Effectiveness of the damping of a pipe during earthquakes

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Abstract

The paper presents two methods to assess the cost effectiveness of a steam generator pipe. The first method is a time series analysis with real earthquake data. The second method is a probabilistic assessment which leads to the evaluation of a life-time mean cost.

1 Introduction

When performing a seismic design of a power plant, an engineer is faced with the following dilemma : damping systems for pipes, vessels, steam generator are very costly equipment but on the other hand they enable the structure to withstand a larger earthquake. Therefore, selecting the minimum damping is a matter of careful evaluation.

1.1 Natural frequency of the harmonic oscillator

This dilemma can be trivially illustrated by a single oscillator connected to a wall by a spring and with a damping system that we will take as parametric.

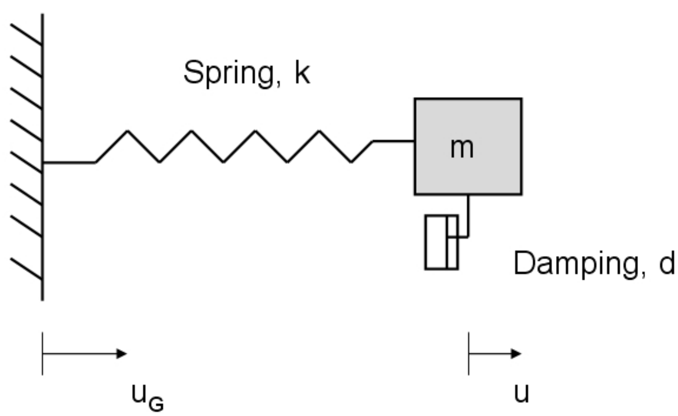

Figure 1: Harmonic oscillator

The fundamental equation governing this system is :

$$m\frac{d^2u}{dt^2} = F(t) - k(u) - d(\frac{du}{dt}) \tag{1}$$

Here F(t) represents the force of the earthquake on the system. Actually the earthquake is not really a force external to the system that is applied to it. An earthquake is really an acceleration of the ground which was supporting the structure. Thus equation (1) can be rewritten assuming that the ground is moving with displacement $u_G(t)$ as :

$$m(\frac{d^2u}{dt^2} - \frac{d^2u_G}{dt^2}) = -k(u - u_G) - d(\frac{du}{dt} - \frac{du_G}{dt}) - m\frac{d^2u_G}{dt^2} \tag{2}$$

Let's define $y(t) = u(t) - u_G(t)$. (2) becomes the simple equation :

$$m\frac{d^2y}{dt^2} + d\frac{dy}{dt} + ky = -m\frac{d^2u_G}{dt^2} \tag{3}$$

Let us define $\omega_0 = \sqrt{\frac{k}{m}}$ and $2Q\omega_0 = \frac{d}{m}$. Then the homogeneous part of (3) can be written simply as :

$$\frac{d^2y}{dt^2} + 2Q\omega_0\frac{dy}{dt} + {\omega_0}^2 y = 0 \tag{4}$$

The characteristic equation is

$$r^2 + 2Q\omega_0 r + {\omega_0}^2 = 0$$

For our problem, we'll consider that ω_0 is fixed whereas Q is the parameter (actually Q is called the quality factor and it is directly related to the damping).

The delta of this equation is then $\Delta = (2Q\omega_0)^2 - 4\omega_0$. Thus $\Delta = 0$ when $Q = 1$ this is called critical damping, $\Delta > 0$ if $Q > 1$ and $\Delta < 0$ if $Q < 1$. In our case Q is always below 1, so $\Delta < 0$.

1.2 Forced excitation: earthquake

We shall now look at the forced oscillation of this system : an earthquake will induce a ground acceleration term that will excite the oscillator at the frequency ω.

