Radiation Interactions with Matter: Energy Deposition

Biological effects are the end product of a long series of phenomena, set in motion by the passage of radiation through the medium.

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Interactions of Heavy Charged Particles

Energy-Loss Mechanisms

- The basic mechanism for the slowing down of a moving charged particle is **Coulombic interactions** between the particle and electrons in the medium. This is common to all charged particles

- A heavy charged particle traversing matter loses energy primarily through the **ionization** and **excitation** of atoms.

- The moving charged particle exerts **electromagnetic forces** on atomic electrons and imparts energy to them. The energy transferred may be sufficient to knock an electron out of an atom and thus **ionize** it, or it may leave the atom in an **excited, nonionized state**.

- A heavy charged particle can transfer only a **small fraction** of its energy in a single electronic collision. Its **deflection in the collision is negligible**.

- All heavy charged particles travel essentially **straight paths** in matter.

[Image removed due to copyright considerations]

[Tubiana, 1990]

Maximum Energy Transfer in a Single Collision

The maximum energy transfer occurs if the collision is head-on.

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Assumptions:

- The particle moves rapidly compared with the electron.
- For maximum energy transfer, the collision is head-on.
- The energy transferred is large compared with the binding energy of the electron in the atom.
- Under these conditions the electron is considered to be initially free and at rest, and the collision is elastic.

Conservation of kinetic energy:

$$\frac{1}{2}MV^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}mv_1^2$$

Conservation of momentum:

$$MV = MV_1 + mv_1.$$

$$Q_{max} = \frac{1}{2}MV^2 - \frac{1}{2}MV_1^2 = \frac{4mME}{(M+m)^2},$$

Where $E = MV^2/2$ is the initial kinetic energy of the incident particle.

Q_{max} values for a range of proton energies.

Except at extreme relativistic energies, the maximum fractional energy loss for a heavy charged particle is small.

Maximum Possible Energy Transfer, Q_{max}, in Proton Collision with Electron

Proton Kinetic Energy E (MeV)	Q_{max} (MeV)	Maximum Percentage Energy Transfer $100Q_{max}/E$
0.1	0.00022	0.22
1	0.0022	0.22
10	0.0219	0.22
100	0.229	0.23
10^3	3.33	0.33
10^4	136	1.4
10^5	1.06×10^4	10.6
10^6	5.38×10^5	53.8
10^7	9.21×10^6	92.1

$$Q_{\max} = \frac{4mME}{(M+m)^2}$$

Single Collision Energy Loss Spectra

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- The y axis represents the **calculated** probability that a given collision will result in an energy loss Q.
- *N.B.*, the maximum energy loss calculated above for the 1 MeV proton, of 21.8 keV is off the scale.
- The most probable energy loss is on the order of 20 eV.
- *N.B.*, energy loss spectra for fast charged particles are very similar in the range of 10 – 70 eV.
- Energy loss spectra for slow charged particles differ, the most probable energy loss is closer to the Q_{max}.

Stopping Power

- The average **linear rate of energy loss** of a heavy charged particle in a medium (MeV cm^{-1}) is of fundamental importance in radiation physics, dosimetry and radiation biology.

- This quantity, designated $-dE/dx$, is called the **stopping power** of the medium for the particle.

- It is also referred to as the **linear energy transfer** (LET) of the particle, usually expressed as keV μm^{-1} in water.

- **Stopping power** and **LET** are closely associated with the dose and with the **biological effectiveness** of different kinds of radiation.

Calculations of Stopping Power

In 1913, Niels Bohr derived an explicit formula for the stopping power of heavy charged particles.

Bohr calculated the energy loss of a heavy charged particle in a collision with an electron, then averaged over all possible distances and energies.

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The Bethe Formula for Stopping Power.

Using relativistic quantum mechanics, Bethe derived the following expression for the stopping power of a uniform medium for a heavy charged particle:

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2 \beta^2} \left[\ln \frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2 \right].$$

k_o = 8.99 x 10^9 N m^2 C^{-2} , (the Boltzman constant)
z = atomic number of the heavy particle,
e = magnitude of the electron charge,
n = number of electrons per unit volume in the medium,
m = electron rest mass,
c = speed of light in vacuum,
β = *V*/*c* = speed of the particle relative to c,
I = mean excitation energy of the medium.

- Only the **charge** *ze* and **velocity** *V* of the heavy charged particle enter the expression for stopping power.

- For the medium, only the **electron density** *n* is important.

