

## Relativity.

### 1) 37.20 Relativistic Relative Velocity

Write differential form of Lorentz transformation:

$$x' = \gamma(x - ut)$$

$$t' = \gamma(t - ux/c^2)$$

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

which is:

$$dx' = \gamma(dx - udt) \tag{1}$$

$$dt' = \gamma(dt - udx/c^2) \tag{2}$$

Divide (2) by (1) you'll get ( $dx/dt = v_x$ ):

$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} \tag{3}$$

Where  $u$  is the velocity of the moving frame. In this problem you want to know the relative velocity so it means that your moving frame has the velocity of one of them and in this frame you want to know velocity of the other:

$$v_x = 0.9520c$$

$$u = -0.9520c$$

Replace this into equation (3) you'll get:

$$v'_x = \frac{0.9520c - (-0.9520c)}{1 - (0.9520c)(-0.9520c)/c^2} = 0.9988c$$

$$\boxed{v_{rel} = 0.9988c}$$

## 2) **37.44** Creating a Particle

a) The total energy of a particle is:

$$E = m\gamma c^2$$

Where  $m$  is the rest mass and

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \quad (4)$$

*Before collision* - incident two protons -:

$$\sum E = E_P + E_P = 2m_P\gamma_P c^2$$

*After collision* - two protons + new particle which are at rest -:

$$\sum E' = E'_P + E'_P + E'_{\eta^0} = (2m_P + m_{\eta^0})c^2$$

From the *energy conservation*:

$$\sum E = \sum E'$$

we get:

$$\gamma_P = \frac{2m_P + m_{\eta^0}}{2m_P}$$

With the numbers given:

$$\boxed{\gamma_P = 1.29}$$

From (4):

$$u/c = \sqrt{1 - \frac{1}{\gamma^2}}$$

With the  $\gamma_P$  we found:

$$u_P = 0.632c$$

**b)** The kinetic energy is the difference of total energy and the rest energy:

$$K_P = (\gamma_P - 1)m_P c^2$$

Plug in the numbers, you'll get

$$K_P = 0.436 \times 10^{-10} \text{ J}$$

Divide it by  $e = 1.60 \times 10^{-19}$  you'll get the energy in " eV" units:

$$K_P = 273 \text{ MeV}$$

**c)** Similarly you get:

$$m_{\eta^0} c^2 = 550 \text{ MeV}$$

**d)** Apart from the "error" of keeping 3 significant figures you may have noticed that the rest energy of  $\eta^0$  is the sum of both protons energy,  $K_P$ . This is indeed expected from the 1) the definition of kinetic energy (difference of total energy and rest energy) 2) conservation of energy 3) the final masses are at rest.

### 3) **37.71** The Pole and Barn Paradox

The reason there is an apparent paradox is that the notion of fitting the moving pole into the barn contains an implicit assumption that the length of the pole (or the barn) is not changed by their motion. Actually the length of a moving object contains the idea of simultaneity - it is the distance between the ends at the same instant of time. Simultaneity is destroyed by special relativity - two people who have a relative velocity will not agree on what is simultaneous! Hence they can (and do) disagree about the relative size of pole and barn.

This paradox is often sharpened by imagining that the doors of the barn are suddenly and simultaneously closed by the farmer. Seeing the door closing, the runner stops. Then does his pole fit? This introduces an extraneous element into the problem: when the runner stops, only the middle of the pole where he is holding it stops - so when do the ends stop? In reality the pole will break. But if the ends magically stopped simultaneously in the frame of the runner, the farmer would see the rear end of the pole stop while the front end kept going. When the pole, which to the farmer appeared shorter than the barn while moving, stops in this manner, it will be longer than the barn. This confirms the original opinion of the runner (who thought the barn much shorter than his pole), but only because the ends stopped simultaneously in the frame of the runner.

#### 4) **37.72** Fizeau Experiment

From the relativistic addition of velocities discussed in the book and derived here (problem 1) we can apply them here. Assume light has velocity  $c/n$  in the direction of water's velocity,  $V$ :

$$v = \frac{\frac{c}{n} + V}{1 + \frac{(c/n)V}{c^2}} = \frac{\frac{c}{n} + V}{1 + \frac{V}{nc}} = (c/n+V)\left[1 - \frac{V}{nc} + \mathcal{O}(V^2/c^2)\right] = c/n+V - V/n^2 + \mathcal{O}(V^2/c^2)$$

Therefore:

$$\boxed{v \cong (c/n) + (1 - 1/n^2)V}$$

Where we threw out  $\mathcal{O}(V^2/c^2)$  assuming it's very small. This is the same as the given formulae with

$$k = 1 - \frac{1}{n^2}$$

With the  $n=1.333$  you'll get:

$$\boxed{k = 0.4372}$$

which is incredibly close to the experimental value.