To find the response spectrum, one usually assume that the solution $y(t)$ is $y(t) = Re(\underline{y}e^{j\omega t})$

Therefore, (4) with the forced term can be written as :

$$-\omega^2\underline{y} + j\omega 2Q\omega_0\underline{y} + \omega_0^2\underline{y} = -\omega^2\underline{u_G} \tag{5}$$

Equation (5) leads to the following expression for the amplification:

$$\frac{y}{u_G} = \frac{(\frac{\omega}{\omega_0})^2}{\sqrt{(1 - (\frac{\omega}{\omega_0})^2)^2 + 4Q^2(\frac{\omega}{\omega_0})^2}} \tag{6}$$

Plotting this ratio with different values of damping on figure (2) suggests that for undamped systems ($Q < 1$) the amplification is larger if Q is smaller.

More important, we can also plot the acceleration as a function of the acceleration at the natural frequency :

$$\frac{F_{dynamic}}{F_{static}} = \frac{1}{\sqrt{(1 - (\frac{\omega}{\omega_0})^2)^2 + 4Q^2(\frac{\omega}{\omega_0})^2}} \tag{7}$$

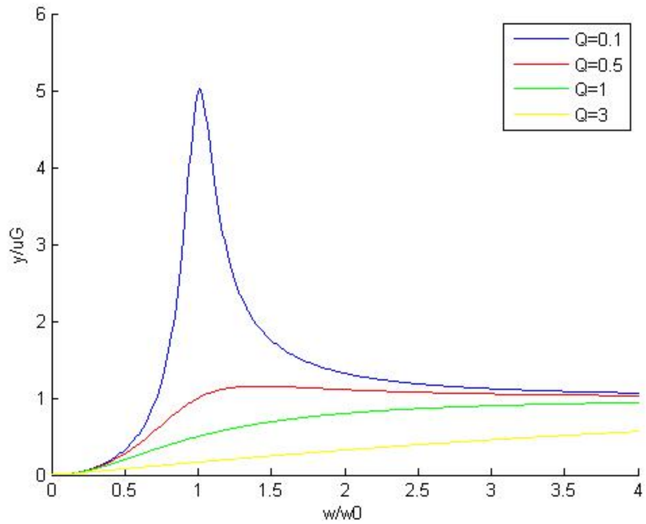

Figure 2: Amplitude amplification

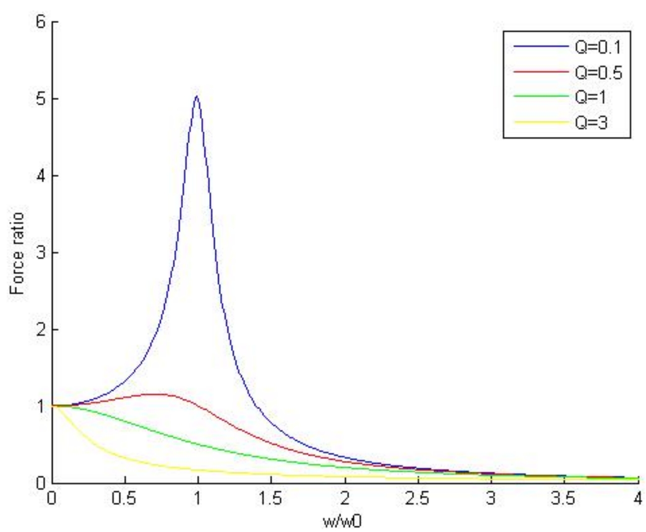

Figure 3: Amplification of the force due to the motion of the ground

Q	$\frac{F_{dynamic}}{F_{static}}$
0.01	50.0
0.02	25.0
0.05	10.0
0.1	5.0
0.2	2.6
0.5	1.2

Table 1: Maximum dynamic amplification variations with Q

Figure (3) is a plot for different values of Q : it is important to notice that for $Q < 1$ the force due to the acceleration of the ground is amplified in the system, which means that the ground acceleration is amplified near the natural frequency ω_0 and can cause more stress than the acceleration itself.

