Problem Set 5 Solutions

1)       MIT is considering building a new car park near Kendall Square.  No university funds are available (overhead rates are under pressure) and the new facility would have to pay for itself from parking fees over a 15 year period.  A minimum rate of return of 10% (before taxes) is required for the project.  The architect has developed four alternative designs for different numbers of parking levels in the structure.  Based on the income and cost data in the Table below, how many levels should be built?

(NOTE:  Prepare your analysis on a before-tax basis.)

| Number of Levels | Cumulative construction costs ($) | Annual operating costs  ($) | Income per year ($) |
|---|---|---|---|
| 1 | 600,000 | 35,000 | 100,000 |
| 2 | 2,200,000 | 60,000 | 350,000 |
| 3 | 3,600,000 | 80,000 | 570,000 |
| 4 | 4,800,000 | 105,000 | 750,000 |

Solution:

Since the lifetime of each facility is 15 years, we can use the PW criterion to compare the alternatives.

We can construct the following generic cash flow diagram
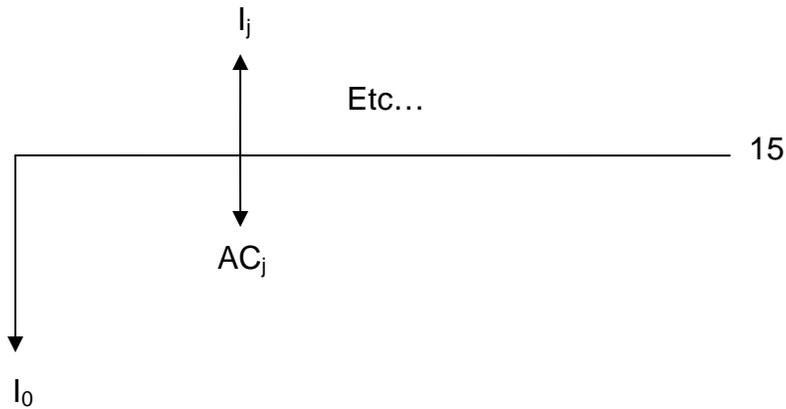
We can calculate the present worth based on the following formula

$$PW = -I_o + (I_j - AC_j)(P/A,10\%,15)$$

The present worth for each option is calculated below

$$PW(1) = -\$600,000 + (\$100,000 - \$35,000)(P/A,10\%,15) = -\$105,605$$
$$PW(2) = -\$2,200,000 + (\$350,000 - \$60,000)(P/A,10\%,15) = \$5,763$$
$$PW(3) = -3,600,000 + (\$570,000 - \$80,000)(P/A,10\%,15) = \$126,979$$
$$PW(4) = -4,800,000 + (\$750,000 - \$105,000)(P/A,10\%,15) = \$105,921$$

Since PW(3) is the greatest it is to our advantage to build the 3 level parking garage.

2)      A municipality is trying to establish a policy on how long to keep economy passenger cars that are used for official business.  The purchase cost of a car at fleet discount is $12,000.  Annual costs for insurance, oil, and repairs for 20,000 miles of annual use are $2,000 for a new car and increase by $100/year with the age of the car.  Fuel consumption is not perceived to change with age and is therefore excluded from the analysis.  It is recognized that as a vehicle becomes older it suffers more breakdowns, causing employees to become less productive.  The average loss of employee productivity is estimated to be 4 days/year for a new car and to increase by 2 days/year with the age of the car.  The average payroll cost of any employee likely to use the car is $200/day.  After 8 years spare parts become difficult to obtain, and the downtimes of the vehicle become unacceptably long and frequent.  Salvage values as a function of age up to 8 years are given below.

Salvage Value Table

| N | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| S | 9000 | 6750 | 5060 | 3800 | 2850 | 2140 | 1600 | 1200 |

Using i = 10%, find the economic life of such a car.  Ignore income taxes, inflation and technological improvements.

Solution:

To calculate the economic lifetime of the car we must use the levelized annual cost (LAC) criterion.

