Newton's Third Law

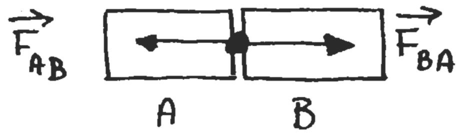
From observations, Newton concluded that all forces in nature act in pairs.

- Impossible to have a single isolated force.

Fig. (a) Tugboat pushes on barge (action). (b) Barge pushes on tugboat (reaction).

Action $\longleftrightarrow$ Reaction

Newton: "To every action (force) there is always opposed an equal reaction (force); or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."



$$\vec{F}_{AB} = -\vec{F}_{BA} \qquad \text{[Action - Reaction Pair]}$$

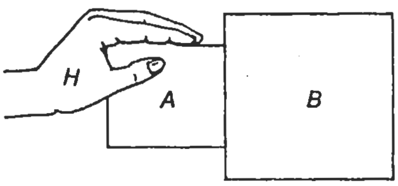
Note: - The two forces in an action-reaction pair act on different objects.

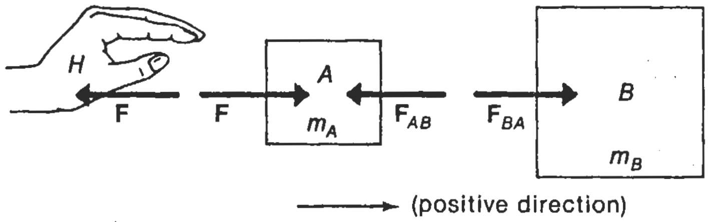
- A body is accelerated <u>only</u> by the forces acting on it. Forces it exerts on another body do not affect its motion.

Example

$F = 2N$ is applied to block A which is in contact with B.

Identify all action-reaction pairs:





$m_A = 1\,kg$
$m_B = 2\,kg$

$\vec{a} = ?$ [Acceleration]
$\vec{F}_{AB} = ?$ [Contact Force]

Choose coordinate system, x-axis positive to right.
Apply Newton's 2ND Law for motion of B:

$$F_{BA} = m_B a_B \qquad (1)$$

Forces acting on A :

$$F - F_{AB} = m_A a_A \quad (2)$$

Since both blocks stay in contact,

$$\therefore a_A = a_B = a \quad \text{[Constraint in problem]}$$

Add Eq. (1) + (2)

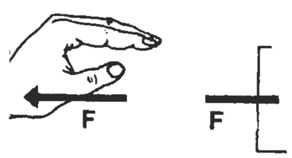
$$F - F_{AB} + F_{BA} = (m_A + m_B)\, a$$

$$|F_{AB}| = |F_{BA}|$$ Magnitudes equal due to Newton's 3rd Law.

$$\therefore F = \underbrace{(m_A + m_B)}\, a$$

Single mass $M = m_A + m_B$ on which only external force F is important.

$$a = \frac{F}{m_A + m_B} = \frac{2N}{(1+2)\,kg} = \frac{2}{3}\, m/s^2$$



<u>Contact Force</u>

$$F_{BA} = m_B a_B = m_B a$$

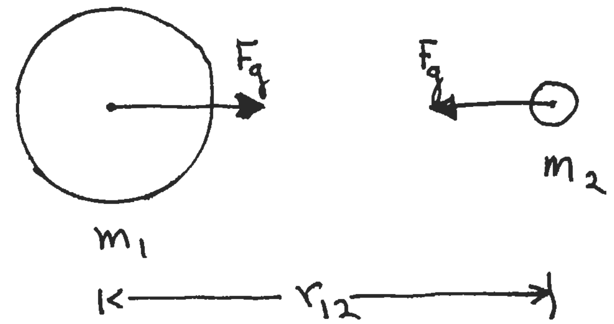
$$= 2 \times \frac{2}{3} = \frac{4}{3}\, N$$

$$\neq F$$

Gravitational Force

Newton's Postulate: Every pair of particles in the universe exerts on one another a motual gravitational force of attraction. This force is proportional to the product of the masses and inverseley proportional to the square of the distance between them.

$$F_g = \frac{G m_1 m_2}{r_{12}^2}$$



$G = 6.673 \times 10^{-11}\ N\cdot m^2/kg^2$

Universal Gravitation Constant.