To be more precise, in table (1) the maximum amplifications of the earthquake acceleration with different values of Q are given.

What is also interesting is that the more damping we add to the system, the less significant is the incremental effect for the amplification (see figure (4)): this means that at some point adding some more damping will prove to cost more than it can benefit the plant.

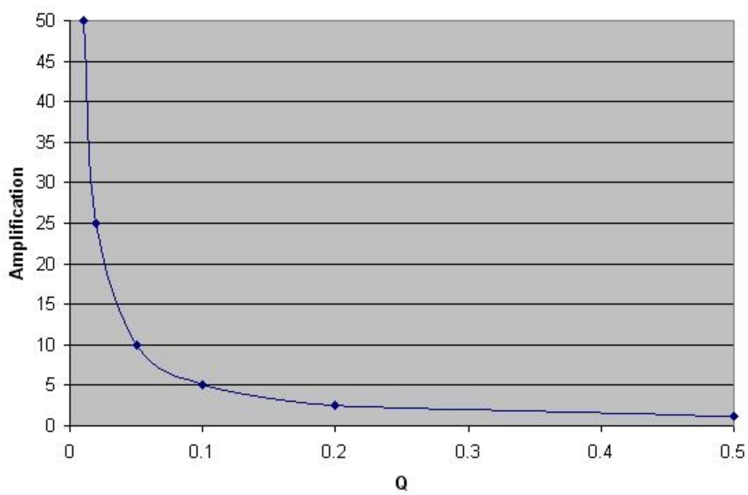

Figure 4: Maximum dynamic amplification

We have thus verified with this simple model the expected conclusion : adding more damping to a system will reduce the amplification in the system, but at some point the marginal efficiency of the damping is smaller compared to the additional cost.

2 Decoupled differential equations of a system

2.1 Lumped masses analysis method

The harmonic oscillator approach gives interesting results for the plant, but it is somehow too simple to capture the complexity of the piping system in a nuclear plant.

The approach broadly used in seismic analysis is the so-called *"lumped analysis"*. The piping system of the nuclear plant is modeled as a succession of harmonic oscillator with mass m_i, coupled to one another by a spring and an absorber. Of course we have $\Sigma m_i = M_{total}$. And the relation between the different oscillators is given by the beam theory. The strength of this approach is that it enables the engineer to reduce the original continuous pipe to a succession of linked masses which only have one degree of freedom (horizontal displacement if the pipe is vertical). The fundamental principle of dynamic applied to the mass m_i yields

$$m_i \frac{d^2 u_i}{dt^2} = -k_{i,i+1}(u_{i+1} - u_i) - k_{i-1,i}(u_{i-1} - u_i) - c_i(\frac{du_i}{dt} - \frac{du_G}{dt}) \qquad (8)$$

As we did before, we will define $y_i = u_i - u_G$ the relative displacement. We also define $Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$, $M = \begin{pmatrix} m_1 & 0 & \dots & 0 \\ 0 & m_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & m_n \end{pmatrix}$, $m = \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{pmatrix}$,$K = \begin{pmatrix} k_{1,1} & k_{1,2} & \dots & k_{1,n} \\ k_{2,1} & k_{2,2} & & k_{2,n} \\ \vdots & & \ddots & \vdots \\ k_{n,1} & k_{n,2} & \dots & k_{n,n} \end{pmatrix}$ is the matrix defined as the static response of the beam to a force $F = \begin{pmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{pmatrix}$ on each mass. So $F = KY$. D is the so-called *"damping matrix"* and represents the ability of the system to transform kinetic energy into heat and thus to dissipate some energy.

Equation (8) can then be written as :

$$M\ddot{Y} + D\dot{Y} + KY = -m\ddot{u_G} \qquad (9)$$

Equation (9) is not trivial to solve as is. But it can be reduced to a linearly independent system of n equations by decomposing the homogeneous equation in terms of harmonic solutions.

