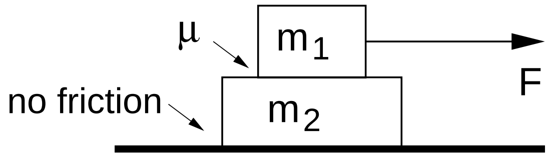
Problem 1: Dynamics (15 pts)

Two blocks of mass m_1 and m_2 are put on a frictionless level surface as shown in the figure below. The static coefficient of friction between the two blocks is μ. A force F acts on the top block m_1.



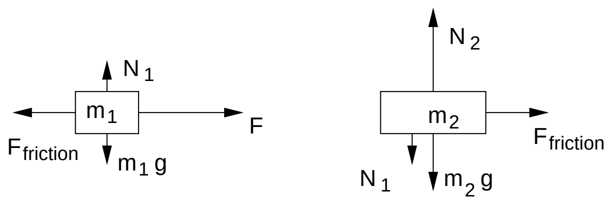
(a) When the force F is small, the two blocks move together. Draw the free-body diagrams of the block m_1 and the block m_2.

(b) Find the acceleration of the two blocks for small F.

(c) Find the magnitude of the force F above which the block m_1 starts to slide relative to the block m_2.

Solution:

(a)



(b) $a = F/(m_1 + m_2)$

(c) m_1 start to slide when

$$F_{friction} = \mu m_1 g = m_2 a = F m_2/(m_1 + m_2).$$

We find

$$F = \frac{\mu m_1 g (m_1 + m_2)}{m_2}$$

Problem 2: Circular motion (15 pts)

A car of mass $m = 1000kg$ is traveling around a flat circular race track of radius $100m$. The static coefficient of friction between the tire and the road (against transverse motion) is $\mu = 0.5$. (Assume $g = 10m/s^2$)

(a) How fast can the car travel before it starts to skid? Express the speed in the units of m/s.

(b) What is the angular velocity ω of the car at the speed calculated in (a).

(c) The driver of the car wants to drive faster. He loads $500kg$ of weight into the car to increase the friction force. Now how fast can the car travel without skidding?

Solution:

(a) The max speed of the car should satisfy

$$m\frac{v^2}{r} = \mu mg$$

We find

$$v = \sqrt{\mu gr} = \sqrt{0.5 * 100 * 10} = 10\sqrt{5}m/s = 22m/s$$

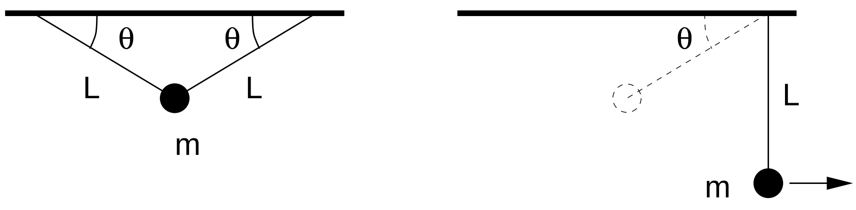
(b) The angular velocity is

$$\omega = v/r = 0.22/s$$

(c) v does not depend on the mass. So the max speed is not changed.

Problem 3: Balance and energy (15 pts)

A block of mass m is tied to two strings as shown in the figure below. Each string has a length L. The angle $\theta = 30°$. ($\sin\theta = 1/2$ and $\cos\theta = \sqrt{3}/2$.) Assume the strings are massless.



(a) Draw the free-body diagram of the block.

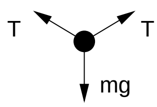
(b) Find the tension of each string.

(c) We cut one string and the block starts to swing down. Find the speed of the block when it reaches the lowest point.

(d) Find the tension in the string when the block reaches the lowest point.

Solution:

(a)



(b) From the balance of the force in the vertical direction, we find

$$mg = 2T\sin\theta = T$$

Thus $T = mg$.

(c) The change in the potential energy is $mg(L - L\sin\theta) = mgL/2$. Thus

$$\frac{1}{2}mv^2 = \frac{mgL}{2}$$

We find

$$v = \sqrt{gL}$$

(d) The tension minus weight should provide the acceleration for the circular motion:

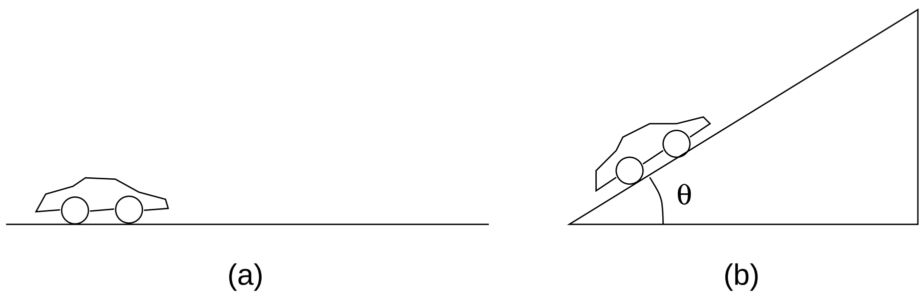
$$T - mg = m\frac{v^2}{L}$$

So

$$T = mg + mg = 2mg$$

Problem 4: Power

A small car's engine can deliver $90kW$ of power (about 120hp). The car's mass is $1000kg$. (Assume $g = 10m/s^2$)



(a) Assume the total resistive force is proportional to the velocity: $F_{friction} = \alpha v$. The drag coefficient α is $\alpha = 100Ns/m$. How fast can the car move on a level road? Express the speed in the units of m/s.

(b) How fast can the car travel up a slope if we ignore all friction? The angle of the slope is θ ($\sin(\theta) = 3/5$ and $\cos(\theta) = 4/5$). Express the speed in the units of m/s.

Solution:

(a) From $P = vF_{friction}$, we find $P = \alpha v^2$ or

$$v = \sqrt{P/\alpha} = \sqrt{90000/100} = 30m/s = 108km/hr$$

(b) From Work=$P * \Delta t = mg\Delta h = mg\Delta x \sin\theta = mgv\Delta t \sin\theta$, we find $P = mgv\sin\theta$ or

$$v = \frac{P}{mg\sin\theta} = \frac{90000}{1000 * 10 * 3/5} = 15m/s = 54km/hr$$