

Physics 8.01 Assignment 1

1.1

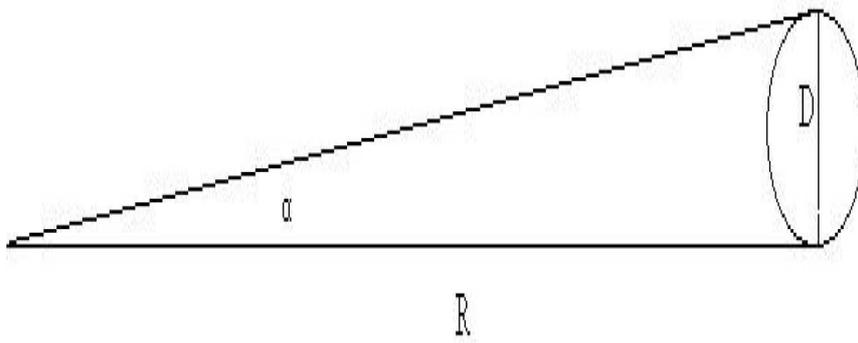


Figure 1: sunset

We treat the sun as an object that is rotating about the earth with uniform angular velocity, and that rises and sets vertically (this is assured by being on the equator on the equinox). We first need to know the angle sun must rotate through during the sunset (α). If we can then get the angular speed of the sun as we see it we can get the time it takes for the sunset.

$$\alpha \text{ is small} \Rightarrow \alpha \simeq \tan \alpha = \frac{D}{R}$$

$$\begin{aligned}D &= 13.92 \times 10^8 m \\R &= 1.50 \times 10^{11} m \\ \alpha &= 9.28 \times 10^{-3} \text{radians}\end{aligned}$$

The angular velocity follows from fact sun will rotate 2π radians during 24 hours:

$$\omega_{sun} = \frac{2\pi}{24 \times 60 \times 60} = 7.27 \times 10^{-5} \frac{\text{radians}}{\text{s}}$$

The time is the angular size of the sun divided by the angular velocity, analogous to the case with distance and linear velocity.

$$T_{sunset} = \frac{\alpha}{\omega_{sun}} = \frac{9.28 \times 10^{-3}}{7.27 \times 10^{-5}} = 128s \approx 2min.$$

1.2

a)

Dimensional consistency of

$$v : \frac{m}{s}$$

$$x : m$$

$$a : \frac{m}{s^2}$$

$$\text{RHS: } \frac{m^2}{s^2}$$

$$\text{LHS: } \frac{m^2}{s^2} + m \times \frac{m}{s^2} \propto \frac{m^2}{s^2} \quad \checkmark$$

b)

Both LHS and RHS have the same dimensions. These are now multiplied by mass with units kg .

$$kg \times \frac{m^2}{s^2} : \text{Joules}$$

1.3

a)