Consider a mass m at the surface of the earth which has a mass $= M_E$ and radius $= R_E$.

$$F_g = m\underbrace{\left(\frac{M_E G}{R_E^2}\right)}_{g} = m\,g$$

Gravitation constant G can be determined by measuring the force with known masses that are separated by a known distance. [Cavendish Expt.]

Gravitational acceleration g is measured by obsering the fall of objects.

$$M_E = \frac{g R_E^2}{G} \qquad \text{"weighing the earth"}$$

Object m resting on surface of the earth.

Object:

Two forces acting.

Earth's attractive gravitational force $\vec{F_g}$. Contact force $\vec{N}$ with ground.

Since object is at rest:

$$\vec{N} + \vec{F_g} = 0$$

$$\vec{N} = -\vec{F_g}$$

Earth:

Two forces acting.

Attractive gravitational force $\vec{f_g}$ due to object.

Contact force $\vec{w}$ with object.

No relative motion:

$$\vec{w} = -\vec{f_g}$$

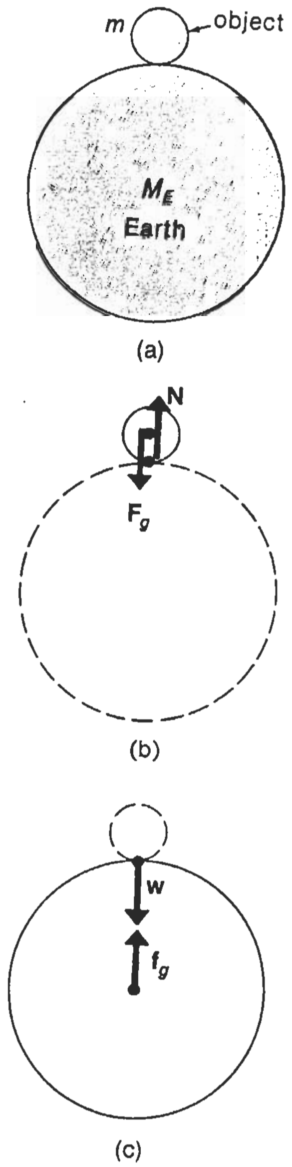


Fig. (a) An object at rest on the Earth's surface. (b) The forces acting on the *object*. (c) The forces acting on the *Earth*. The forces shown together in (b) and (c) are *not* action-reaction pairs.

Action - Reaction Force Pairs :

$$\vec{F_g} = -\vec{f_g}$$

$$\vec{N} = -\vec{w}$$

All four forces have equal magnitudes

$$F_g = f_g = N = w = mg$$

<u>Weight</u>

The contact force $\vec{w}$ that an <u>object exerts</u> on whatever is <u>supporting</u> it is called the weight of the object.

For above example $\vec{F_g}$ acts on the object and $\vec{w}$ acts on the earth. For this case of zero acceleration

$$\vec{w} = m\vec{g}$$

Accelerating Elevator

- Object on floor of elevator that is accelerating upward with acceleration $\vec{a}$.

For forces acting on m:

$$N - F_g = ma$$
$$N = F_g + ma = mg + ma = m(g+a)$$

N: Force elevator exerts on object
F_g: Force earth exerts on object

$$\vec{N} = -\vec{w} \quad \text{[Action - Reaction]}$$
$$\therefore w = m(g+a) \quad \text{[Magnitude]}$$

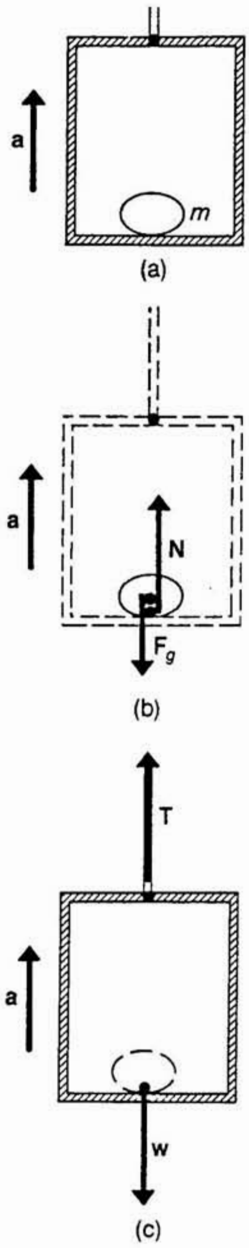
- Weight is increased by the amount ma over weight at rest.

