Law of Universal Gravitation

In 1687 Newton published his work on the universal law of gravity in his "Mathematical Principles of Natural Philosophy".

"every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inverseley proportional to the square of the distance between them."

He pulled together a whole body of astronomical observations over the centuries with a simple unifying description.

Ancient astronomers recognized and attempted to explain the orderliness of the movement of the sun and planets against the background of the stars.

- day-night cycle
- annual progression of seasons
- periodic return of the planets to regions of the sky.

Detailed observations were carried out:

- accurate calendars
- star maps
- wanderings of planets

1. Pythagoras (582-497 BC)
Plato (427-347 BC)

- spherical stationary earth
- heavenly bodies move in circles
- stars fixed on a shell surrounding the earth
- sun, moon, planets revolve on spheres such as to match their periods.

Problems:

- some planets relative to background of stars showed retrograde motion.
- sun and planets did not appear to move uniformly.
- planets varied in brightness.

Epicycles: Planets move in a circle (deferent) and execute small circular motions along the orbit.

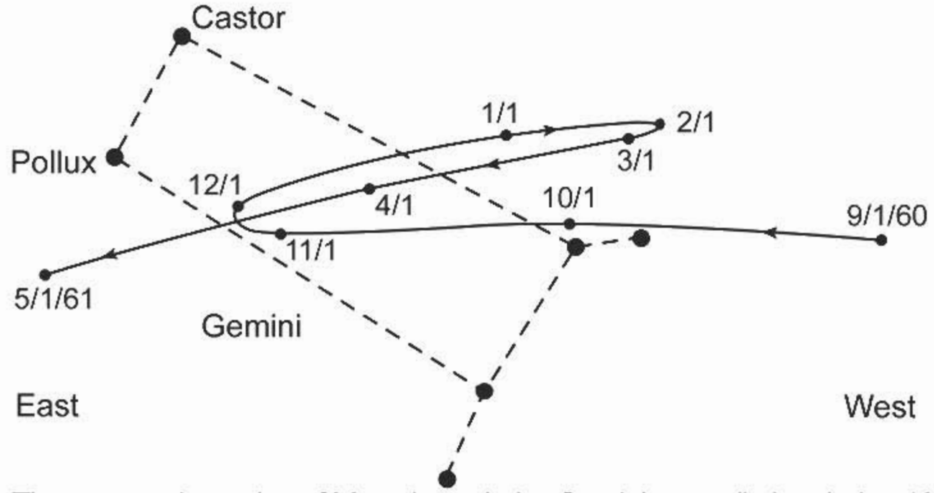
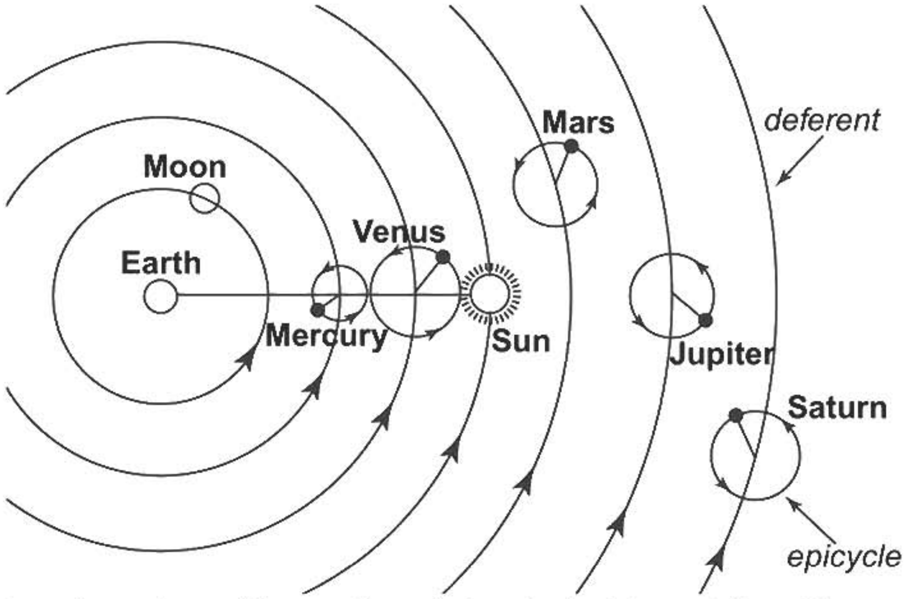


Fig. The retrograde motion of Mars through the Gemini constellation during 1960 and early 1961. The position of the planet on the first of each moon is shown. The path has the shape of a loop because the planes in which the Earth and Mars revolve do not exactly coincide.

