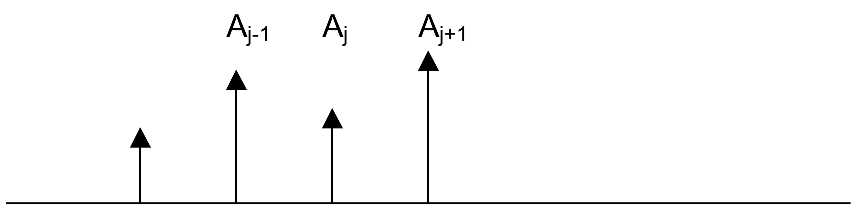
Classnote – March 1, 2004

The Levelized Cost of Production and the Annual Carrying Charge Factor

First, define levelized cash flows:

1. Discrete cash flows

Consider the non-uniform cash flow series:

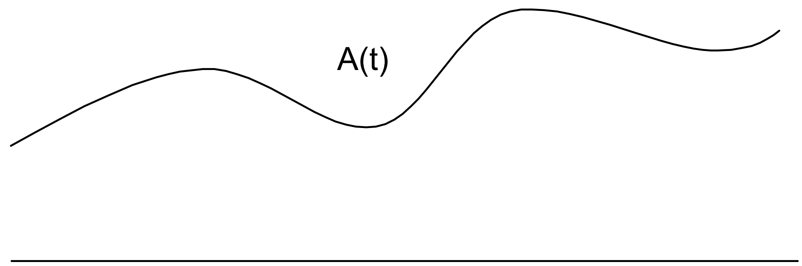


We can define an 'equivalent levelized' cash flow, A_L, such that the uniform series PW is equal to the PW of the actual series:

$$\sum_{n=1}^{N} A_L\,(P/F,i,n) = \sum_{n=1}^{N} A_n(P/F,i,n)$$

$$A_L = \frac{\sum_{n=1}^{N} A_n(P/F,i,n)}{\sum_{n=1}^{N} (P/F,i,n)}$$

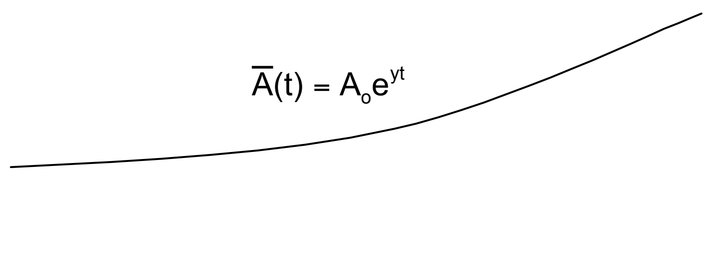
2. Continuous cash flow rate



We obtain, by analogy,

$$\bar{A}_L = \frac{\int_0^T A_o e^{-rt}\,dt}{\int_0^T e^{-rt}\,dt}$$

For the special case of an exponential increase in $\bar{A}$



$$\bar{A}_L = \frac{\int_0^T A_o e^{(y-r)t}\,dt}{\int_0^T e^{-rt}\,dt} = A_o \frac{r}{r-y}\left[\frac{1-e^{(y-r)T}}{1-e^{-rT}}\right]$$

And expanding the exponentials as Taylor series and retaining terms through second order, yielding, to first order,

$$\frac{\overline{A}_L}{A_o} \approx \frac{1 - \frac{(r - y)}{2}T + ...}{1 - \frac{rT}{2} + ...} \approx 1 + {yT}/{2}$$