For elevator moving doward with acceleration a′:

$$w = m(g - a')$$

- weight is decreased by ma′.

Free Fall: $a' = g \Rightarrow w = 0$
Object is weightless — exerts no contact force on support.



(a) An object with a mass m is on the floor of an elevator that is accelerating upward.
(b) Forces acting on the object.
(c) Forces acting on the elevator.

Example: Astronaut/Satellite

An artificial satellite is in a circular orbit around the Earth. At position of orbit acceleration due to gravity is g'.

$$g' = \frac{v^2}{R} \qquad \text{[Free-fall motion]}$$

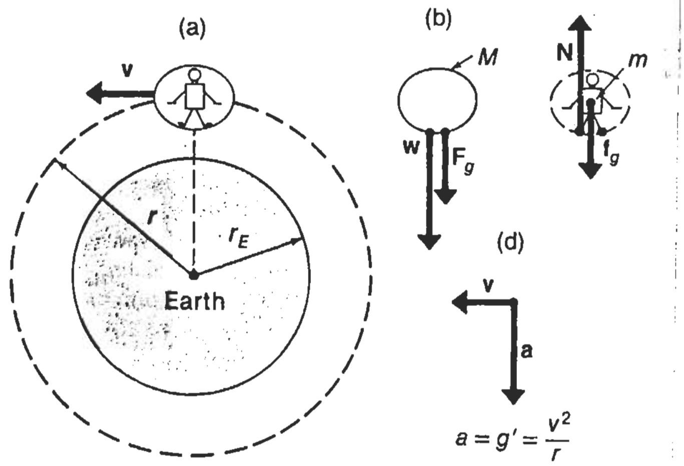


Fig. An astronaut in an orbiting artificial satellite is *weightless*.

Force acting on satellite:

$$F_S = F_g + w = Mg' \qquad [F = ma]$$

Forces acting on astronaut:

$$f_a = f_g - N = mg' \qquad [F = ma]$$

$$\left.\begin{aligned} F_g &= Mg' \\ f_g &= mg' \end{aligned}\right\} \text{Gravitational Force on each object by definition.}$$

$$\therefore \quad w = N \equiv 0 \qquad \text{[No contact force / No weight]}$$

Astronaut is weightless – can exert no net force on satellite.

Note: Weight of an object is always equal to the vector force exerted on the supporting object or system.

Gravitational / Inertial Mass

Mass plays essentially two different roles in mechanics.

1. Inertial Mass:

In Newton's Laws the quantity m connects the force applied to an object with its acceleration.

$$\vec{F} = m_I \vec{a}$$

The mass measured in this way is called the inertial mass, M_I.

2. Gravitational Mass:

Mass is also the property of matter that causes bodies to exert attractive gravitational forces on each other. eg. Measure force on object due to the earth.

$$\vec{F} = \frac{G M_E M_G}{R_E^2} = \vec{g} M_G$$

This mass is called the gravitational mass.

Question: Is $M_I = M_G$ for all bodies?

Experiment: Must provide the ultimate answer. Do all bodies fall due to gravity with the same acceleration?

Body #1: $M_I(1)\, a(1) = \frac{G M_E}{R_E^2} M_G(1) = g\, M_G(1)$ ①

Body #2: $M_I(2)\, a(2) = g\, M_G(2)$ ②

①/②
$$\frac{M_I(1)\, a(1)}{M_I(2)\, a(2)} = \frac{M_G(1)}{M_G(2)}$$

$$\frac{M_I(1)}{M_G(1)} = \frac{M_I(2)}{M_G(2)} \times \frac{a(2)}{a(1)}$$

Experiment finds that $a(2)/a(1)$ = constant. Tested for many different materials.

Galileo
Eötvös
Dicke
} $< 1 \times 10^{-12}$ difference.

Ratio $\frac{M_I}{M_G} = 1$ by a proper choice of G.