2.2 Modeling the pipe

After this brief theoretical overview of the modeling method, we will in this section effectively model a steam generator pipe. We will afterwards study the effect of an earthquake on the pipe with different damping values.

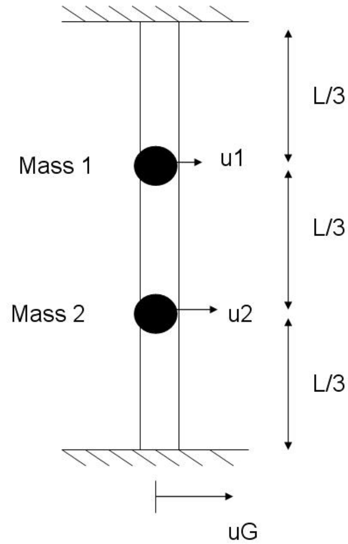

Figure 5: Modeling the pipe: two lumped masses, fixed at z=0 and z=L

The pipe will be considered to be fixed between two walls. We will model the pipe as two lumped masses, each of mass $\frac{M_{total}}{2}$ at a distance $\frac{L_{total}}{3}$ from the wall. See figure (5).

Some usual figures for the pipe's geometry and properties are summarized below :

- Pipe length: $L_{total} = 3\ m$
- Pipe outside diameter 21 mm, material thickness of 1.5 mm
- Pipe composed of stainless steel grade 304: $\rho = 8000\ kg/m^3$, $E = 193\ GPa$
- Pipe filled and surrounded with water at 70 MPa ($\rho_{water} = 750\ kg/m^3$)
- Virtual mass coefficient of 1.1

From these properties, we can directly compute some useful information on the pipe. First its mass is $M_{total} = M_{pipe} + M^w_{inside} + M^w_{moved}$. Where $M_{pipe} = \pi(R^2_{out} - R^2_{in})L_{total}\rho$, $M^w_{inside} = \pi R^2_{in} L \rho_{water}$ and $M^w_{moved} = C_{virtual}\pi R^2_{out} L \rho_{water}$. This gives $M_{pipe} = 2.545\ kg$, $M^w_{inside} = 0.779\ kg$, $M^w_{moved} = 1.120\ kg$. We end up having a total mass M_{total} of $4.444\ kg$. Thus each lumped mass will have a mass $m = 2.222\ kg$.

Another parameter of importance is I the moment of inertia of the pipe around its x-axis. It is defined as $I = \int_A x^2 dA$. In our case, using polar coordinates we get $x = r.cos\theta$ and $dA = rdrd\theta$. Thus $I = \int_0^{2\pi} \int_{R_i}^{R_o} r^3 cos^2\theta drd\theta$, $I = \int_0^{2\pi} cos^2\theta d\theta \int_{R_i}^{R_o} r^3 dr$. So we get $I = 6.739 \times 10^{-9}\ m^4$.

2.3 Differential equation governing the beam motion

Recalling equation (9) we have to find the matrix K. K is related to the response in displacements of the beam when a force F is applied at masses' locations. We introduce the matrix A defined as $Y = AF$ where $Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ and $F = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}$.

Given the linearity of this equation, we can study the effect of F_1 alone and F_2 alone.

For instance F_1 will induce a displacement at $z = L/3$ and also at $z = 2L/3$ proportional to F_1.

To find this relation, one has to solve the fundamental equation of the beam dynamic motion. I is the moment of inertia of the pipe around the x-axis.

$$\frac{\partial^4 u}{\partial z^4} + \frac{M_{total}}{EI}\frac{\partial^2 u}{\partial t^2} = -\frac{\Sigma f}{EI}$$

In our case, we are interested in static response ($\frac{\partial}{\partial t} = 0$) and the linear forces f is simply $F_1 \delta(z = L/3)$ where $\delta(z = L/3)$ is the Dirac function.