2. Ptolemy (150 AD)

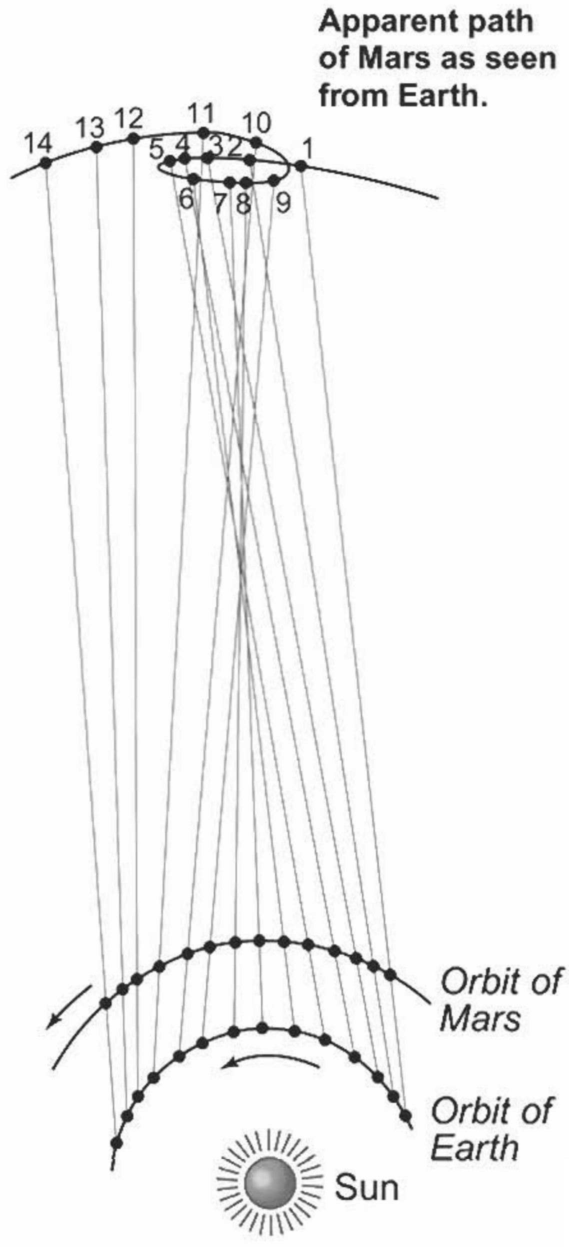
- Generalized all the Greek work.
- Earth centered system
- Epicycles
- Inner planets locked to move in-line with sun
- Explained brightness changes.
- Could predict future positions of planets.



The Ptolemaic system of the motion of planets (not to scale), put forward in about 150 A.D. Each planet moves on a circular *epicycle* and the center of each epicycle moves on a circular *deferent*. The centers of the epicycles of Mercury and Venus lie on a straight line connecting the Earth and the Sun.

3. Copernicus (1473-1543)

- Radical new theory of solar system.
- Sun is at the center
- Earth and planets revolve about the sun in circles.
- Epicycles to maintain uniform <u>circular motion</u>
- Accounted naturally for observed retrograde motion



The apparent retrograde motion of the planets on the basis of the Copernican heliocentric model.

4. Kepler (1571-1630)

- Elliptic orbits
- Abandoned circular orbits
- Major advance in the theory of planetary motion.
- Quantified the precise data of Tycho Brahe (1546-1601)

Kepler's Laws

HOW??

i) All planets move in elliptical orbits with the sun at one of the focal points.

ii) The radius vector drawn from the sun to any planet sweeps out equal areas in equal time intervals.

iii) The square of the orbital period of any planet is proportional to the cube of the semi-major axis of the elliptical orbit.

