} \cdot \mathbf{B}$  is a scalar. If I take the time derivative of  $\mathbf{A} \cdot \mathbf{B}$ , then I apply the chain rule-- that is  $\frac{d\mathbf{A}}{dt} \cdot \mathbf{B}$  plus  $\mathbf{A} \cdot \frac{d\mathbf{B}}{dt}$ . It is a scalar, and  $\mathbf{A} \times \mathbf{B}$  is a vector. The time derivative of the cross product  $\mathbf{A}$  and  $\mathbf{B}$ -- again, I apply the chain rule-- would be  $\frac{d\mathbf{A}}{dt} \times \mathbf{B}$  plus  $\mathbf{A} \times \frac{d\mathbf{B}}{dt}$ .

I can give you an example of a cross product, and take the time derivative. This is angular momentum relative to point  $Q$  of a moving object-- this is point  $Q$ . Here is an object with mass  $m$ , that is moving with velocity  $\mathbf{v}$ , and this is the position vector  $\mathbf{r}$  of  $Q$ .  $\mathbf{L}$  of  $Q$ , the angular momentum relative to point  $Q$ , is defined as the position vector relative to point  $Q$  cross  $\mathbf{p}$ .

Notice that this vector is very different if you choose different points of  $Q$ , but I'm not going to address that issue now-- you will see that somewhere else in this course.  $\mathbf{p}$  equals  $m\mathbf{v}$ -- that's an intrinsic property of the motion of an object. Angular momentum is not-- angular momentum depends on where I choose my point  $Q$ . I take  $\frac{d\mathbf{L}_Q}{dt}$ , which now according to my chain rule, equals  $\frac{d\mathbf{r}_Q}{dt} \times \mathbf{p}$  plus  $\mathbf{r}_Q \times \frac{d\mathbf{p}}{dt}$ .

Now we see something very interesting. For one thing,  $\frac{d\mathbf{p}}{dt}$  is force, and  $\frac{d\mathbf{r}}{dt}$  is velocity of that object  $m$ , but  $\mathbf{p}$  equals  $m\mathbf{v}$ . So, the cross product between the vector  $\mathbf{v}$  and  $m\mathbf{v}$  must always be 0 because they are in the same direction--  $\theta$  is 0, and so the sine of  $\theta$  is 0.

What we end up with is an extremely famous equation, which is that the time derivative of angular momentum equals  $\mathbf{r} \times \mathbf{F}$ , relative to point  $Q$ --  $\mathbf{r}$  relative to  $Q$ -- cross  $\mathbf{F}$ . That is the definition of the torque  $\boldsymbol{\tau}$  relative to point  $Q$ . This is a very, very important equation, and we're going to see this many times in Newtonian mechanics. It is not an easy concept to apply it properly.

Change of angular momentum and torques, believe me, is one of the most difficult subjects in Newtonian mechanics.