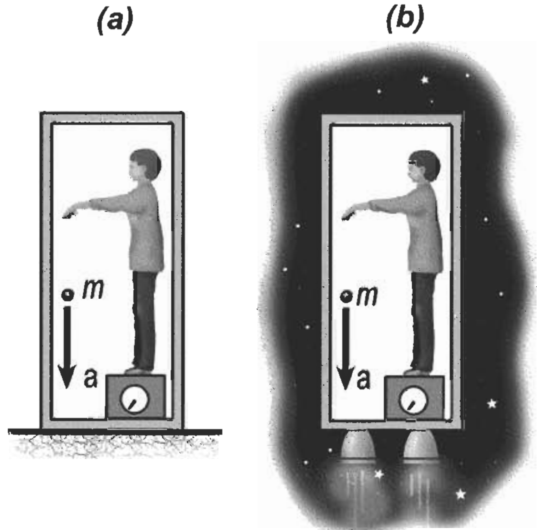
Newtonian Theory: $M_I \propto M_G$ not required
General Relativity: $M_I \propto M_G$ absolutely a must

→ 'Principle of Equivalence'
-gravitation is equivalent to acceleration.

Principle of Equivalence

- A fundamental postulate of Einstein's General Theory of Relativity. In this theory gravity is interpreted as a curvature in space and time

- Equivalence
  - Physicist in a box cannot tell the difference between gravity and acceleration.



(a) A physicist-in-a-box, resting on the earth.

(b) The same, accelerating in deep space at 9.8 m/s^2. It is not possible, by doing experiments within the box, for the physicist to tell which box he is in.

Ex: Suppose $\left(\frac{M_g}{M_I}\right)_A = 2\left(\frac{M_g}{M_I}\right)_B$. Release both objects in an Einstein elevator. If they fall with same acceleration then elevator must be accelerating upwards. If A falls with twice acceleration of B then it must be due to gravitational field. Violation of Principle of Equivalence!! Predicted bending of light by a massive object.

Hooke's Law - Springs

All bodies are elastic to varying degrees. When a stretching or compressing force is applied they deform.

- steel balls
- rubber bands
- springs

A body resists deformation by means of a <u>restoring force</u>.

Pull on a spring - it pulls back.

In many cases the relationship between the resulting restoring force and the deformation obeys a simple empirical law known as <u>Hooke's Law.</u>

"The magnitude of the restoring force is directly proportional to the deformation"

- approximate
- empirical description
- good for small deformations

Coil Spring

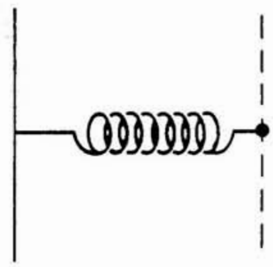
$+x$ - spring stretched
$-x$ - spring compressed

Hooke's Law:

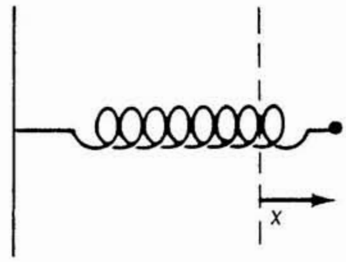
$$F = -kx$$

spring constant

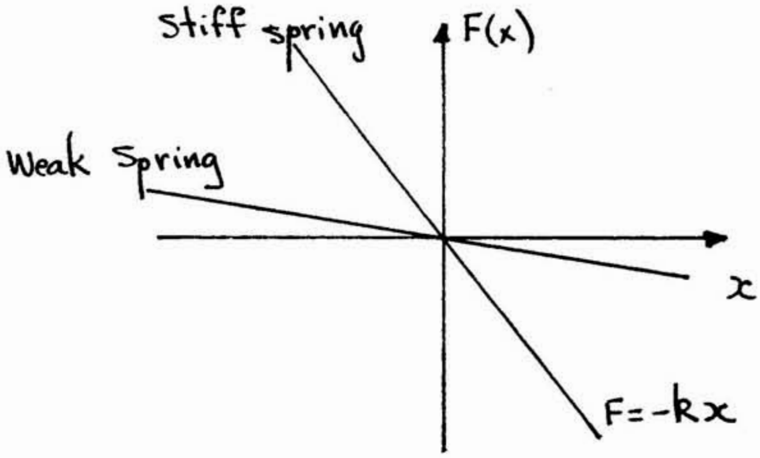
Negative sign means the restoring force opposes the deformation.



