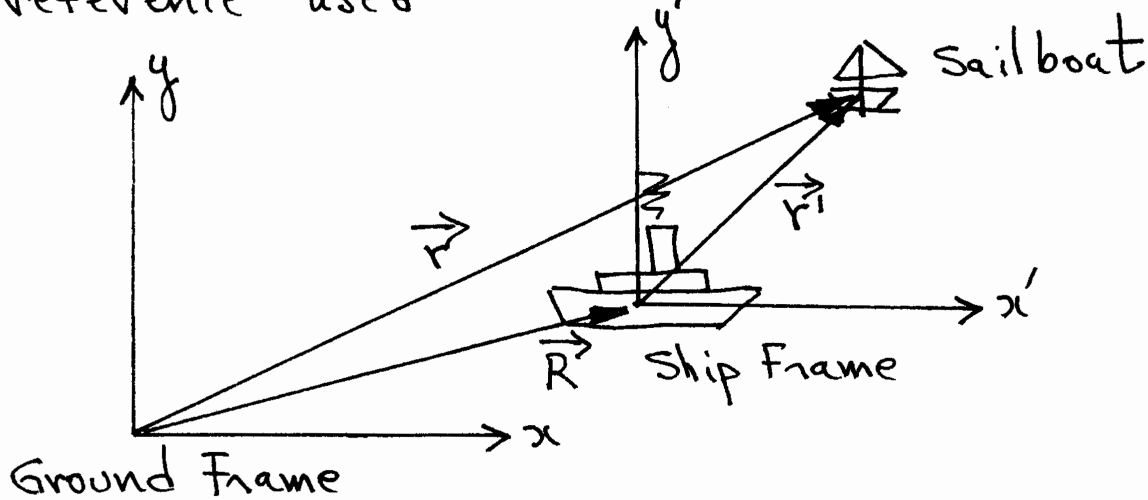
Special Relativity

Newtonian Mechanics : Galilean Transformation

- Motion is relative
- Velocity/acceleration depend on the frame of reference used



- Consider a rest system, 'ground', and a moving system, 'ship'.
- Axes aligned and the ship moves with a velocity V relative to the ground.
- Systems coincide at $t=0$.

Ground reference frame : x, y, z, t
Ship reference frame : x', y', z', t'

Assume time is absolute :

$t = t'$ $\qquad v \ll c$.

$\vec{r}$: position vector of sailboat rel. to ground
$\vec{r}'$: position vector of sailboat rel. to ship

- Ship moves with velocity $\vec{V}$ along $\vec{R}$

Then at any time t:

$$\vec{r} = \vec{r}' + \vec{R} \qquad ①$$

$\vec{R}$ ← ground frame

$\vec{r}'$ ← measured in ship frame

- Adding vectors
- Length is absolute
- Same length scales apply.

$$\vec{R} = \vec{V} t \qquad ②$$
$$\vec{r} = \vec{r}' + \vec{V} t \qquad ③$$
$$\vec{r}' = \vec{r} - \vec{V} t \qquad ④$$

Special Case :

$\vec{V} = V \hat{i}$ velocity along x-axis

$$\therefore \vec{r}' = \vec{r} - V t \hat{i}$$

$$\left.\begin{aligned} x' &= x - Vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \text{Galilean Transformation } v \ll c$$

Velocities

Differentiate Eq. 4

$$\vec{v}' = \frac{d\vec{r}'}{dt'} = \frac{d\vec{r}'}{dt} = \frac{d}{dt}(\vec{r} - \vec{V}t) = \frac{d\vec{r}}{dt} - \vec{V}$$

$$\therefore \vec{v}' = \vec{v} - \vec{V}$$

$\vec{V}$ ← Vel. of ship rel. to ground

$\vec{v}$ ← vel. of sailboat rel. to ground

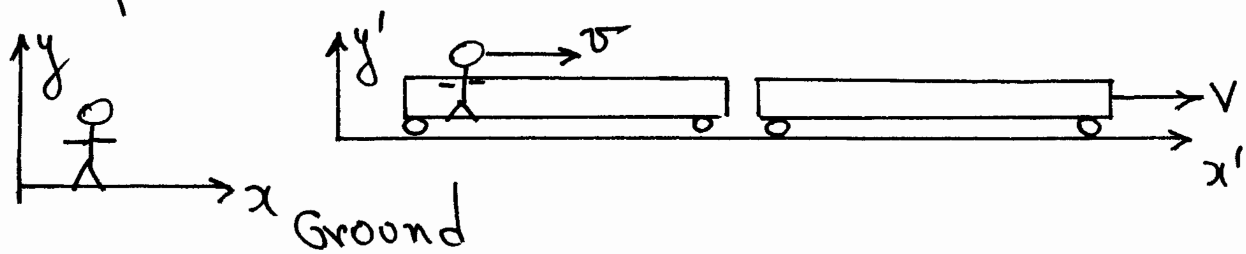
- Velocity of sailboat rel. to ship is the difference between the velocity of the sailboat relative to the ground and the velocity of the ship relative to the ground.