From the problem we are given the following information

$$I_0 = \$12,000$$
$$M = M_0 + M_n(n-1) = \$2,000 + (n-1)\$100$$
$$L = 4(\$200) + (n-1)(2)(\$200) = L_0 + L_n(n-1) = \$800 + (n-1)\$400$$

We can now calculate the levelized annual cost based on the investment, the operating costs, and the productivity loss.  For the gradient terms we use the annuity factor for a gradient increase.

Let us first list the annuity factors we will need to use to complete this problem

$$(A/F, i\%, N) = \frac{i}{(1+i)^N - 1}$$

$$(A/P, i\%, N) = \frac{i(1+i)^N}{(1+i)^N - 1}$$

$$(A/G, i\%, N) = (A/P, i\%, N)(P/G, i\%, N) = (A/P, i\%, N)\left(\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right)$$

$$= \left(\frac{i(1+i)^N}{(1+i)^N - 1}\right)\left(\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right)$$

The levelized cost can be calculated as thus

$$LAC = I_0(A/P, 10\%, N) - I_N(A/F, 10\%, N) + M_0 + M_n(A/G, 10\%, N) + L_0 + L_n(A/G, 10\%, N)$$

$$LAC = \$12{,}000(A/P, 10\%, N) - I_N(A/F, 10\%, N) + \$2{,}800 + \$500(A/G, 10\%, N)$$

The following page shows a table which calculates the LAC given that the car operates for any number of years between 1 and 8

| N | (A/P,10%,N) | (A/F,10%,N) | (A/G,10%,N) | I0 | IN | G | C | LAC |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.1000 | 1.0000 | 0.0000 | $12,000.00 | $9,000.00 | $500.00 | $2,800.00 | $7,000.00 |
| 2 | 0.5762 | 0.4762 | 0.4762 | $12,000.00 | $6,750.00 | $500.00 | $2,800.00 | $6,738.10 |
| 3 | 0.4021 | 0.3021 | 0.9366 | $12,000.00 | $5,060.00 | $500.00 | $2,800.00 | $6,564.95 |
| 4 | 0.3155 | 0.2155 | 1.3812 | $12,000.00 | $3,800.00 | $500.00 | $2,800.00 | $6,457.44 |
| 5 | 0.2638 | 0.1638 | 1.8101 | $12,000.00 | $2,850.00 | $500.00 | $2,800.00 | $6,403.81 |
| 6 | 0.2296 | 0.1296 | 2.2236 | $12,000.00 | $2,140.00 | $500.00 | $2,800.00 | $6,389.71 |
| 7 | 0.2054 | 0.1054 | 2.6216 | $12,000.00 | $1,600.00 | $500.00 | $2,800.00 | $6,407.02 |
| 8 | 0.1874 | 0.0874 | 3.0045 | $12,000.00 | $1,200.00 | $500.00 | $2,800.00 | $6,446.63 |

The Economic Lifetime is given by the option where the LAC is the smallest, in this case the economic lifetime is 6 years.

3) A new automobile with a list price of $10,000 is being considered as a replacement for a used automobile presently owned by the company. The new automobile can be acquired by trading in the old one plus $6000 cash. Alternatively, the company can sell outright the present auto for $2000 and purchase the new car outright for $8000 cash. Current remaining book value (unallocated cost) of the present auto is $3000.

   a. If AEC of the present auto is based on its remaining book value, the resulting AEC tends to be (a) overstated, (b) understated, or (c) correctly stated

   b. If AEC of the new automobile is based on the $10,000 list price and if AEC of the used auto is based on the trade-in allowance of $4000, the study tends to be biased in favor of (a) the proposed, (b) the present, or (c)neither automobile.

   c. If AEC of the new automobile is based on the $6000 cash paid and if AEC of the used auto is based on the fact that because "already owned" it requires zero outlay and therefore has zero present basis, the study tends to be biased in favor of (a) the proposed, (b) the present, or (c) neither auto.

Solution:

Assuming that AEC stands for the Annual Equivalent Capital Cost, we can evaluate the options given the interest rates and tax rates, however, we are only asked to evaluate the trends in the AEC based on the methods of calculation.