Tables for Computation of Stopping Powers

If the constants in the Bethe equation for stopping power, dE/dX, are combined, the equation reduces to the following form:

$$-\frac{dE}{dx}=\frac{5.08\,x10^{-31}z^2 n}{\beta^2}[F(\beta)-\ln I_{ev}] \quad \text{MeV cm}^{-1}$$

where, $F(\beta)=\ln\frac{1.02\,x10^6\beta^2}{1-\beta^2}-\beta^2$

TABLE 5.2. Data for Computation of Stopping Power for Heavy Charged Particles

Proton Kinetic Energy (MeV)	β^2	$F(\beta)$ Eq. (5.34)
0.01	0.000021	2.179
0.02	0.000043	3.775
0.04	0.000085	4.468
0.06	0.000128	4.873
0.08	0.000171	5.161
0.10	0.000213	5.384
0.20	0.000426	6.077
0.40	0.000852	6.771
0.60	0.001278	7.175
0.80	0.001703	7.462
1.00	0.002129	7.685
2.00	0.004252	8.376
4.00	0.008476	9.066
6.00	0.01267	9.469
8.00	0.01685	9.753
10.00	0.02099	9.972
20.00	0.04133	10.65
40.00	0.08014	11.32
60.00	0.1166	11.70
80.00	0.1510	11.96
100.0	0.1834	12.16
200.0	0.3205	12.77
400.0	0.5086	13.36
600.0	0.6281	13.73
800.0	0.7088	14.02
1000.	0.7658	14.26

[Turner]

Conveniently,…..

For a given value of β, the kinetic energy of a particle is proportional to the rest mass,

Table 5.2 can also be used for other heavy particles.

Example:
The ratio of kinetic energies of a deuteron and a proton **traveling at the same speed** is

$$\frac{\frac{1}{2}M_d V^2}{\frac{1}{2}M_p V^2} = \frac{M_d}{M_p} = 2$$

Therefore the value of F(β) of 9.972 for a 10 MeV proton, also applies to a 20 MeV deuteron.

Mean Excitation Energies

Mean excitation energies, *I*, have been calculated using the quantum mechanical approach or measured in experiments. The following approximate empirical formulas can be used to estimate the I value in eV for an element with atomic number Z:

$I \approx 19.0$ eV; Z = 1 (hydrogen)

$I \approx 11.2 \text{ eV} + (11.7)(Z) \text{ eV}; 2 \le Z \le 13$

$I \approx 52.8 \text{ eV} + (8.71)(Z) \text{ eV}; Z > 13$

For compounds or mixtures, the contributions from the individual components must be added.

In this way a composite lnI value can be obtained that is weighted by the electron densities of the various elements.

The following example is for water (and is probably **sufficient for tissue**).

$$n \ln I = \sum_i N_i Z_i \ln I_i,$$

Where n is the total number of electrons in the material ($n = \Sigma_i N_i Z_i$)

When the material is a pure compound, the electron densities can be replaced by the electron numbers in a single molecule.

Example:
Calculate the mean excitation energy of H_2O

Solution:
I values are obtained from the empirical relations above.
For H, $I_H = 19.0$ eV, for O, $I_O = 11.2 + 11.7 \times 8 = 105$ eV.
Only the ratios, $N_i Z_i/n$ are needed to calculate the composite I.
Since H_2O has 10 electrons, 2 from H and 8 from O, the equation becomes

$$\ln I = \frac{2x1}{10}\ln 19.0 + \frac{1x8}{10}\ln 105 = 4.312 \qquad \text{giving } I = 74.6 \text{ eV}$$

Stopping power versus distance: the Bragg Peak

$$-\frac{dE}{dx}=\frac{5.08\,x10^{-31}\,z^2 n}{\beta^2}[F(\beta)-\ln I_{ev}] \quad \text{MeV cm}^{-1}$$

- At low energies, the factor in front of the bracket increases as $\beta \to 0$, causing a peak (called the Bragg peak) to occur.

- The linear rate of energy loss is a maximum as the particle energy approaches 0.

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Rate of energy loss along an alpha particle track.

- The peak in energy loss at low energies is exemplified in the Figure, above, which plots -dE/dx of an alpha particle as a function of distance in a material.

- For most of the alpha particle track, the charge on the alpha is two electron charges, and the rate of energy loss increases roughly as 1/E as predicted by the equation for stopping power.

- Near the end of the track, the charge is reduced through electron pickup and the curve falls off.

Stopping Power of Water for Protons

$$-\frac{dE}{dx} = \frac{5.08 x 10^{-31} z^2 n}{\beta^2}[F(\beta) - \ln I_{ev}] \quad \text{MeV cm}^{-1}$$

What is needed to calculate **stopping power**, - dE/dX?

n	the electron density
z	the atomic number
lnI	the mean excitation energy

For protons, z = 1,

The gram molecular weight of water is 18.0 g/mole and the number of electrons per molecule is 10.
One m^3 of water has a mass of 10^6 g.

The density of electrons, n, is:

$$n = 6.02 \text{ x } 10^{23} \text{ molecules/mole x } \frac{10^6 \, g \, m^{-3}}{18.0 \, g / mole} \text{ x } 10 \text{ e}^-\text{/molecule} =$$

$3.34 \text{ x } 10^{29}$ electrons/m^3

As found above, for water, ln I_{ev} = 4.312. Therefore, eq (1) gives

$$-\frac{dE}{dx} = \frac{0.170}{\beta^2}[F(\beta) - 4.31] \quad \text{MeV cm}^{-1}$$

At 1 MeV, from Table 5.2, β^2 = 0.00213 and F(β) = 7.69, therefore,

$$-\frac{dE}{dx} = \frac{0.170}{0.00213}[7.69 - 4.31] \; = \; 270 \text{ MeV cm}^{-1}$$

The stopping power of water for a 1 MeV proton is 270 MeV cm^{-1}

Mass Stopping Power

- The **mass stopping power** of a material is obtained by dividing the stopping power by the density ρ.