$$\vec{v} = \vec{v}' + \vec{V}$$

For $\vec{V} = V\hat{i}$ along x-axis

$$\left.\begin{aligned} v'_x &= v_x - V \\ v'_y &= v_y \\ v'_z &= v_z \end{aligned}\right\} \text{Velocity Transformations}$$

Example : Man + Train



- Slow train : $V = 10$ km/h
- Fast walker: $v = 5$ km/h

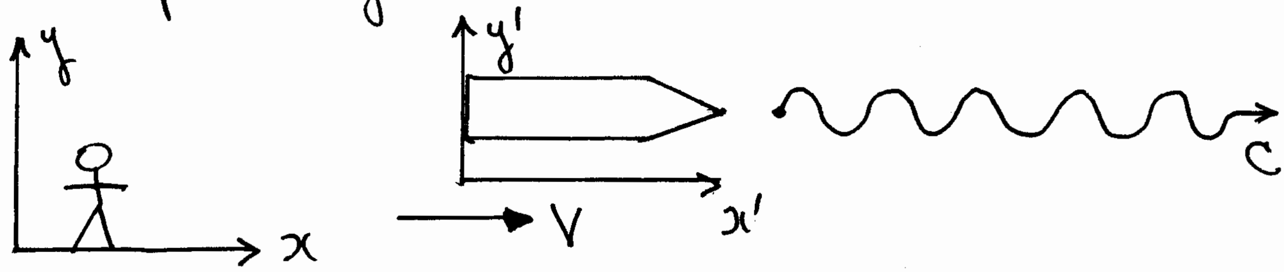
$$v_x = v_x' + V$$
$$= 5 + 10 = 15 \text{ km/h}$$ person rel. to ground

In 1-hr train travels 10 km to right
In 1-hr person walks 5 km along train

At end of 1-hr man is 15 km from person on ground.
Vel. rel. to gnd = 15 km/1 hr = 15 km/hr.

Makes sense!!

Example : Light Beam



- Rocket moving fast relative to the ground:

$$V = \frac{3}{4} c$$

- Rocket fires laser beam to the right
- What is speed of light beam as measured by person on the ground?

$$v_x = v_x' + V$$
$$= c + \frac{3}{4} c = \frac{7}{4} c$$

$\Rightarrow$ almost twice light speed!

- If beam were directed to the left along $-x$ we would have

$$v_x = -c + \frac{3}{4} c = \frac{1}{4} c$$

Light has almost stopped moving!

Is this analysis correct?
No !!!

Galilean Accelerations

$$\vec{a}' = \frac{d\vec{v}'}{dt'} = \frac{d\vec{v}'}{dt} = \frac{d}{dt}(\vec{v} - \vec{V})$$

Assume constant Inertial Frame

$$\vec{a}' = \frac{d\vec{v}}{dt} = \vec{a}$$

- Acceleration is absolute relative to inertial frames.

$\therefore\ \vec{F} = \vec{F}'$

$$a_x' = a_x$$
$$a_y' = a_y$$
$$a_z' = a_z$$

The accelerations are identical because the relative velocity, $\vec{V}$, of the two frames is constant. If the relative velocities were not a constant then the accelerations would differ by $\frac{d\vec{V}}{dt}$.

Suppose $v \sim c$. What then? This analysis is incorrect. Will show.