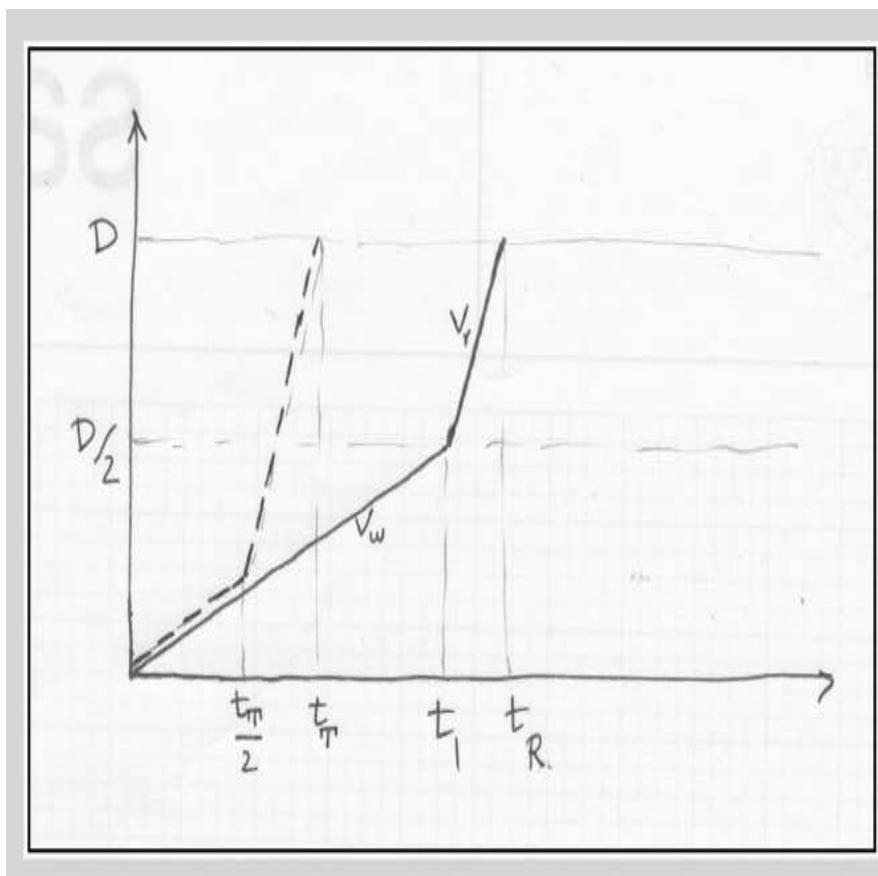


Figure 2: Problem 3

b)

The Person who spends ***less time walking and more time running*** will win. Initially they walk along together, but Tim starts running sooner because he has only to walk 1/2 the time it will take him, not 1/2 the distance which makes Rick take longer to reach D as the graph shows (note the specific times designated on the graph).

c)

Rick

$$t_1 = \frac{D/2}{V_w} \quad (t_1 / : \text{the time that Rick reaches } D/2)$$

$$t_R - t_1 = \frac{D/2}{V_r}$$

$$t_R = \frac{D}{\frac{2V_w V_r}{V_r + V_w}}$$

d)

$$\bar{V}_R = \frac{D}{t_R} = \frac{2V_r V_w}{V_r + V_w}$$

e)

Tim

$$\frac{t_T}{2} V_r + \frac{t_T}{2} V_w = D$$

$$t_T = \frac{D}{(V_r + V_w)/2}$$

1.4

I:

$\bar{a} = \frac{v_f - v_i}{\Delta t}$ Definition of average acceleration

$v_i = 0$ (starts at rest)

$\Delta t = 6s$

$v_f = 90m/s$

II:

Distance \sim area under $v(t)$ curve

III:

$a_{max} = 45 \Rightarrow D_{min}$ archived by a straight line from a point on t axis to $v_f = 90m/s$ at $t_f = 6$. $\Delta t = 90/45 = 2 \Rightarrow$ starts moving at 4.

Any line before 4 to 90 m/s has a lower slope so we wouldn't violate the a_{max} condition. To get twice the distance I look for a line that has twice the area under it. The *easiest* choice is the line from 2s to 90 m/s. The triangle under it has twice the base and the same altitude, hence twice the area.

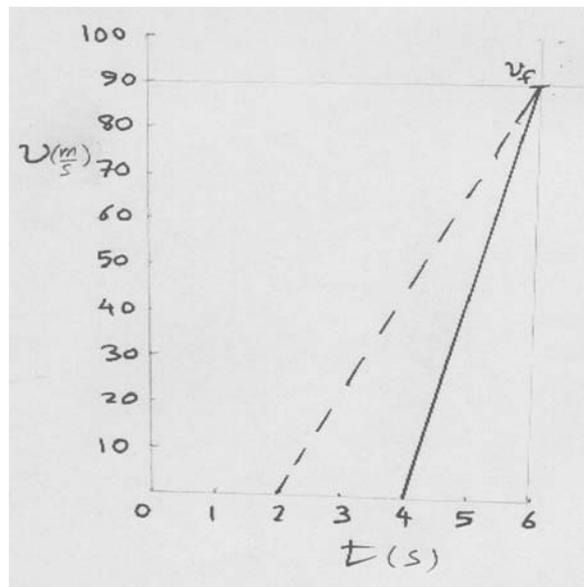


Figure 3: Problem 4

1.5

$$a = -10m/s^2$$

$$\Delta t = 2.0$$

g: hitting the ground r: reflection from the ground

$$f = 0.71$$

These are the three basic kinematic equations written with arbitrary time for the initial conditions (instead of the conventional $t=0$).

$$\text{Eq. 1: } x(t) = x(t_0) + v(t')(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

$$\text{Eq. 2: } v(t) = v(t_0) + a(t - t_0)$$

$$\text{Eq. 3: } v^2(t) - v^2(t_0) = +2a[x(t) - x(t_0)]$$

We should pay attention that t_0 in these equations is the initial time, not necessarily zero. So for each region as you see in the picture there are different initial conditions for v_0 and t_0 . We should first write the appropriate $y(t)$ for each region. Setting the two $y(t)$'s for the two balls gives us the time of the collision time t_c .

