

HW Solutions # 9 - 8.01 MIT - Prof. Kowalski

Rigid Body Rotation and Angular Momentum.

1) **9.78**

Please refer to figure 9.29 p.358.

From the definition:

$$I = \int r_{\perp}^2 dm$$

we observe that if an object is **more concentrated** around the line has **less** moment of inertia. (Of course this conclusion is not right if they have different masses)

a) The cylinder's mass (object **A** in the figure) is the most concentrated near its axis \Rightarrow Object **A** has the smallest moment of inertia.

b) The cylinder with a hole has greater I than a cylinder (of the same mass) but slightly less than a hoop. With the assumption that it's a **thin** cylinder we assume it's very close to I of a hoop:

$$I_{thin\ cylinder} \approx MR^2.$$

The cube has moment of inertia $I_{cube} = \frac{1}{12}M(a^2 + b^2) = \frac{2}{3}MR^2$ which is less than the thin cylinder. So, the thin cylinder (object **B** in the figure) has the largest moment of inertia.

c) The sphere is more concentrated near its axis than even the cylinder so it will replace solid cylinder as the smallest moment of inertia.

2) 9.80

The key to this problem is to set up the energy conservation equation having in mind that no non-conservative force is present here:

$$K_1 + U_1 = E_1 = E_2 = K_2 + U_2 \quad (1)$$

U of the disk will not change due to rotation around its center. There are two parts contributing to the kinetic energy: movement of center of mass of the mass m , and of rotation of the disk:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I_{disk}\omega^2 \quad (2)$$

$$I_{disk} = \frac{1}{2}mR^2$$

Mass m is attached to the disk so it rotates with the same ω as the disk:

$$v = R\omega$$

Hence,

$$K_2 = \frac{3}{4}mR^2\omega^2$$

We will measure the gravitation potential energy with respect to the height of the horizontal line passing through the center of the disk.

$$E_1 = 0 + 0 + 0 = 0$$

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$$E_2 = \frac{3}{4}mR^2\omega^2 - mgR$$

$$E_1 = E_2 \Rightarrow \frac{3}{4}mR^2\omega^2 - mgR = 0$$

$$\boxed{\omega = \sqrt{\frac{4g}{3R}}}$$

3) 9.86

a) Let's denote L for the left mass, R for the right and P for the pulley. The key to this problem is to set up the energy conservation equation having in mind that no non-conservative force is present here:

$$\begin{aligned} K_{L_1} + U_{L_1} + K_{R_1} + U_{R_1} + K_{P_1} &= E_1 \\ &= E_2 = K_{L_2} + U_{L_2} + K_{R_2} + U_{R_2} + K_{P_2} \end{aligned} \quad (3)$$

We will measure the gravitation potential energy with respect to the ground. You don't need to write U_p because it's constant throughout this process. The length of the rope is constant so $|v_L| = |v_R| = V$. The rope doesn't slip on the pulley rim so $V = R\omega$. Where ω represents angular velocity of the pulley. The problem asks for V so use $\omega = \frac{V}{R}$: $K_P = \frac{1}{2}I_P\omega^2 = \frac{I_P V^2}{2R^2}$.

$$V_1 = 0; \quad y_{R_1} = 0$$

$$V_2 = V; \quad y_{L_2} = 0$$

$$y_{L_1} = y_{R_2} := y_0$$

Substituting in (3):

$$0 + m_L g y_0 + 0 + 0 + 0 = \frac{1}{2} m_L V^2 + 0 + \frac{1}{2} m_R V^2 + m_R g y_0 + \frac{I_P V^2}{2R^2}$$

$$\frac{1}{2} V^2 (m_L + m_R + \frac{I_P}{R^2}) = g y_0 (m_L - m_R)$$

$$\boxed{V = \sqrt{\frac{2g y_0 (m_L - m_R)}{m_L + m_R + \frac{I_P}{R^2}}}}$$

Plugging in the numbers given in the problem, you'll get $V = 2.81 \text{ m/s}$.

b) Please refer to figure 1 for the diagram of forces.