Light : Theory of Waves

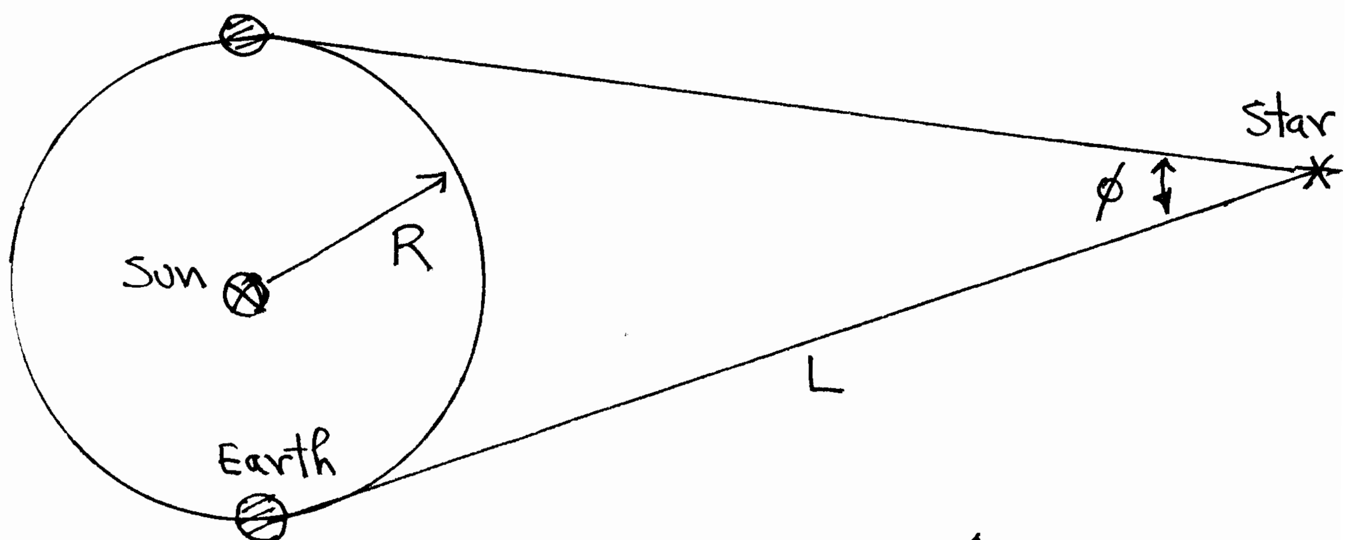
- Young (~ 1801-04)
- Light is a wave phenomenon
- Interference effects
- Diffraction, etc.
- Polarization

- Analagous to sound waves. Sound needs a medium, air, to propagate.
- Light should have a medium too!
  - luminiferous ether
- Speed through medium should be independant of velocity of source!

Stellar Aberration

Bradley : 1726

- Careful measurements of star positions
- Wanted to measure distance to nearest star using parallax and diameter of earth's orbit.



$$\varphi \sim \frac{2R}{L} = \frac{2 \times 1.5 \times 10^{11}}{9.46 \times 10^{15}} = 0.32 \times 10^{-4} \text{ rad}$$

L = 1 light-year

$\varphi = 6.5''$ arc.

$\varphi = 0.65''$ [L = 10 l.y]

Observations showed :

$\varphi = 41''$; Much larger !!

- Aberration angle correlated to earth's velocity and not just to location.