The AEC is calculated using the following formula for an initial investment of $I_0$ and a salvage value of $I_N$ over an economic lifetime of N, without accounting for depreciation allowances towards tax deductions.

$$AEC = (A/P, i\%, N)\left[I_0 + I_N (P/F, i\%, N)\right]$$

(a) From the outsider's point of view to decide between investing in the new auto, one would have to consider purchasing the defending auto at the book value and calculating the AEC. This is compared to the outsider purchasing a new auto at the list price and computing the AEC.

If we assume that the new and old automobiles are effectively the same, where no new technology has been employed, using the BV of the automobile as the market price does not bias the decision in either direction.

(b) Since the trade-in allowance of the old automobile is higher than the BV, it is effectively making the defender more expensive to the outsider relative to the case in (a) thus biasing the decision towards purchasing the new automobile.

Equivalently, this option can be thought of from the insider's perspective of making the new automobile cheaper.

(c) If it is assumed that because the vehicle is 'already owned', it has zero present basis, then using the AEC as a decision criterion is an invalid technique. It will bias the decision maker in favor of the present, because it assumes that the AEC of the present option of the automobile does not include the investment cost, whereas the AEC of the proposed is based on a $6000 purchase price. It is also noteworthy that the $6000 purchase price is obtained from combining the salvage value of the defender with the investment cost of the defender, thus shifting part of the worth of the defender to the worth of the challenger.

4) You have calculated the economic life of a facility. Now you are asked to indicate for each factor below whether an increase in that factor would *shorten* or *lengthen* that economic life.
   a. The amount of increase in the positive gradient pattern of year-by-year costs
   b. Rate of return requirement
   c. Estimated future salvage values
   d. First cost of the facility
   e. Rate of technological progress
   f. Applicable income tax rate
   g. Emphasis on prestige and image of the firm

Solution:

We will use problem 2 as a prototype to answer these questions by changing the values in the spread sheet and seeing the effect on the economic lifetime.

The following is the LAC equation as shown in problem 2

$$LAC = I_0(A/P,10\%,N) - I_N(A/F,10\%,N) + M_0 + M_n(A/G,10\%,N) + L_0 + L_n(A/G,10\%,N)$$
$$LAC = \$12,000(A/P,10\%,N) - I_N(A/F,10\%,N) + \$2,800 + \$500(A/G,10\%,N)$$

(a) The gradient pattern is in the Mn and Ln terms. If these are increased the economic lifetime changes as follows mathematically

| N | (A/P,10%,N) | (A/F,10%,N) | (A/G,10%,N) | I0 | IN | G | C | LAC |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.1000 | 1.0000 | 0.0000 | $12,000.00 | $9,000.00 | $1,000.00 | $2,800.00 | $7,000.00 |
| 2 | 0.5762 | 0.4762 | 0.4762 | $12,000.00 | $6,750.00 | $1,000.00 | $2,800.00 | $6,976.19 |
| 3 | 0.4021 | 0.3021 | 0.9366 | $12,000.00 | $5,060.00 | $1,000.00 | $2,800.00 | $7,033.23 |
| 4 | 0.3155 | 0.2155 | 1.3812 | $12,000.00 | $3,800.00 | $1,000.00 | $2,800.00 | $7,148.03 |
| 5 | 0.2638 | 0.1638 | 1.8101 | $12,000.00 | $2,850.00 | $1,000.00 | $2,800.00 | $7,308.87 |
| 6 | 0.2296 | 0.1296 | 2.2236 | $12,000.00 | $2,140.00 | $1,000.00 | $2,800.00 | $7,501.49 |
| 7 | 0.2054 | 0.1054 | 2.6216 | $12,000.00 | $1,600.00 | $1,000.00 | $2,800.00 | $7,717.83 |
| 8 | 0.1874 | 0.0874 | 3.0045 | $12,000.00 | $1,200.00 | $1,000.00 | $2,800.00 | $7,948.87 |

We've increased the gradient from 500 to 1000 and we can see that the economic lifetime shortens from 6 to 2 years. This is because the LAC increases as the higher costs in subsequent years are added.