For the first ball :

$$\text{Eq.1} \Rightarrow t_{g1} = \sqrt{2} = 1.41s$$

$$\text{Eq.3} \Rightarrow v_{g1} = \sqrt{200} = 14.1m/s$$

$$v_{r1} = fv_{g1} = 10m/s$$

$$v_{g2} = -v_{r1} \wedge \text{Eq.2} \Rightarrow t_{g2} - t_{g1} = \frac{2 \times 10}{10} = 2.0$$

Because the time it takes that second ball hit the ground is again 1.41 and because $2.0 + 1.41 = 2.0 + 1.41!$ The second hits the first ball when the first ball hits the ground itself.

However, if you change Δt say 1.8 you would get a different answer. I also write the *systematic* way of solving this problem:

Between the first hit and the second one ball #1 is at $y_1(t)$ and ball 32 is at $y_2(t)$

$$y_1(t) = 0 + 14.1f(t - 1.41) - 5(t - 1.41)^2$$

$$y_2(t) = 10 - 5(t - 2)^2$$

Collision time: t_c

$$y_2(t_c) = y_1(t_c)$$

Solving this with will give you 3.41 for \tilde{t} as expected! (But you have to be careful that your solution is in the region of validity of both y_1 and y_2 .)

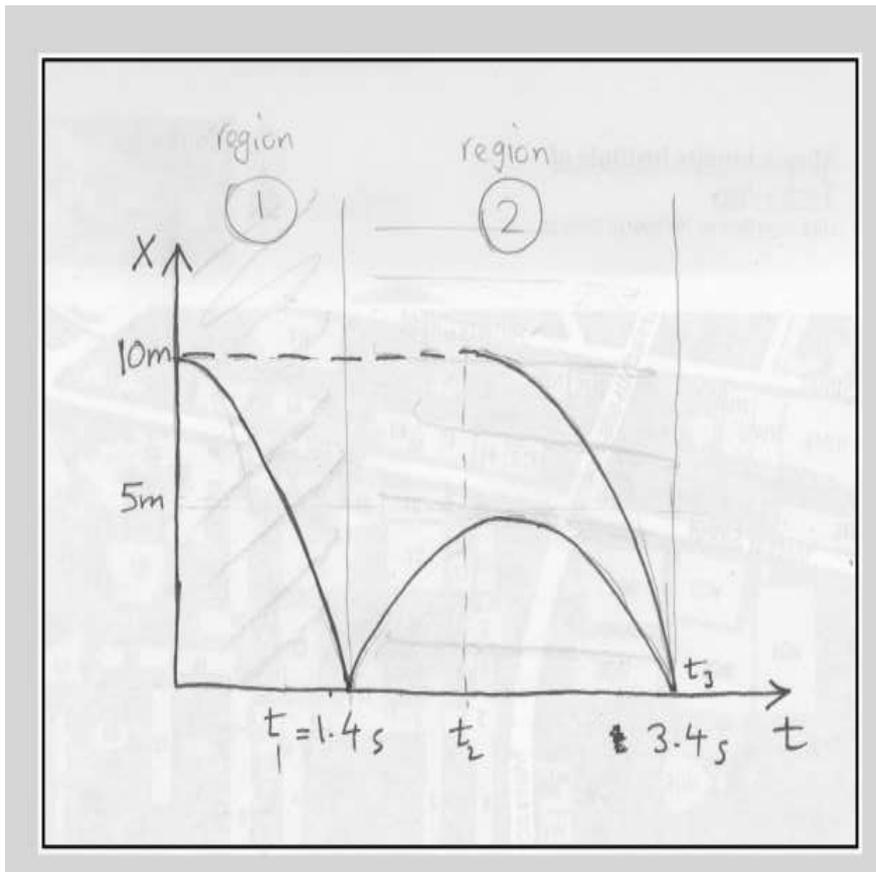


Figure 4: Collision