Using the rotational analogue of Newton's second law:

$$\sum \tau_z = I\alpha_z$$

we get

$$T_2R - T_1R = I_P\alpha = I_P\dot{\omega} = I_P\frac{\dot{v}}{R} = I_P\frac{a}{R}$$

$$T_2 - T_1 = \frac{I_P}{R^2}a \quad (4)$$

Writing Newton's law $\sum F_y = ma_y$ (Let's omit the subscript y afterwards)

$$a_R = -a_L = a \quad (5)$$

$$T_1 - m_Rg = m_Ra \implies T_1 = m_R(a + g) \quad (6)$$

$$T_2 - m_Lg = -m_La \implies T_2 = m_L(g - a) \quad (7)$$

Combining (4) with (6) and (7):

$$m_L(g - a) - m_R(g + a) = \frac{I_P}{R^2}a$$

$$a = g \frac{m_L - m_R}{m_L + m_R + \frac{I_P}{R^2}}$$

NOTE: The pulley does not contribute to the imbalance, but only increases the inertia.

c) In the case $I_P = 0$ we get the familiar result of :

$$a_0 = g \frac{m_L - m_R}{m_L + m_R}$$

d) Let's denote

$$\frac{I_P}{R^2} := \frac{m_P}{2}$$

The result of part **b** shows that, the pulley does not contribute to the imbalance, but only increases the inertia. This suggests the 1st configuration shown in the figure 2.

There are of course other possibilities:

By writing a as:

$$a = g \frac{(m_L + m_P/4) - (m_R + m_P/4)}{(m_L + m_P/4) + (m_R + m_P/4)}$$

Which is the familiar result of two masses for ideal pulley problem with masses: $m_1 = m_L + \frac{m_P}{4}$, $m_2 = m_R + \frac{m_P}{4}$. You can set up the 2nd configuration shown in the figure 3.

4) **10.35**

a) Using the definition:

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \vec{\mathbf{r}} \times m \vec{\mathbf{v}}$$

The magnitude of $\vec{\mathbf{L}}$ is:

$$L = rp \sin \phi$$

where ϕ is the angle between the two vectors $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$. the direction of $\vec{\mathbf{r}} \times \vec{\mathbf{p}}$ is perpendicular to the plane of the two vectors being multiplied, as given by the right-hand rule. Choose the z direction to be perpendicular to the plane shown in the figure and pointing *up*.

Plugging in the numbers you'll get:

$$\boxed{\vec{\mathbf{L}} = -115 \text{ kgm}^2/\text{s} \hat{\mathbf{z}}}$$

b) Using the definition:

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \vec{\mathbf{r}} \times m \vec{\mathbf{g}}$$

With the same method you'll get:

$$\vec{\tau} = +125 \text{ kgm}^2/\text{s}^2 \hat{\mathbf{z}}$$

This is the rate of change of $\vec{\mathbf{L}}$,

$$\boxed{\frac{d\vec{\mathbf{L}}}{dt} = \vec{\tau} = +125 \text{ kgm}^2/\text{s}^2 \hat{\mathbf{z}}}$$