(b) The rate of return requirement.  The rate of return is 10%, if we increase the RR to 15%, the LAC table changes as follows.

| N | (A/P,15%,N) | (A/F,15%,N) | (A/G,15%,N) | I0 | IN | G | C | LAC |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.1500 | 1.0000 | 0.0000 | $12,000.00 | $9,000.00 | $500.00 | $2,800.00 | $7,600.00 |
| 2 | 0.6151 | 0.4651 | 0.4651 | $12,000.00 | $6,750.00 | $500.00 | $2,800.00 | $7,274.42 |
| 3 | 0.4380 | 0.2880 | 0.9071 | $12,000.00 | $5,060.00 | $500.00 | $2,800.00 | $7,052.12 |
| 4 | 0.3503 | 0.2003 | 1.3263 | $12,000.00 | $3,800.00 | $500.00 | $2,800.00 | $6,905.30 |
| 5 | 0.2983 | 0.1483 | 1.7228 | $12,000.00 | $2,850.00 | $500.00 | $2,800.00 | $6,818.49 |
| 6 | 0.2642 | 0.1142 | 2.0972 | $12,000.00 | $2,140.00 | $500.00 | $2,800.00 | $6,774.97 |
| 7 | 0.2404 | 0.0904 | 2.4498 | $12,000.00 | $1,600.00 | $500.00 | $2,800.00 | $6,764.67 |
| 8 | 0.2229 | 0.0729 | 2.7813 | $12,000.00 | $1,200.00 | $500.00 | $2,800.00 | $6,777.45 |

The economic lifetime in this case is 7 years.  Increasing the RR lengthens the economic lifetime.  The initial investment is always greater than the salvage value, ignoring the effects of the gradient term, one can see that increasing the RR effectively increases the contribution of the initial investment and salvage value to the LAC.  The initial investment contribution will outweigh the salvage value, and the economic lifetime increases because you must earn more income from the asset to counter this effective increase in the investment cost of the asset.

(c) Estimated Future Salvage values were increased by 25% and the economic life compared to the case in problem 2

| N | (A/P,10%,N) | (A/F,10%,N) | (A/G,10%,N) | I0 | IN | G | C | LAC |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.1000 | 1.0000 | 0.0000 | $12,000.00 | $11,250.00 | $500.00 | $2,800.00 | $4,750.00 |
| 2 | 0.5762 | 0.4762 | 0.4762 | $12,000.00 | $8,437.50 | $500.00 | $2,800.00 | $5,934.52 |
| 3 | 0.4021 | 0.3021 | 0.9366 | $12,000.00 | $6,325.00 | $500.00 | $2,800.00 | $6,182.78 |
| 4 | 0.3155 | 0.2155 | 1.3812 | $12,000.00 | $4,750.00 | $500.00 | $2,800.00 | $6,252.75 |
| 5 | 0.2638 | 0.1638 | 1.8101 | $12,000.00 | $3,562.50 | $500.00 | $2,800.00 | $6,287.10 |
| 6 | 0.2296 | 0.1296 | 2.2236 | $12,000.00 | $2,675.00 | $500.00 | $2,800.00 | $6,320.37 |
| 7 | 0.2054 | 0.1054 | 2.6216 | $12,000.00 | $2,000.00 | $500.00 | $2,800.00 | $6,364.86 |
| 8 | 0.1874 | 0.0874 | 3.0045 | $12,000.00 | $1,500.00 | $500.00 | $2,800.00 | $6,420.40 |

As one can see the economic lifetime is dramatically reduced (1 year), this is because if you can get more money for the asset at the end of life, selling it sooner is advantageous.

(d) First Cost of the Facility, the initial investment was increased by 50%.