R1-9

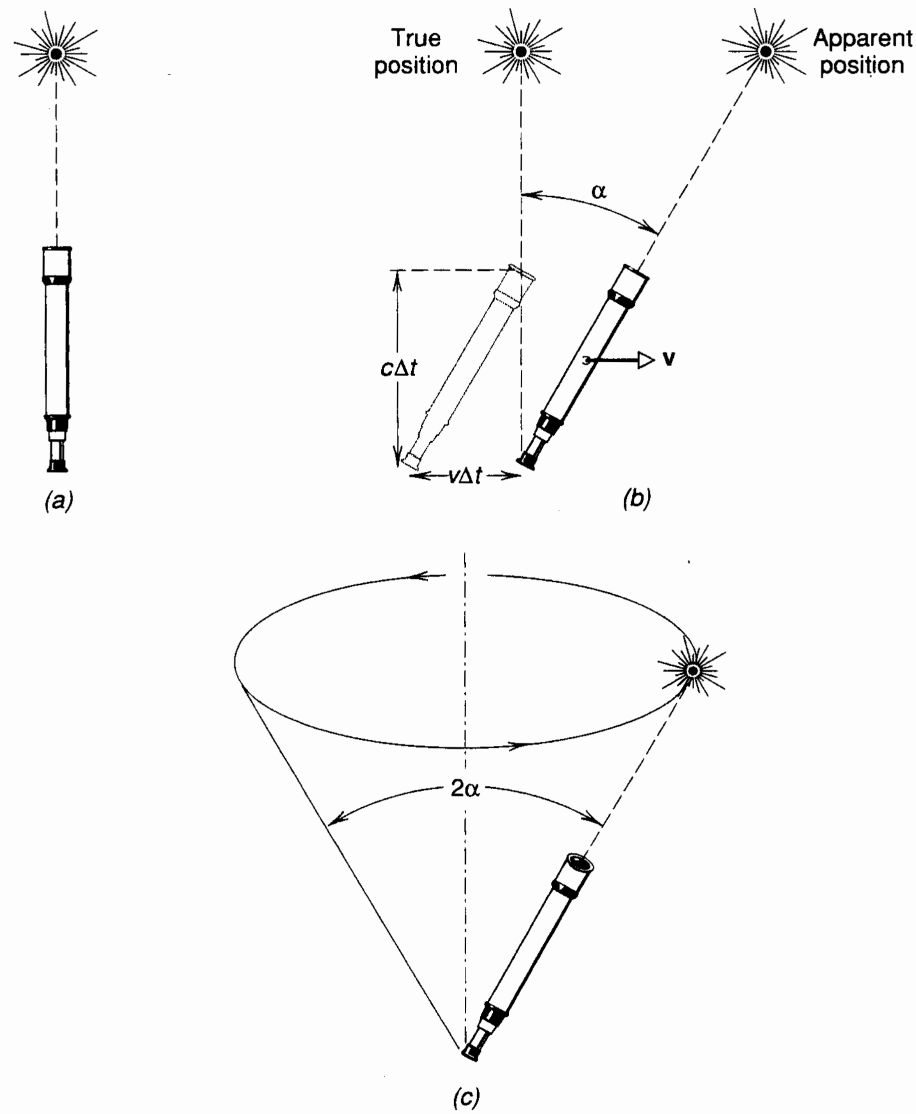


FIGURE 1-9. (*a*) The star and telescope have no relative motion (that is, both are at rest in the ether); the star is directly overhead. (*b*) The telescope now moves to the right at speed v through the ether; it must be tilted at an angle α (greatly exaggerated in the drawing) from the vertical to see the star, whose apparent position now differs from its true position. ("True" means with respect to the sun, that is, with respect to an earth that has no motion relative to the sun.) (*c*) A cone of aberration of angular diameter 2α is swept out by the telescope axis during the year.

- Light has a speed c
- Earth moves in orbit around sun with velocity v:

$$v = \frac{2\pi \times 1.5 \times 10^{11}}{365 \times 24 \times 3600} = 30\,\mathrm{km/s}$$

Explanation

- star directly overhead
- Telescope would point straight up if earth were at rest in ether. Light would travel straight down.
- Suppose earth moving to right with velocity v through ether. Telescope must be tilted to right to keep light from hitting sides. i.e. to stay on axis from eyepiece to objective.

Let Δt = time to travel down telescope

$$\Delta t = \frac{\Delta l}{c}$$

Let Δs = distance telescope moves in Δt.

$$\Delta s = v\,\Delta t$$

Angle of Tilt:

$$\tan\alpha = \frac{\Delta s}{\Delta l} = \frac{v\Delta t}{c\Delta t} = v/c$$

For earth;

$$\alpha = 20.5'' \text{ of arc.}$$

In a year, earth velocity reverses and star appears to circle a cone with opening,