TABLE 14.2 Useful Planetary Data

Body	Mass (kg)	Mean Radius (m)	Period (s)	Distance from Sun (m)	$\frac{T^2}{r^3}\left[10^{-19}\left(\frac{s^2}{m^3}\right)\right]$
Mercury	3.18×10^{23}	2.43×10^{6}	7.60×10^{6}	5.79×10^{10}	2.97
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}	2.99
Earth	5.98×10^{24}	6.37×10^{6}	3.156×10^{7}	1.496×10^{11}	2.97
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}	2.98
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}	2.97
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}	2.99
Uranus	8.68×10^{25}	2.33×10^{7}	2.64×10^{9}	2.87×10^{12}	2.95
Neptune	1.03×10^{26}	2.21×10^{7}	5.22×10^{9}	4.50×10^{12}	2.99
Pluto	$\approx 1 \times 10^{23}$	$\approx 3 \times 10^{6}$	7.82×10^{9}	5.91×10^{12}	2.96
Moon	7.36×10^{22}	1.74×10^{6}	—	—	—
Sun	1.991×10^{30}	6.96×10^{8}	—	—	—

Newton's Law of Gravity

WHY ??

If the particles have masses m_1 and m_2 are separated by the distance r, the magnitude of the force is

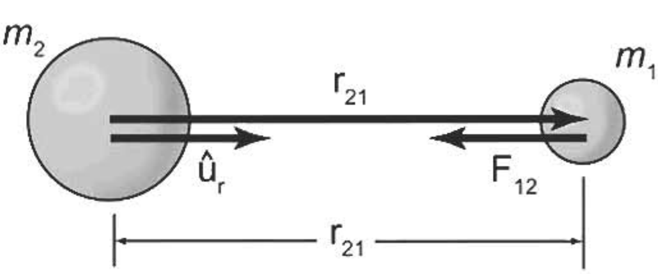
$$F = \frac{G m_1 m_2}{r^2}$$

$G = 6.672 \times 10^{-11}\ N \cdot m^2/kg^2$ Gravitational Constant. "Cavendish Experiment"

- Inverse square law force
- Always attractive
- Applys to all objects having mass
- Effects extend to infinity.

Writing the force in vector form,

$$\vec{F}_{12} = -\frac{G m_1 m_2}{r_{21}^2}\,\hat{u}_r$$



The gravitational force on m_1 due to m_2 is F_{12}

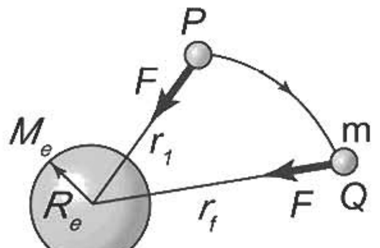
$\vec{F}_{12}$ = force on m_1 due to m_2.

$\vec{F}_{12} = -\vec{F}_{21}$ (Action-Reaction Pair)

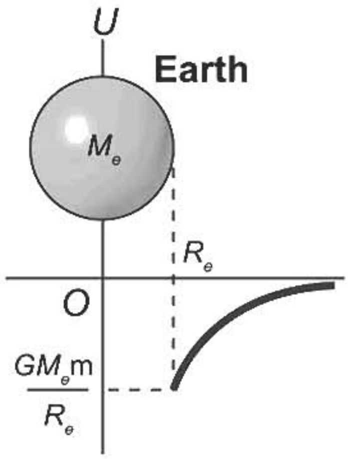
Gravitational Potential Energy

We defined previously that the change in potential energy associated with a displacement is the negative of the work done by the gravitational force during the displacement.

$$\Delta U = U_f - U_i = -\int_{r_1}^{r_2} \vec{F}(r) \cdot d\vec{r}$$



As a particle of mass m moves from P to Q above the earth's surface, the potential en-



Graph of the gravitational potential energy, U, versus r for a particle above the earth's surface. The potential energy goes to zero as r approaches ∞

For a particle m above the earth's surface

$$\vec{F} = -\frac{G M_e m}{r^2}\hat{r}$$

$$U_f - U_i = G M_e m \int_{r_i}^{r_f} \frac{dr}{r^2} = G M_e m \left[-\frac{1}{r} \right]_{r_i}^{r_f}$$

$$U_f - U_i = -G M_e m \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

Choice of reference point is arbitrary.

Let $U_i = 0$ at $r_i = \infty$ $\quad F(\infty) = 0$

$$\therefore U(r) = -\frac{G M_e m}{r} \qquad r > R_e$$

For any two particles m_1 and m_2,

$$U(r) = -\frac{G m_1 m_2}{r}$$

Principle of Superposition

We had that the net effect on an object of a number of forces, $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$ is simply given by

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$$

– principle of superposition.

