Introduction to Plasma Physics I

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1. Consider a collisionless plasma in equilibrium for which $\partial/\partial y = \partial/\partial z = 0$, $\mathbf{B} = B\hat{\mathbf{z}}$ is uniform, in which density and temperature gradients exist in the x direction. Particles at position $\mathbf{r}$ are assumed to have a Maxwellian distribution of y-velocity that depends only on their gyro-center: $\mathbf{r}_0$

$$f(\mathbf{r}, v_y) = n(\mathbf{r}_0)\left(\frac{m}{2\pi T(\mathbf{r}_0)}\right)^{1/2} \exp[-mv_y^2/2T(\mathbf{r}_0)].$$

This varies only with the x component of the gyrocenter $\mathbf{r}_0$ (and with v of course). Expand the distribution function about the (instantaneous) particle position, to first order in the (assumed small) gyroradius, remembering that the position of the gyro-center relative to the particle can be written

$$\mathbf{r}_0 - \mathbf{r} = -\mathbf{r_L} = -\frac{\mathbf{B} \wedge m\mathbf{v}_\perp}{qB^2}.$$

Hence show that there is a mean y-velocity of the particles when integrated over the distribution. Show that it is consistent with a fluid expression for diamagnetic velocity.

2. Show that the single fluid (MHD) momentum equation can be written in the conservative form

$$\frac{\partial \mathbf{G}}{\partial t} = -\nabla . \overset{\leftrightarrow}{\Pi},$$

where $\mathbf{G} \equiv \rho_m \mathbf{V} + \epsilon_0 \mathbf{E} \wedge \mathbf{B}$ is the total momentum density, and the momentum flux tensor is

$$\overset{\leftrightarrow}{\Pi} \equiv \rho_m \mathbf{V}\mathbf{V} + p - [\frac{\mathbf{B}\mathbf{B}}{\mu_0} + \epsilon_0 \mathbf{E}\mathbf{E} - \frac{B^2}{2\mu_0} - \frac{\epsilon_0 E^2}{2}].$$

3. A certain tokamak plasma with pure hydrogen ions has uniform electron and ion temperatures equal to 1 keV, density $n = 5 \times 10^{19}$ m³, density-gradient in the minor radial direction $dn/dr = n/L$ with $L = 0.2$ m, toroidal field $B_\phi = 2$ T, poloidal field $B_\theta = 0.2$ T, current density, $j = 10^6$ A/m^2, and time dependence such that for any quantity g, $dg/dt = g/\tau$ with $\tau = 1$ s.

(a) If the current density is in the toroidal direction and the plasma velocity is zero, evaluate to one significant figure the magnitude of each of the terms in the full generalized Ohm's law.

(b) Repeat this calculation for those terms that are different, under the alternative assumption that the current density flows in a direction that ensures that $\nabla p = \mathbf{j} \wedge \mathbf{B}$.