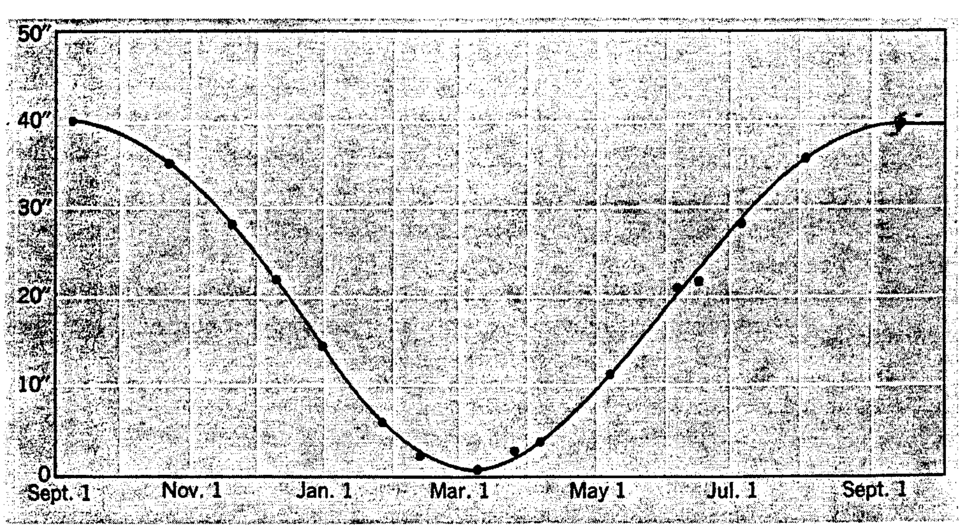
$$2\alpha = 41'' \text{ of arc!!}$$

Experiment in perfect agreement!!

Conclusion: Ether is NOT dragged around with the earth.

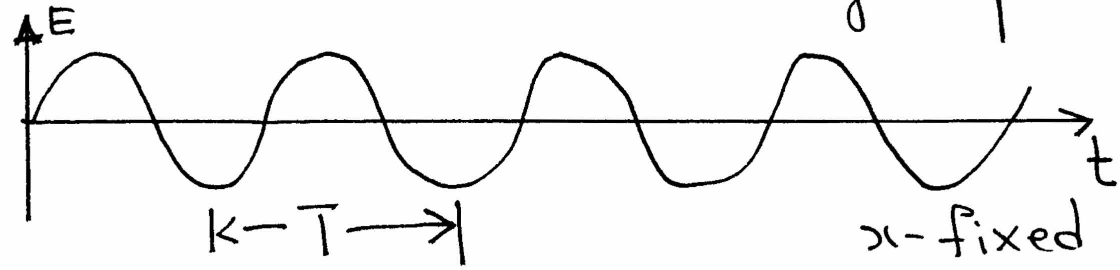
If ether was dragged with earth light would be swept along with ether similar to sound swept with wind.

Fig. 2–3 Bradley's data on the north-south component of the aberration of γ-Draconis (1727–1728).



Michelson - Morley

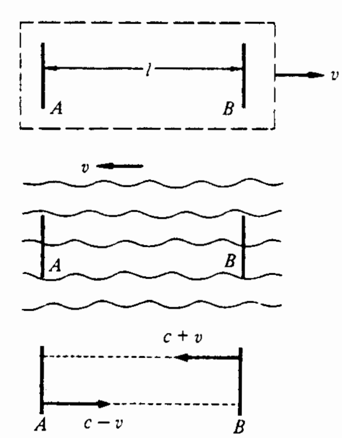
- Sound : medium = air } wave
- Light : medium = ether } propagation
  - electromagnetic wave disturbance
  - oscillates and travels through space



f: frequency, oscillations / s
T: period of oscillation
λ = wavelength
$c = f\lambda = 3 \times 10^8$ m/s

- Consider light propagating through an ether which fills all space
- Earth moves through ether with velocity v.

- Light makes a round trip: $A \rightarrow B \rightarrow A$ between AB separated a distance L.



Time: $A \rightarrow B$ $\quad t_1 = \dfrac{\ell}{c-v}$

Time: $B \rightarrow A$ $\quad t_2 = \dfrac{\ell}{c+v}$

If, $v = 0$; $\quad t_0 = \dfrac{\ell}{c}$

- Effect of earth's motion delays the return of the light by

$$\Delta t = t_1 + t_2 - 2t_0$$

$$= \frac{\ell}{c-v} + \frac{\ell}{c+v} - \frac{2\ell}{c}$$

$$= \frac{\ell}{c}\left[\frac{1}{1-v/c} + \frac{1}{1+v/c} - 2\right]$$

$$= \frac{2\ell}{c}\left[\frac{1}{1-v^2/c^2} - 1\right]$$

$$\simeq \frac{2\ell}{c}\frac{v^2}{c^2}$$

For $v/c = 10^{-4}$

$\ell = 1\,m$

$\Delta t = 7 \times 10^{-17} s.$

Impossible to measure !!