What about the potential $U(r)$? Consider three particles. Each pair contributes a term

$$U(r) = -\frac{G m_1 m_2}{r}$$

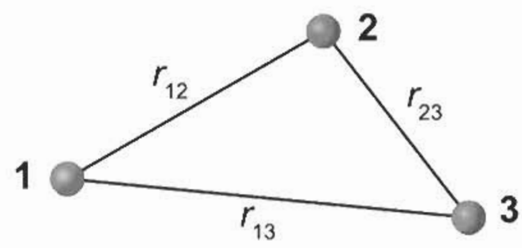


Diagram of three interacting particles.

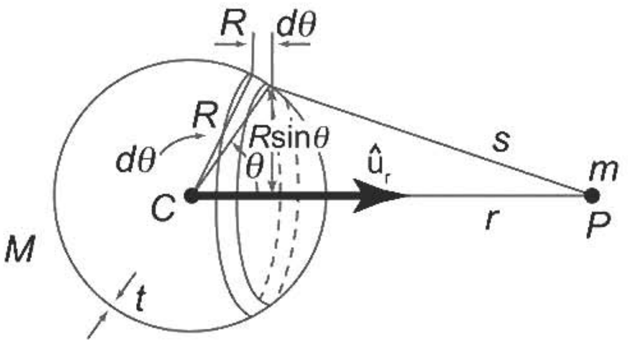
The total potential energy is given by a sum over all pairs of particles.

$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$= -G\left(\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \frac{m_2 m_3}{r_{23}}\right)$$

Potential Energy of a Spherical Shell and Particle

- particle of mass m, pointlike
- spherical shell
  - mass M
  - thickness t
  - radius R
- separation from center of shell to particle $= r$



Geometry for the calculation of the potential energy of a thin spherical shell and a particle. The thickness of the shell is *t*.

Particle at point P. Consider a ring of material with density ρ every part of which is same distance s from P.
Width of ring is $R\,d\theta$, thickness is t and its radius is $R\sin\theta$.

Mass dM of ring is

$$dM = \underbrace{(R\,d\theta)}_{\text{width}} \cdot \underbrace{2\pi R\sin\theta}_{\text{circumference}} \cdot t \cdot \underbrace{\rho}_{\text{density}}$$

(width · circumference: volume of Ring.)

$$dM = 2\pi R^2 t \rho \sin\theta \, d\theta$$

The potential energy of the ring and the particle m is

$$dU = -\frac{Gm\,dM}{s} = \frac{-2\pi Gm R^2 t\rho \sin\theta\, d\theta}{s}$$

To obtain total PE for shell need to integrate this over entire shell.
Easier to integrate over s instead of θ.
Change variables.

$$s^2 = r^2 + R^2 - 2rR\cos\theta \qquad \text{(Law of Cosines)}$$

$$2s\frac{ds}{d\theta} = 2rR\sin\theta$$

$$\frac{\sin\theta}{s}\,d\theta = \frac{ds}{rR}$$

$$\therefore\ dU = -\frac{2\pi GmRt\rho}{r}\,ds$$

Case I: $r > R$ [m is outside of shell]

$$\theta = 0 \qquad s = r - R$$

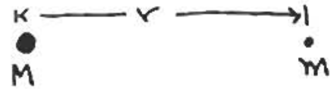
$$\theta = \pi \qquad s = r + R$$

$$U(r>R) = \int dU = -\frac{2\pi GmRt\rho}{r}\int_{r-R}^{r+R} ds$$

$$U = -\frac{Gm}{r}\left(4\pi R^2 t\rho\right) \qquad (r>R)$$

$$M = V\rho = (4\pi R^2 t)\rho$$

$$\therefore\ \mathcal{U} = -\frac{GmM}{r} \qquad (r > R)$$



Potential due to a shell of mass M with radius R is exactly the same as that of a pointlike mass M located at the center. [For points outside the shell].

$$\vec{F}(\vec{r}) = -\frac{d\mathcal{U}}{dr}\,\hat{r} = -\frac{GMm}{r^2}\,\hat{r} \qquad (r > R)$$

A thick shell or a solid sphere is a summation of a large number of thin shells.