| N | (A/P,10%,N) | (A/F,10%,N) | (A/G,10%,N) | I0 | IN | G | C | LAC |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.1000 | 1.0000 | 0.0000 | $18,000.00 | $9,000.00 | $500.00 | $2,800.00 | $13,600.00 |
| 2 | 0.5762 | 0.4762 | 0.4762 | $18,000.00 | $6,750.00 | $500.00 | $2,800.00 | $10,195.24 |
| 3 | 0.4021 | 0.3021 | 0.9366 | $18,000.00 | $5,060.00 | $500.00 | $2,800.00 | $8,977.64 |
| 4 | 0.3155 | 0.2155 | 1.3812 | $18,000.00 | $3,800.00 | $500.00 | $2,800.00 | $8,350.27 |
| 5 | 0.2638 | 0.1638 | 1.8101 | $18,000.00 | $2,850.00 | $500.00 | $2,800.00 | $7,986.59 |
| 6 | 0.2296 | 0.1296 | 2.2236 | $18,000.00 | $2,140.00 | $500.00 | $2,800.00 | $7,767.35 |
| 7 | 0.2054 | 0.1054 | 2.6216 | $18,000.00 | $1,600.00 | $500.00 | $2,800.00 | $7,639.46 |
| 8 | 0.1874 | 0.0874 | 3.0045 | $18,000.00 | $1,200.00 | $500.00 | $2,800.00 | $7,571.30 |

The economic lifetime is increased (8 years).  This is because it takes more time earning money from the asset to make up for the larger initial investment.

(e) Rate of technological progress

The economic lifetime is reduced, because the asset will likely become obsolete compared with the challenging technologies available. The LAC of the newer technologies will be less due to the progress, as the rate of progress increases the challenger becomes economically viable as a replacement sooner.

(f) Applicable Income Tax Rate

The depreciation allowance per year increases as the lifetime of the asset decreases typically. Likewise as shown with the example with no taxes, the LAC without tax increases as the economic lifetime is shortened. The firm will receive a tax deduction for the costs of operation and the depreciation allowance. The true cost to the firm for the asset can be thought of as

$$-\tau D - \tau LAC + LAC = (1-\tau)LAC - \tau D$$

From the standpoint of the firm it is worthwhile to minimize this quantity, where LAC is the levelized annual cost without tax. If the LAC and D are larger in earlier years, and the tax rate is large, then it is advantageous for the firm to shorten the lifetime of the asset.

(g) Emphasis on Prestige and Image of the Firm

Newer assets increase the prestige of the firm; therefore the firm that emphasizes this image will replace assets frequently, thus shortening the economic lifetime if a dollar value can be attached to image.

5) (PSB 16.11)  A defender has a current salvage value of $15000, a remaining life of 5 years with zero salvage thereafter, and O&M costs of $26,000 per year.  The challenger costs $50,000, has a life of 12 years with $5,000 salvage value thereafter, and O&M costs of $20,000/year.  MARR = 15%

    a.  Use AE with the outsider point of view to make a decision.
    b.  Use AE with the cash flow approach to make a decision.
    c.  Use PV with the outsider point of view to make a decision.
    d.  Use PV with the cash flow approach to make a decision.
    e.  Reconcile any conflicting answers.
(Hint:  What is the study period?)

Solution:

We will examine this problem by regrouping the problem segments; parts (a) and (c) share similar cash flow diagrams, whereas parts (b) and (d) share similar diagrams that are both different from (a) and (c).

Outsider Point of View (a) and (c)

Defender: The outsider buys the defender for the book value of the defender and operates the defender for the rest of its life. Therefore the book value appears as an expense.

$$\begin{array}{c} \text{5} \end{array}$$

$15k

Etc...

$26k
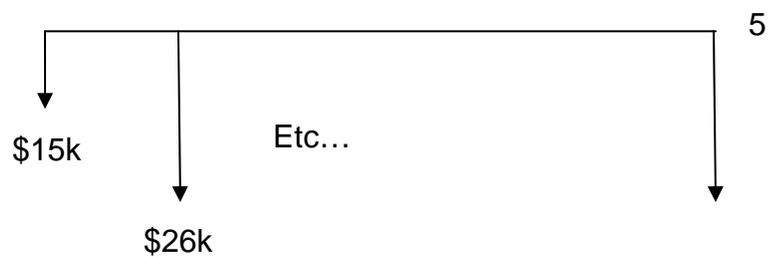
The mathematical expression for the annual expenditure is:

$$AE_D = I_0(A/P,15\%,5) + OC = \$15,000(A/P,15\%,5) + \$26,000$$
$$= \$30,474$$

and the present value of this option is

$$PV_D = I_0 + OC(P/A,15\%,5) = \$15,000 + \$26,000(P/A,15\%,5)$$
$$= \$102,156$$

Challenger: The aspects of the defender are not included from the outsider's point of view.
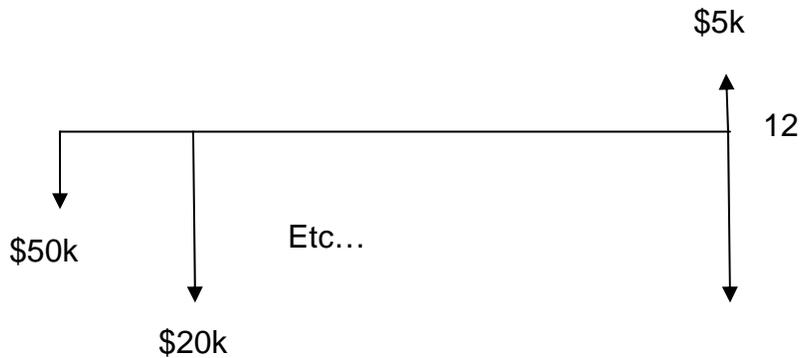


The analogous expressions for annual expenditure and present value, respectively:

$$AE_C = I_0(A/P,15\%,12) + OC - I_N(A/F,15\%,12) = \$50{,}000(A/P,15\%,12) + \$20{,}000 - \$5{,}000(A/F,15\%,12)$$
$$= \$29{,}032$$

$$PV_C = I_0 + OC(P/A,15\%,12) - \$5{,}000(P/F,15\%,12) = \$50{,}000 + \$20{,}000(P/A,15\%,12) - \$5{,}000(P/F,15\%,12)$$
$$= \$157{,}478$$

So from the point of view of the Outsider using the AE method, the challenger (AE=$29,032) is a better option than the defender (AE=$30,474).

From the point of view of the Outsider using the PV method, the defender (PV=$102,156) is a better option than the challenger (PV=$157,478)

Cash Flow Point of View (b) and (d)

Defender: There is no cost associated with keeping the defender from the cash flow point of view.



5

Etc…

$26k
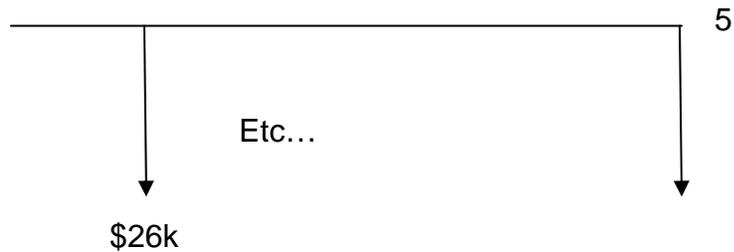
The mathematical expression for the annual expenditure is:

$$AE_D = OC = \$26,000$$
$$= \$26,000$$

and the present value of this option is

$$PV_D = OC(P/A,15\%,5) = \$26,000(P/A,15\%,5)$$
$$= \$87,156$$

Challenger: from the cash flow perspective, the firm sells the defender for the book value and then purchases the challenger.
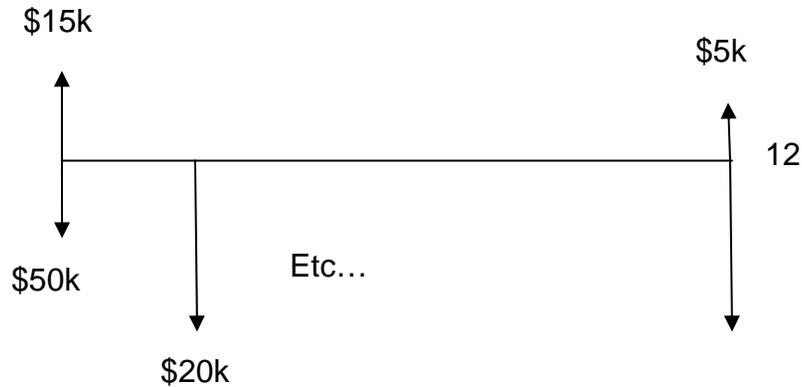
$15k