So the differential equation we have to solve is:

$$\frac{\partial^4 u}{\partial z^4} = \frac{F_1 \delta(z = L/3)}{EI} \tag{10}$$

Given the geometry of the pipe, there is no displacement at the boundaries, and there is no angular deviation as well. We have the following boundary conditions:

$$\begin{cases} u(z=0) = 0 \ and \ u(z=L) = 0 \\ \frac{\partial u}{\partial z}(z=0) = 0 \ and \ \frac{\partial u}{\partial z}(z=L) = 0 \end{cases}$$

By solving equation (10) we get figure (6). So $A * \begin{pmatrix} F_1 \\ 0 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0.0988\frac{F_1}{EI} \\ 0.0679\frac{F_1}{EI} \end{pmatrix}$

Given the symmetry of the system we can predict that similar results will be found for F_2. Thus putting all the results together we get A.

$$A = \frac{1}{EI}\begin{pmatrix} 0.0988 & 0.0679 \\ 0.0679 & 0.0988 \end{pmatrix}$$

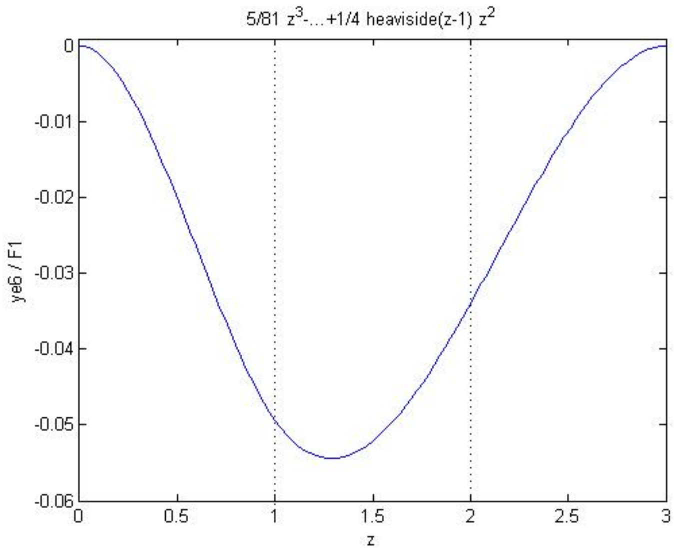

Figure 6: Displacement response of the pipe at force F_1

It is important to note that A is symmetric, thus A^{-1} is defined. The way A is defined it is clear that $K = A^{-1}$.

$$K = EI \begin{pmatrix} 19.1806 & -13.1818 \\ -13.1818 & 19.1806 \end{pmatrix} \tag{11}$$

2.4 Natural frequencies of the undamped system

The fundamental equation of the undamped system with no excitation is the following :

$$M\ddot{Y} + KY = 0 \tag{12}$$

To obtain the natural frequencies, we will solve equation (12) for $Y = e^{\omega t}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$. Equation (12) becomes

$$-\omega^2 My + Ky = 0$$

In order to have a non trivial solution ($i.e.$ $y \neq 0$) ω shall verify:

$$Det(-\omega^2 M + K) = 0 \tag{13}$$

Solving (13) we get

$$\begin{cases} \omega_1 = 59.3\ rad/s\ or\ f_1 = 9.4\ Hz \\ \omega_2 = 137.6\ rad/s\ or\ f_2 = 21.9\ Hz \end{cases}$$

Corresponding to these values, we get the Y_1 and Y_2 vector solutions.

$$\begin{cases} Y_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ Y_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{cases}$$

This two modes represents the two fundamentals oscillation modes of the beam, and can be represented on figure (7)