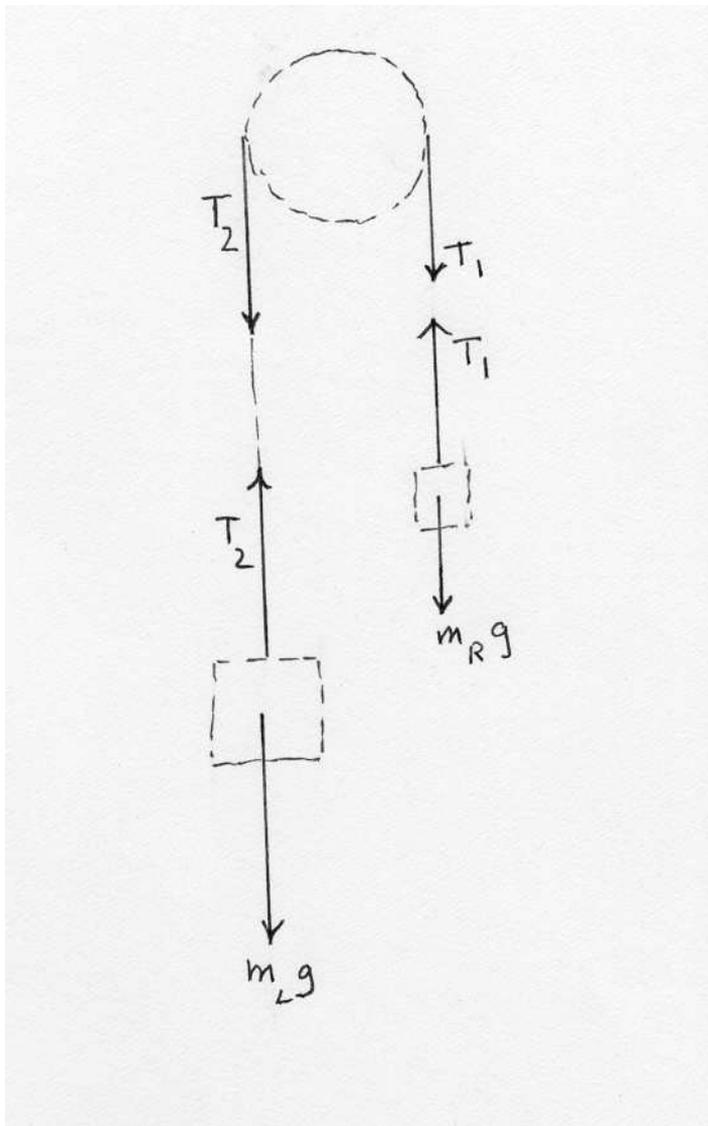


Figure 1: 9.80 Force Diagram

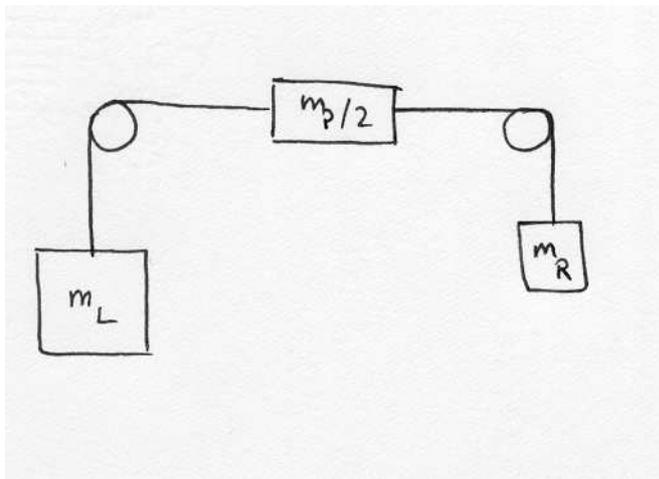


Figure 2: 9.80 part d #1

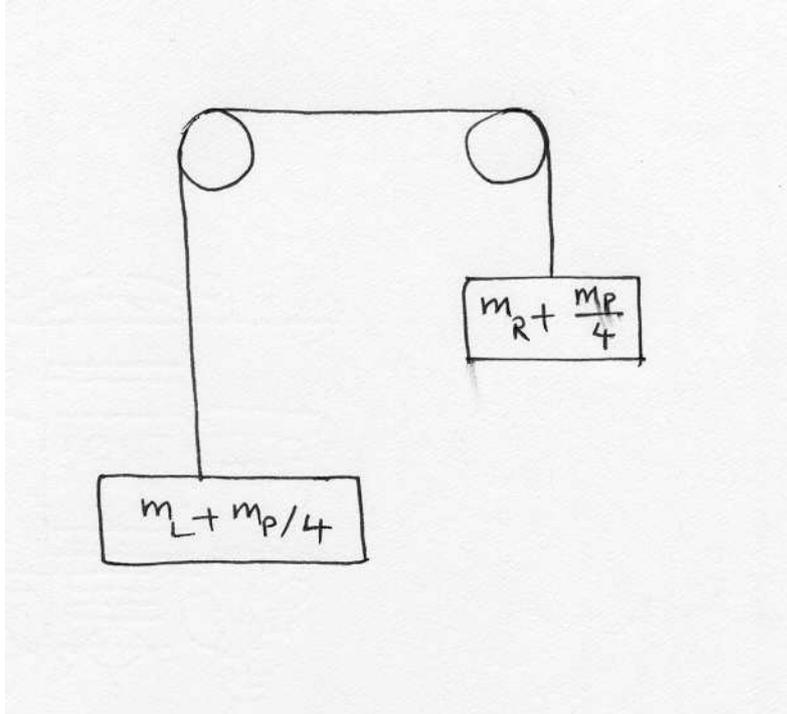


Figure 3: 9.80 part d #2