$5k

12

$50k

Etc…

$20k

The analogous expressions for annual expenditure and present value, respectively:

$$AE_C = -I_{N,Defender}(A/P,15\%,12) + I_0(A/P,15\%,12) + OC - I_{N,Challenger}(A/F,15\%,12)$$
$$= -\$15,000(A/P,15\%,12) + \$50,000(A/P,15\%,12) + \$20,000 - \$5,000(A/F,15\%,12)$$
$$= \$26,284$$

$$PV_C = -I_{N,Defender} + I_0 + OC(P/A,15\%,12) - I_{N,Challenger}(P/F,15\%,12) = -\$15,000 + \$50,000 + \$20,000(P/A,15\%,12) - \$5,000(P/F,15\%,12)$$
$$= \$142,478$$

So according to the cash flow approach using the AE method, the defender (AE=$26,000) is a better option than the challenger (AE=$26,284).

According to the cash flow approach using the PV method, the defender (PV=$87,156) is still a better option than the challenger (PV=$142,478).

The investigation has been summarized in the following table:

| Approach | Method | Defender Cost | Challenger Cost | Decision |
|---|---|---|---|---|
| (a) | AE, Outsider | $30,474 | $29,032 | Challenger |
| (b) | AE, Cash Flow | $26,000 | $26,284 | Defender |
| (c) | PV, Outsider | $102,156 | $157,478 | Defender |
| (d) | PV, Cash Flow | $87,156 | $142,478 | Defender |

(e)  There is a discrepancy for the outsider between the AE and PV evaluations.  This discrepancy is due to the fact that the PV evaluation compares a 5-year project to a 12-year project.  Although the PV of the 5-year project is much less, it is only running for five years, whereas the other project displays costs that run for 12 years.  The PV is thus an invalid comparison, and from the outsider's point of view, the challenger should be the preferred option.

The difference from cash flow to outsider can be attributed to the levelization of the salvage value of the defender; for the defender, it is levelized over five years.  The challenger, by contrast, levelizes the salvage value over twelve years.

6) (Sullivan et al, Q 13-6)A small dam is being planned for a river tributary that is subject to frequent flooding. From past experience, the probabilities that water flow will exceed the design capacity of the dam during a year, plus relevant cost information, are as follows:

| Design | Probability of greater flow during a year | Capital investment |
|---|---|---|
| A | 0.100 | -180,000 |
| B | 0.050 | -195,000 |
| C | 0.025 | -208,000 |
| D | 0.015 | -214,000 |
| E | 0.006 | -224,000 |

Estimated annual damages that occur if water flows exceed design capacity are $150000, $160000, $175000, $190,000, and $210000 for design A, B, C, D, and E, respectively. The life of the dam is expected to be 50 years, with negligible salvage value. For an interest rate of 8% per year, determine which design should be implemented. What nonmonetary considerations might be important to the selection?

The design to be chosen should have the minimum PW. The present worth of the total cost includes the initial investment and the annual damages resulting from the probability of the overflow.

The present worth can be calculated according to the following formula

$$PW = I_0 + pA(P/A, i\%, N)$$

Where i% is the interest rate, N is the lifetime of the project, A is the annual repair cost, p is the probability of an overflow and Io is the initial investment.

$PW(A) = I_A + p_A A_A (P/A, i\%, N) = \$180,000 + 0.1(\$150,000)(P/A, 8\%, 50) = \$363,502$

$PW(B) = \$195,000 + 0.05(\$160,000)(P/A, 8\%, 50) = \$292,868$

$PW(C) = \$208,000 + 0.025(\$175,000)(P/A, 8\%, 50) = \$261,521$

$PW(D) = \$214,000 + 0.015(\$190,000)(P/A, 8\%, 50) = \$248,865$

$PW(E) = \$224,000 + 0.006(\$210,000)(P/A, 8\%, 50) = \$239,414$

Since the PW of the E design is the smallest, this is the design option that should be pursued assuming that each generates the same annual revenue.

Dams disrupt the local ecosystem and this is not taken into account through the monetary analysis.  We should also consider potential loss of human life or quality thereof due to water flow excess if it is not already included in the annual damage cost estimates.