Michelson - Morley Experiment (1882)

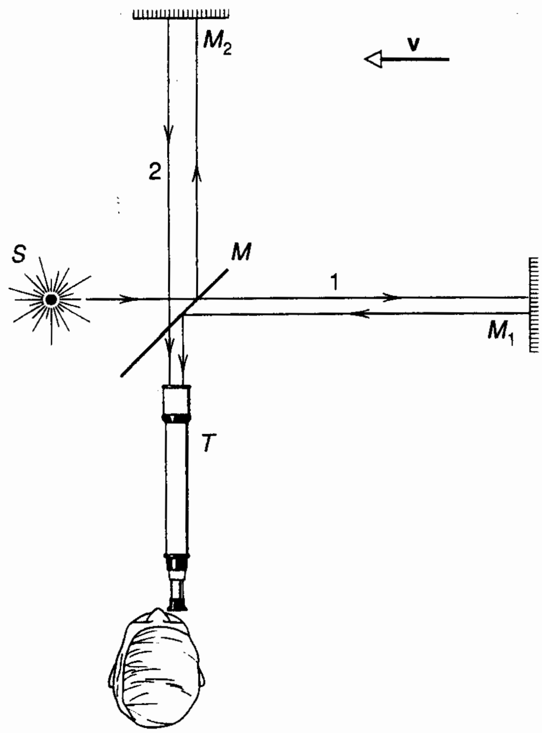


FIGURE 1-4. A simplified version of the Michelson interferometer showing how the beam from the source S is split into two beams by the partially silvered mirror M. The beams are reflected by mirrors M_1 and M_2, returning to the partially silvered mirror. The beams are then transmitted to the telescope T, where they interfere, giving rise to a fringe pattern. In this figure, **v** *is the presumed velocity of the ether with respect to the interferometer.*

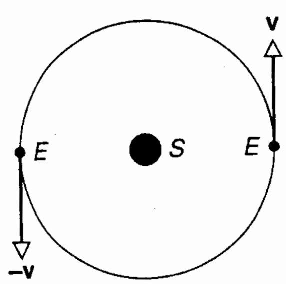
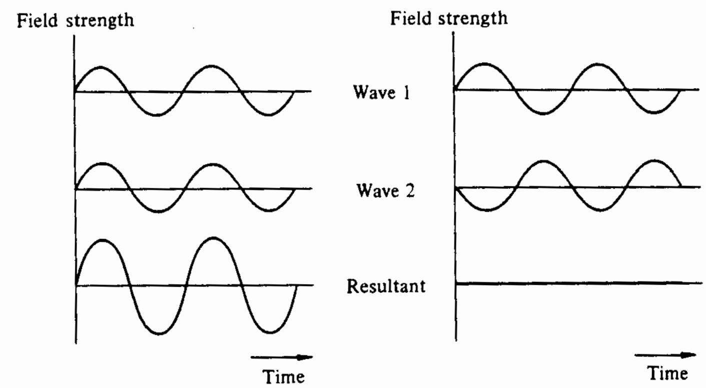


FIGURE 1-5. The earth E moves at an orbital speed of 30 km/s along its nearly circular orbit about the sun S, reversing the direction of its velocity every six months.

- Light from a source is split into two beams by a half-silvered mirror at M.
- Half the light travels to mirror M1 before it is reflected back to mirror M and then to the observer

- Half the light is diverted up the second arm and strikes mirror M2, which reflects it to the observer.

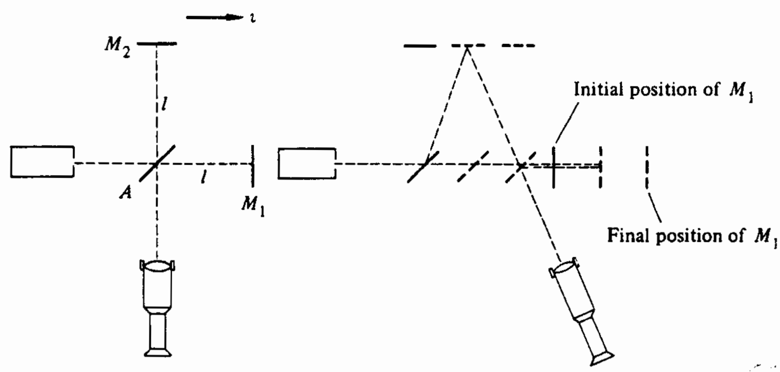


In phase
(*a*)

Out of phase
(*b*)

- What the observer sees depends on the delay between the two beams.
- If $\Delta t = 0$ or NT the observer will see a bright fringe.
- If $\Delta t = T/2$, exactly one half cyle of oscillation, the waves arrive in opposite phase (180^0) and cancel each other out. The observer's field is dark.
- Obervers mirrors are slightly tilted so he sees pattern of fringes.
- If time along either arm is changed - fringe pattern shifts.
- Light traverses each path twice so changing length by $\lambda/2$ shifts pattern by one fringe.