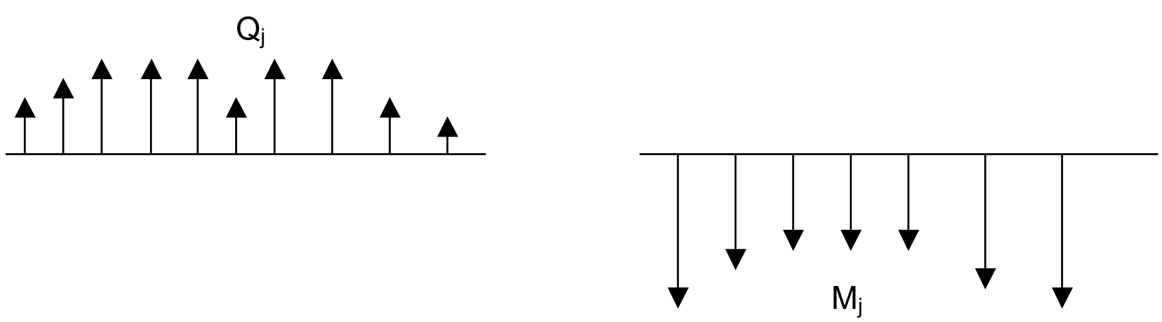
Levelized Unit Cost of Product

The lifetime levelized cost, the constant cost that is equivalent in a present worth sense to the relevant time-varying cost, is a useful benchmark for comparisons of facilities which might otherwise be difficult to compare (e.g., windmills versus gas turbines.)

Example – manufacturing facility

Consider a factory with initial investment cost I_o at t=0, which operates for N years after which it is salvaged at I_N.

Suppose that during this period the factory produces Q_j units per year at an annual operating cost of M_j dollars per year.



What is the levelized cost of a unit of product – i.e., the uniform cost which, if recovered on every unit produced, will provide lifetime revenues just sufficient to cover all capital and operating costs?

Case I: No Taxes

Write the levelized unit cost, c, as the sum of operating and capital components:

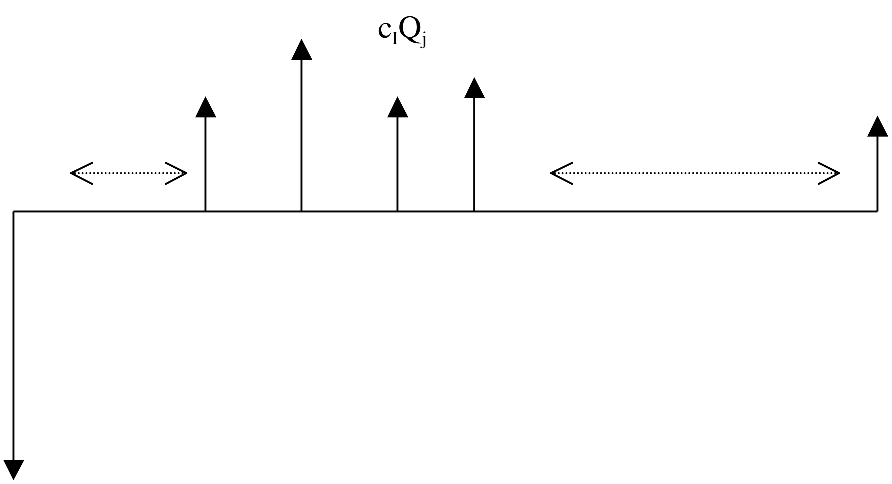
$$c = c_M + c_I$$

1. Operating cost component, c_m

$$\sum_{j=1}^{N} c_M Q_j (P/F,i,j) = \sum_{j=1}^{N} M_j (P/F,i.j)$$

$$c_M = \frac{\sum_{j=1}^{N} M_j (P/F,i.j)}{\sum_{j=1}^{N} Q_j (P/F,i,j)}$$

2. Capital cost component, c_I



$$I_o - \frac{I_N}{(1+i)^N} = \sum_{j=1}^{N} c_l Q_j (P/F,i,j) \qquad (1)$$

Define: Average (levelized) production rate Q_L

$$\sum_{j=1}^{N} Q_L (P/F,i,j) = \sum_{j=1}^{N} Q_j (P/F,i,j)$$

$$Q_L = \frac{\sum_{j=1}^{N} Q_j (P/F,i,j)}{(P/A,i,N)}$$

and substituting for Q_L in (1)