- Common units for mass stopping power, -dE/ρdx, are MeV cm^2 g^{-1}.

- The mass stopping power is a useful quantity because it expresses the rate of energy loss of the charged particle per g cm^{-2} of the medium traversed.

- In a gas, for example, -d*E*/dx depends on pressure, but –d*E*/ρdx does not, because dividing by the density exactly compensates for the pressure.

- Mass stopping power does not differ greatly for materials with similar atomic composition.

- Mass stopping powers for water can be scaled by density and used for tissue, plastics, hydrocarbons, and other materials that consist primarily of light elements.

For Pb (Z=82), on the other hand, -d*E*/ρdx = 17.5 MeV $cm^2 g^{-1}$ for 10-MeV protons. (water ~ 47 MeV cm^2 g^{-1} for 10 MeV protons)

**Generally, heavy atoms are less efficient on a g cm^{-2} basis for slowing down heavy charged particles, because many of their electrons are too tightly bound in the inner shells to participate effectively in the absorption of energy.

Range

The **range** of a charged particle is the distance it travels before coming to rest.

The range is **NOT** equal to the energy divided by the stopping power.

Table 5.3 [Turner] gives the mass stopping power and range of protons in water. The range is expressed in g cm^{-2}; that is, the range in cm multiplied by the density of water (ρ = 1 g cm^{-3}).

Like mass stopping power, the range in g cm^{-2} applies to all materials of similar atomic composition.

A useful relationship…..

For two heavy charged particles ***at the same initial speed*** $\boldsymbol{\beta}$, the ratio of their ranges is simply

$$\frac{R_1(\beta)}{R_2(\beta)} = \frac{z_2^2 M_1}{z_1^2 M_2},$$

where:

R_1 and R_2 are the ranges
M_1 and M_2 are the rest masses and
z_1 and z_2 are the charges

If particle number 2 is a proton (M_2 = 1 and z_2 = 1), then the range R of the other particle is given by:

$$R(\beta) = \frac{M}{z^2} R_p(\beta),$$

where $R_p(\beta)$ is the proton range.

TABLE 5.3. Mass Stopping Power $-dE/\rho dx$ and Range R_p for Protons in Water

Kinetic Energy (MeV)	β^2	$-dE/\rho dx$ (MeV cm^2 g^{-1})	R_p (g cm^{-2})
0.01	.000021	500.	3×10^{-5}
0.04	.000085	860.	6×10^{-5}
0.05	.000107	910.	7×10^{-5}
0.08	.000171	920.	9×10^{-5}
0.10	.000213	910.	1×10^{-4}
0.50	.001065	428.	8×10^{-4}
1.00	.002129	270.	0.002
2.00	.004252	162.	0.007
4.00	.008476	95.4	0.023
6.00	.01267	69.3	0.047
8.00	.01685	55.0	0.079
10.0	.02099	45.9	0.118
12.0	.02511	39.5	0.168
14.0	.02920	34.9	0.217
16.0	.03327	31.3	0.280
18.0	.03731	28.5	0.342
20.0	.04133	26.1	0.418
25.0	.05126	21.8	0.623
30.0	.06104	18.7	0.864
35.0	.07066	16.5	1.14
40.0	.08014	14.9	1.46
45.0	.08948	13.5	1.80
50.0	.09867	12.4	2.18
60.0	.1166	10.8	3.03
70.0	.1341	9.55	4.00
80.0	.1510	8.62	5.08
90.0	.1675	7.88	6.27
100.	.1834	7.28	7.57
150.	.2568	5.44	15.5
200.	.3207	4.49	25.5
300.	.4260	3.52	50.6
400.	.5086	3.02	80.9
500.	.5746	2.74	115.
600.	.6281	2.55	152.
700.	.6721	2.42	192.
800.	.7088	2.33	234.
900.	.7396	2.26	277.
1000.	.7658	2.21	321.
2000.	.8981	2.05	795.
4000.	.9639	2.09	1780.

Figure 5.7 shows the ranges in g cm^{-2} of protons, alpha particles, and electrons in water or muscle (virtually the same), bone, and lead.

For a given proton energy, the range in g cm^{-2} is greater in Pb than in H_2O, consistent with the smaller mass stopping power of Pb.

[Image removed due to copyright considerations]

References:
This material was taken largely from J.E. Turner, Atoms, Radiation, and Radiation Protection, Wiley, New York, 1995, Chapter 5 “Interaction of heavy charged particles with matter”.

Additional reading:
E.L Alpen, Radiation Biophysics, Prentice Hall, Englewood Cliffs, New Jersey, 1990.

M. Tubiana, J. Dutrice, A. Wambersie, Introduction to Radiobiology, Taylor and Francis, New York, 1990, Chapter 1.