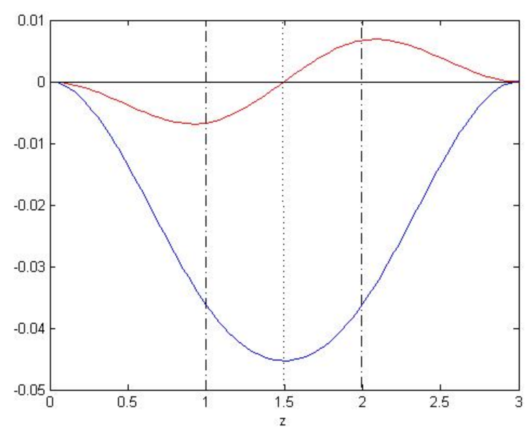

Figure 7: Fundamental modes of the beam: for ω_1 in blue, for ω_2 in red.

2.5 Uncoupled differential equation

Now that we have the two fundamental modes of the beam, we will rewrite our initial problem in this coordinate system. To do so, we define the modal matrix associated with this base :

$P = \begin{pmatrix} Y_1 & Y_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

The new coordinate vector X is such that $Y = PX$. We also have a critical damping matrix D_{cr}. So the fundamental equation $M\ddot{Y} + D_{cr}\dot{Y} + KY = -m\ddot{u_G}$ can be rewritten as

$$P^t MP\ddot{X} + P^t D_{cr} P\dot{X} + P^t KPX = -P^t m\ddot{u_G} \tag{14}$$

We find that $P^t MP = M$. We define a new critical damping matrix $D'_{cr} = P^t D_{cr} P$ as: $D'_{cr} = \begin{pmatrix} d'_1 & 0 \\ 0 & d'_2 \end{pmatrix}$. Computing $K' = P^t KP$ we get $K' = 10^4 \begin{pmatrix} 0.7802 & 0 \\ 0 & 4.2091 \end{pmatrix}$. We also have $m' = P^t m = \begin{pmatrix} 3.1424 \\ 0 \end{pmatrix}$

So equation (14) becomes simply

$$M\ddot{X} + D'_{cr}\dot{X} + K'X = -m'\ddot{u_G}$$

And if we take $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, then we can write the new set of equation as the uncoupled following system of equations:

$$\begin{cases} m\ddot{x_1} + d'_1\dot{x_1} + 7.80 \times 10^3 x_1 = -3.14\ddot{u_G} \\ m\ddot{x_2} + d'_2\dot{x_2} + 4.21 \times 10^4 x_2 = 0 \end{cases} \tag{15}$$

We now need to get values for d'_1 and d'_2. We can find the critical damping values, and then express the total damping of the system as a fraction of the critical damping. By definition of the critical damping of a single harmonic oscillator we get $d'_{cr,1} = \sqrt{4m7.80 \times 10^3} \approx 263.3\ kg/s$ and $d'_{cr,2} = \sqrt{4m4.21 \times 10^4} \approx 263.3\ kg/s$.

$$D'_{cr} = \begin{pmatrix} 263.3 & 0 \\ 0 & 611.6 \end{pmatrix}$$

3 Earthquake data

The next step is to use our model and to compute the displacement when the pipe is subject to an earthquake. To do this, we will use data obtained from real earthquake (taken from [1]) and apply it to our model.

3.1 Hector mine, Joshua tree earthquake, CA

This earthquake occurred on October 16, 1999 in California. The data come from the online data-base COSMOS Strong Motion Program. The seismograph was located at 48.4 km from the epicenter (vertical projection of the focus), and the earthquake was rated 7.1 on the Richter scale (which goes up to 9.0).

Figure 8: Joshua tree: earthquake location

The data consists of a list of 3000 acceleration recorded during 60 seconds (*i.e.* one record every 0.02 s).

Also of interest is the Fourier spectrum of this signal. The acquisition frequency is $\frac{1}{0.02} = 50\ Hz$. The Power Spectral Distribution (or Fourier Spectrum) is defined as $\frac{|F(f)|^2}{2\pi}$ where $F(f)$ is the fourier transform of the ground acceleration.