Spring, relaxed.



Spring, stretched by a length x.

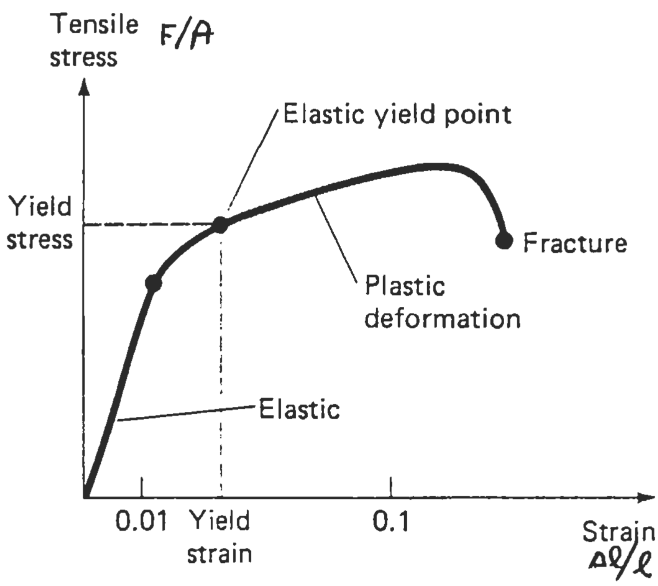


$[k] = N/m$

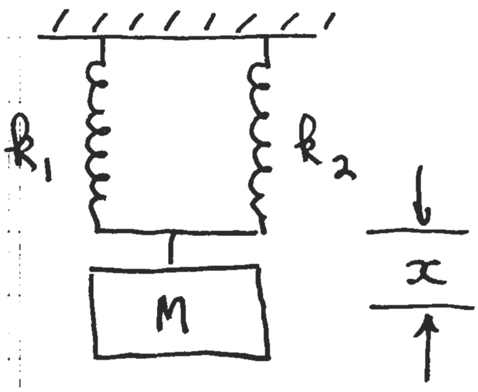
k large: stiff spring
large force/unit displacement

k small: soft spring
small force/unit displacement

+x -elongation ; Force negative opposes stretching
-x -compression ; Force positive opposes compression



Parallel Springs



$$-k_1 x - k_2 x + mg = 0$$

$$x(k_1 + k_2) = mg$$

$$x = \frac{mg}{(k_1 + k_2)} = \frac{mg}{k_{eff}} \qquad k_{eff} = k_1 + k_2$$

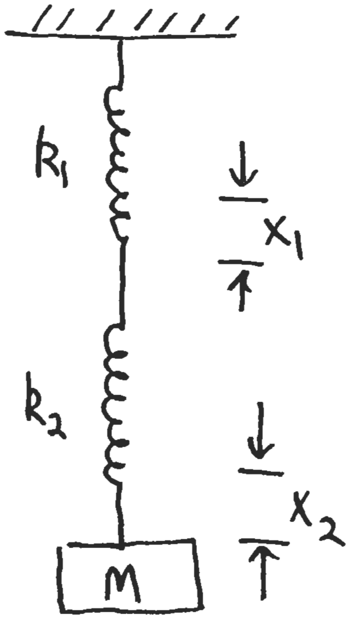
$k_{eff} \Rightarrow$ Stiffer Spring

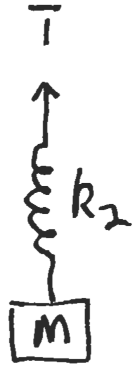
If $k_1 = k_2 = k$

$$k_{eff} = 2k$$

Twice as stiff

Series Springs





$$T = Mg$$

$$\therefore k_2 X_2 = Mg$$

Also $\quad k_1 x_1 = Mg$

$$x = x_1 + x_2 = \frac{Mg}{k_1} + \frac{Mg}{k_2}$$

$$x = Mg\left[\frac{1}{k_1} + \frac{1}{k_2}\right]$$

Let $\quad \dfrac{1}{k_{eff}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$ $\quad$ Weaker Spring

If $k_1 = k_2 = k$

$k_{eff} = \frac{1}{2} k$ $\quad$ weaker