"The gravitational effect of a spherically symmetric distribution of matter on a particle is the same as that of a particle (point like) with mass M located at the center of the sphere"

Case II: $r<R$ [Inside Shell of Mass]

- Particle is inside the shell

$\theta = 0 \qquad s = R - r$

$\theta = \pi \qquad s = R + r$

$$u(r<R) = -\frac{2\pi G m t \rho R}{r}\int_{R-r}^{R+r} ds = -\frac{2\pi G m R t \rho}{r}\Big[(R+r)-(R-r)\Big]$$

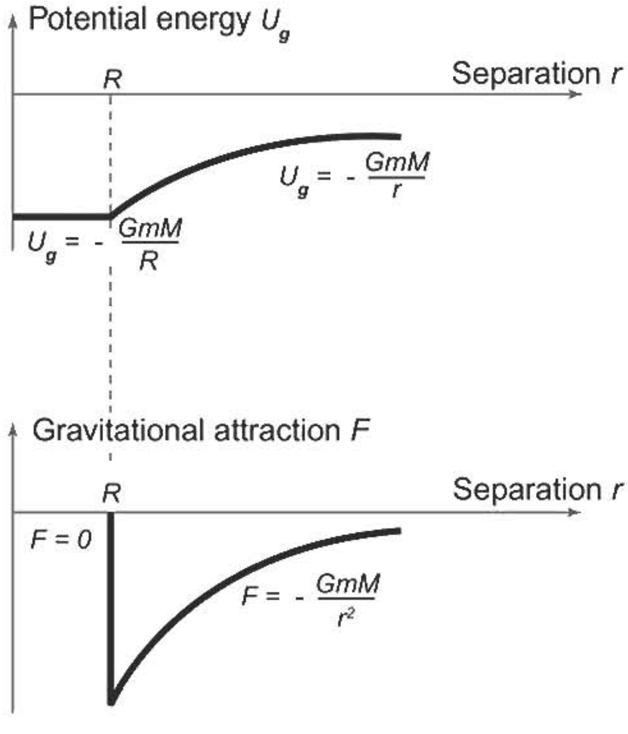
$$= -\frac{Gm}{R}\left(4\pi R^2 t \rho\right) = -\frac{GmM}{R} \qquad (r<R)$$

Potential is a constant everywhere inside the shell

Force on the particle m:

$$\vec{F}(\vec{r}) = -\frac{du}{dr}\hat{r} = 0 \qquad (r<R)$$

Force Vanishes !! By symmetry particle equally attracted all around.



(a) U_g as a function of r

(b) F as a function of r

Potential energy and gravitational attraction of a particle m by a spherical shell of mass M and radius R, where r is the distance from the center of the shell

Cavendish Experiment : G

Force between masses M and m a distance r apart

$$F = \frac{GmM}{r^2}$$

Torque due to couple about the suspension wire is

$$\vec{\tau} = \vec{L} \times \vec{F}$$

$$= \frac{l\,GmM}{r^2}$$

Twisted wire resists torque such that

$\tau = -K\phi$ Hooke's Law for a twisted wire. ϕ is angle of twist. K is property of material and resists twist.

$T = 2\pi\sqrt{\frac{I}{K}}$ Period for a torsional pendulum.

$I = \frac{ml^2}{2}$ Moment of inertial of masses on the rod.

$\Theta = 2\phi$ Reflection of laser from plane mirror.

$$-k\phi = \frac{l\,GmM}{r^2}$$ Relating torques.

$$\phi = \frac{l\,GmM}{kr^2}$$

$$\phi' = \frac{2l\,GmM}{kr^2}$$ Total deflection for reversed direction of torque.

$$\frac{\theta}{2} = \frac{2l\,GmM}{kr^2}$$

$r = 4.65$ cm [Balls center-to-center]

$M = 1.5$ kg

$m = .015$ kg.

$l = 10$ cm.

$L =$ m [Distance mirror → wall]

$k = 8.5 \times 10^{-9}$ N·m/rad

$\Delta S = L_1 \theta$ $G = 6.672 \times 10^{-11}$ N·m^2/kg^2

$T \sim 10$ min.

$$\Delta S = \frac{4 l L_1 GmM}{kr^2}$$

$$= \frac{4 \times 0.10 L_1 \times 6.672 \times 10^{-11} \times .015 \times 1.50}{8.5 \times 10^{-9} \times (.0465)^2}$$

$$\Delta S = .0327 L_1 \text{ (m)}$$

	$L = 19.7$ m	$L = 5.7$ m	
$(\Delta S)_{Th} =$	64.4 cm	18.6 cm	$\underline{\underline{2.6\%}}$
$(\Delta S)_{Exp} =$		19.1 cm	