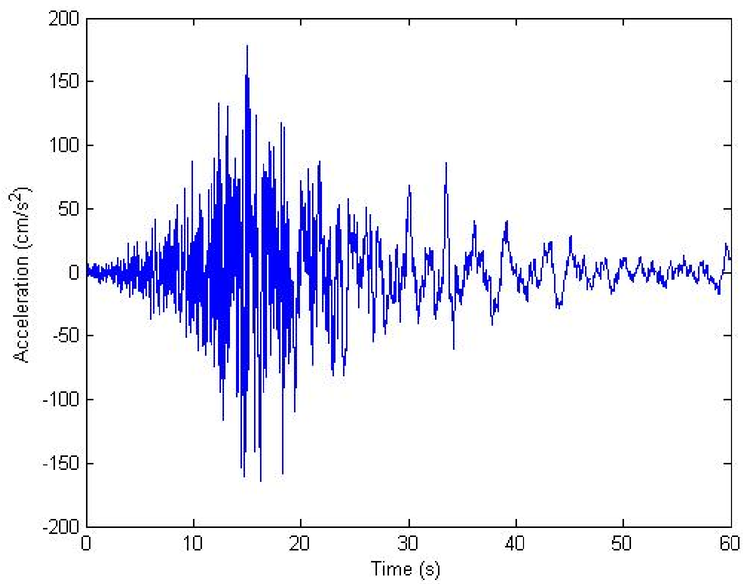

Figure 9: Est-West acceleration of the ground during Hector Mine's Earthquake

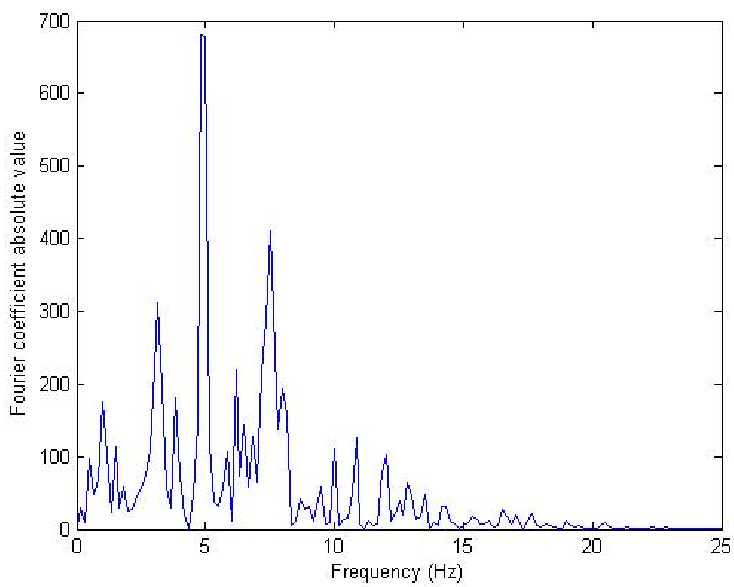

Figure 10: Fourier spectrum of the Hector Mine's seismograph

3.2 New Hampshire's 1982 earthquake

This earthquake occurred on January 19, 1982 in New Hampshire. It was rated 4.5 on the Richter scale, so it was much more modest than the precedent one, but every cannot be sitting right on the San Anrea's Fault zone! The accelerograph we will use was recorded at Franklin Falls Dam, NH at 10.8 km from the epicenter (note that the data were acquired much closer to the epicenter).