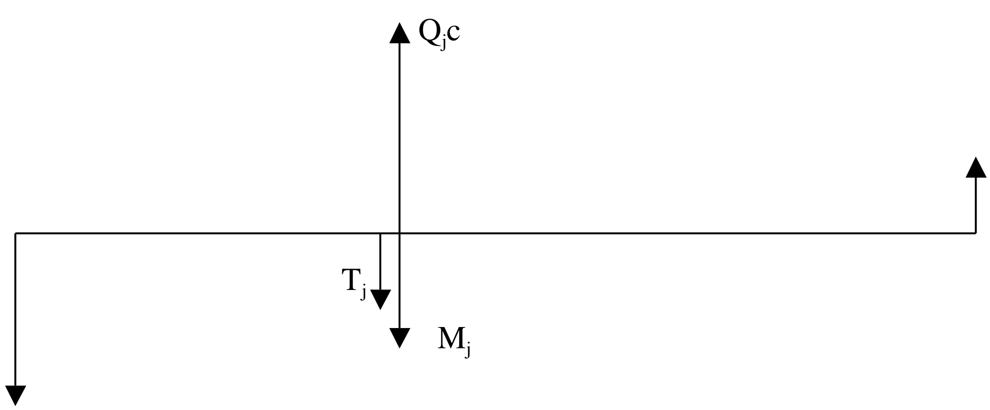
$$c_l = \frac{1}{Q_L (P/A,i,N)} \left[I_o - I_N (P/F,i,N) \right]$$

$$= \frac{1}{Q_L} [I_o (A/P,i,N) - I_N (A/F,i,N)]$$

i.e.,

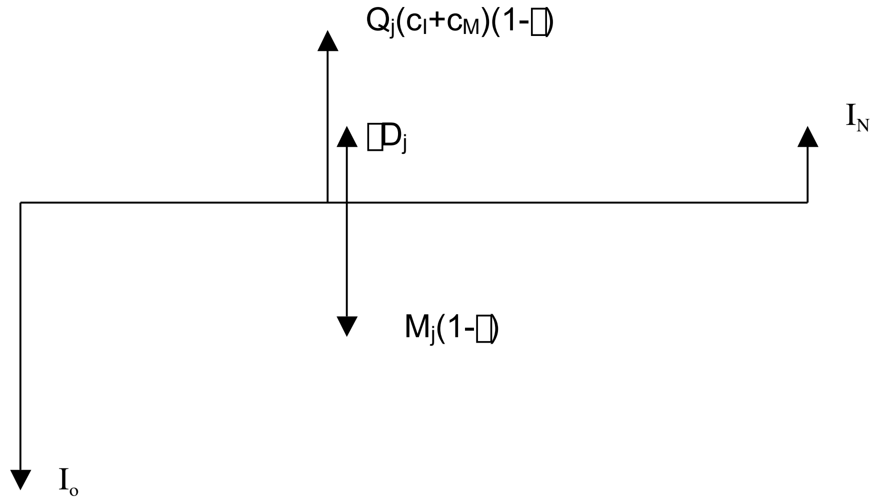
$$\text{levelized unit cost} = \frac{1}{\text{levelized production rate}} \left[I_o \times \text{capital recovery factor} - I_N \times \text{sinking fund fa} \right.$$