- Careful measurements can see changes of $\lambda/100$.
- High precision measurement.



- Align apparatus so $M \to M1$ is along earth's velocity
- The time for light to travel from $M \to M1$ and back is,

$$T_1 = \frac{\ell}{c-v} + \frac{\ell}{c+v}$$

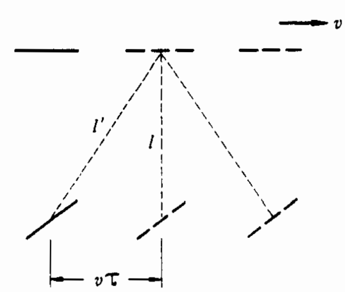
$$= \frac{\ell}{c}\left[\frac{1}{1-v/c} + \frac{1}{1+v/c}\right]$$

$$T_1 = \frac{2\ell}{c}\left[\frac{1}{1 - v^2/c^2}\right]$$
$$\simeq \frac{2\ell}{c}\left[1 + \frac{v^2}{c^2}\right]$$

ℓ = length of each arm.

- There is a time delay in arm-2 also.
- Mirror M2 is moving to the right
- Light must travel angular path!
- Let $\tilde{\tau}$ be time to travel M–M2

- The actual distance is



$$\ell' = (\ell^2 + v^2\tilde{\tau}^2)^{1/2}$$
$$\ell' = c\tilde{\tau}$$

$$\tilde{\tau} = \frac{(\ell^2 + v^2\tilde{\tau}^2)^{1/2}}{c}$$

or
$$\tilde{\tau}^2 = \frac{\ell^2}{c^2} + \frac{v^2}{c^2}\tilde{\tau}^2$$

$$\tilde{\tau} = \frac{\ell}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

For return trip

$$T_2 = 2\tilde{\tau} = \frac{2\ell}{c} \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\simeq \frac{2\ell}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$\Delta T = T_1 - T_2 = \frac{\ell}{c}\frac{v^2}{c^2}$$: delay due to earth's motion

- Apparatus has no zero so getting both arm lengths the same is impossible.
- Trick was to line up M1 along $\vec{v}$ then after a while to rotate the apparatus by 90° so M2 aligns with $\vec{v}$.
- Exchanges arms $1 \to 2$.
- change in delay is $2\Delta T$.

- Time delay of $\lambda/2$ shifts pattern by 1-fringe.
- Time delay $2\Delta T$ shifts pattern N fringes,

$$N = \frac{2\Delta T}{\lambda/c} = \frac{2\ell}{\lambda}\frac{v^2}{c^2}$$

$\ell \simeq 10\text{m}$
$\lambda = 5.5 \times 10^{-7}\text{ m}$ [yellow-light]
$\frac{v}{c} = 10^{-4}$ [earth's velocity]

$$N = \frac{2 \times 10 \times (10^{-4})^2}{5.5 \times 10^{-7}} = 0.36 \text{ Fringes!}$$

· Small shift; but possible

· 1887: No shift observed
· Something wrong!
· Maybe ether carried by earth then there would be no shift.
· Big problem!! Stellar aberration says ether stands still!!

· Classical Experiment finally resolved by Special Relativity

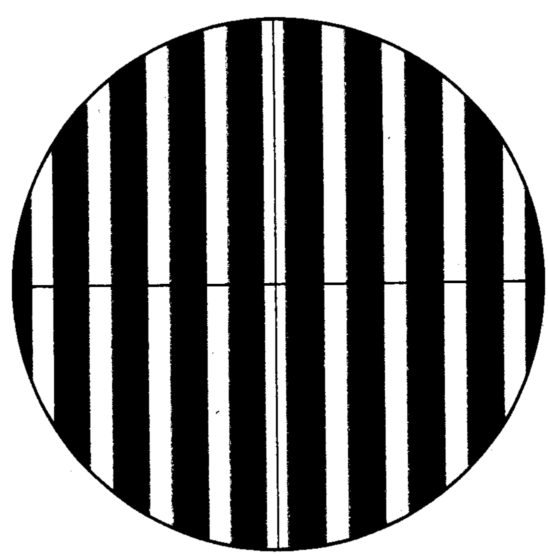


FIGURE 1-6. A typical fringe system seen through the telescope T when M_1 and M_2 are not quite at right angles.