Case II: With Taxes

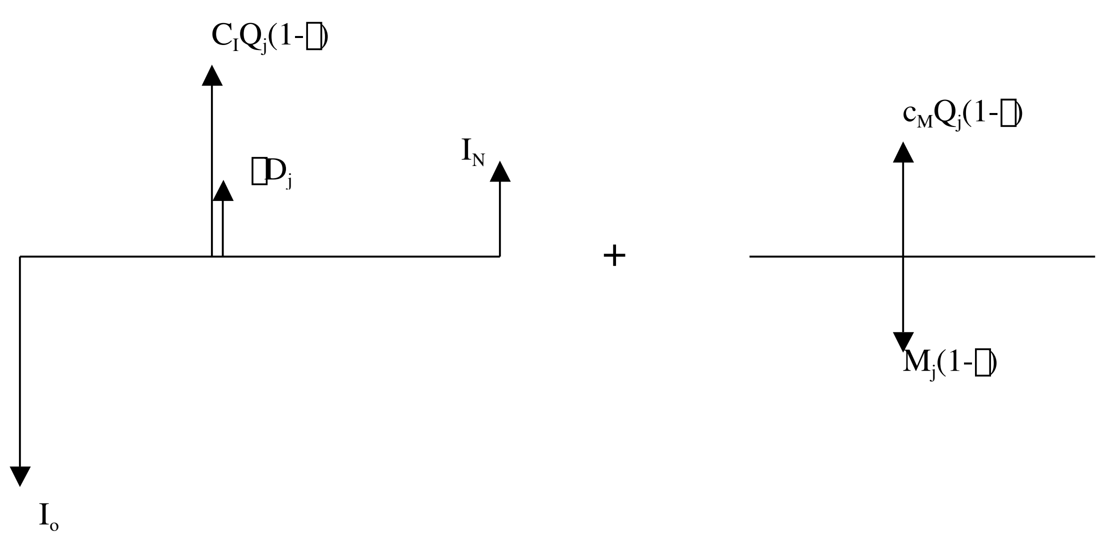


As before, write $c = c_m + c_I$

Next, transform the cash flow problem into an equivalent tax-implicit problem



And, decomposing into capital and operating components,



Then solve separately for c_I and c_M.

a. <u>c_M</u>

$$(1-\tau)\sum_{j=1}^{N} c_M Q_j (P/F,x,j) = (1-\tau)\sum_{j=1}^{N} M_j (P/F,x,j)$$

$$c_M = \frac{\sum_{j=1}^{N} M_j (P/F,x,j)}{\sum_{j=1}^{N} Q_j (P/F,x,j)}$$

b. <u>c_I</u>

$$(1-\tau)\sum_{j=1}^{N} c_I Q_j (P/F,x,j) = I_o - I_N (P/F,x,N) - \tau\sum_{j=1}^{N} D_j (P/F,x,j)$$

For the case of straight line depreciation:

$$D_j = \frac{I_o - I_N}{N}$$

and

$$c_I = \frac{1}{1-\tau}\left[\frac{I_o - I_N (P/F,x,N) - \tau\frac{(I_o - I_N)}{N}(P/A,x,N)}{\sum_{j=1}^{N} Q_j (P/F,x,j)}\right] \qquad (2)$$

as before, define a levelized production rate, Q_L

$$Q_L = \frac{\sum_{j=1}^{N} Q_j (P/F,x,j)}{\sum_{j=1}^{N} (P/F,x,j)} = \frac{\sum_{j=1}^{N} Q_j (P/F,x,j)}{(P/A,x,N)}$$

And substituting in (2) above

$$c_I = \frac{1}{(1-\tau)Q_L}\left[I_o(A/P,x,N) - I_N(A/F,x,N) - \tau\frac{I_o - I_N}{N}\right]$$

$$= \frac{I_o}{Q_L}\left\{\frac{1}{1-\tau}\left[(A/P,x,N) - \frac{\tau}{N}\left(1-\frac{I_N}{I_o}\right) - \frac{I_N}{I_o}(A/F,x,N)\right]\right\} \qquad (3)$$

$$c_I = \frac{I_o}{Q_L}\phi$$

where ϕ, the term in square brackets, is the <u>annual carrying charge factor</u> (with units of yr^{-1})

<u>Notes</u>

1. I_o is the PW of the initial investment <u>at the start of operation</u>.
2. In a tax-free environment (τ=0), the annual carrying charge factor ϕ reduces to the capital recovery factor, adjusted for NSV.
3. In the limit of large N ($N \to \infty$)

   $$(A/P,x,N) = \frac{x(1+x)^N}{(1+x)^N - 1} \to x$$

   $$(A/F,x,N) = \frac{x}{(1+x)^N - 1} \to 0$$

   $$\phi(N \to \infty) = \frac{x}{1-\tau}$$

   This is a good approximation for large N.
4. The form of the annual capital charge factor in equation (3) applies to the case of straight-line depreciation. Equivalent expressions can be derived for other depreciation schedules.