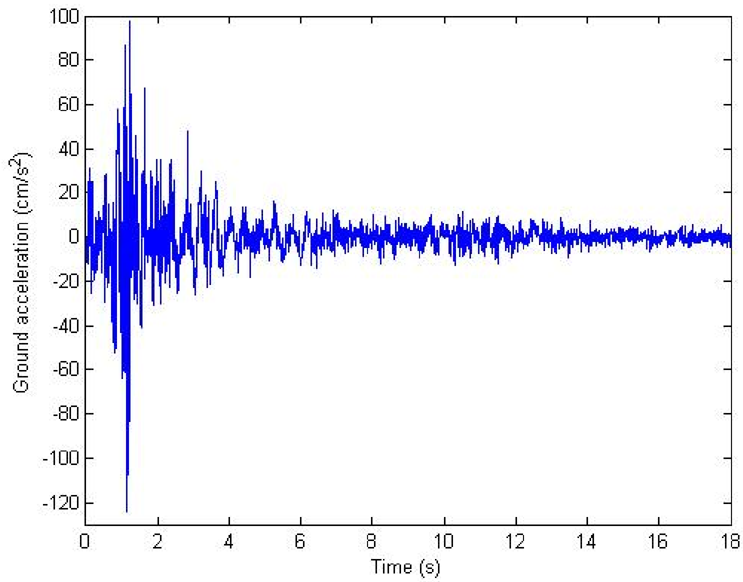

Figure 11: Est-West acceleration of the ground during NH's Earthquake

Even though the earthquake had some large acceleration peaks, its duration (about 20 seconds) is much shorter than the previous earthquake. The acquisition frequency is 200 Hz.

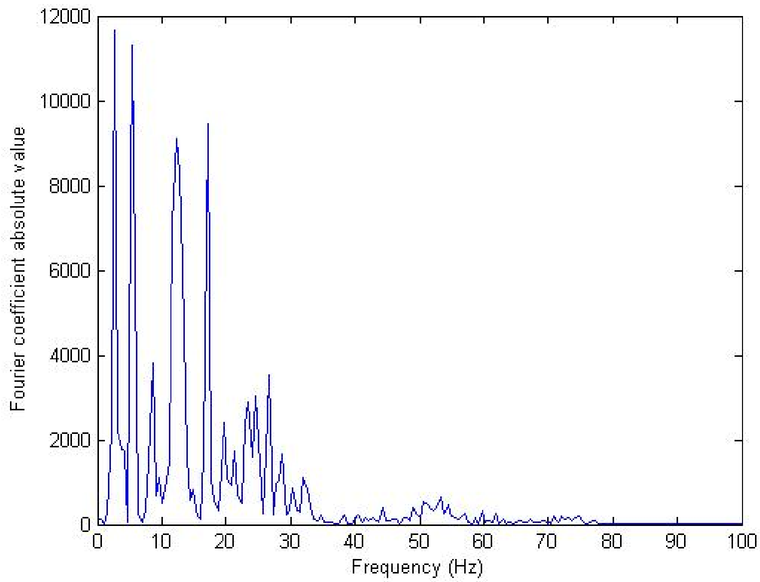

Figure 12: Fourier spectrum of New Hampshire 1982 earthquake's seismograph

4 Response of the pipe to an earthquake

Now that we both have the uncoupled equation and some sample of ground motion we can solve (15) with the data of the two different earthquakes and study the evolution of the displacement of the lumped masses and the force on the support caused by the ground motion as a function of the damping.

4.1 Methodology

For each earthquake we will proceed as follow:

- Pick a damping value (α percent of critical damping)
- Solve the uncoupled differential equation using a step-by-step algorithm (we only know $\ddot{u_G}$ at discreet times) : MatLab's ode45 solver is used
- Compute the maximum displacement of the lumped masses
- Deduce the forces applied on the support for that particular α
- Increment α and start over...

This will give us a curve of amplification versus damping ratio.

4.2 Calculations for the Joshua Tree's earthquake

4.2.1 $\alpha = 0.01$

First of all we note that the second equation of equation set (15) is an unexcited harmonic oscillator. If we assume that the pipe was originally at rest (initial displacement and velocity are zero) then $x_2(t) = 0$ all along the earthquake. This means that $y_1(t) = y_2(t)$. The only variable of interest is $x_1(t)$.

A MatLab solve of this problem gives the following solution for $x_1(t)$.

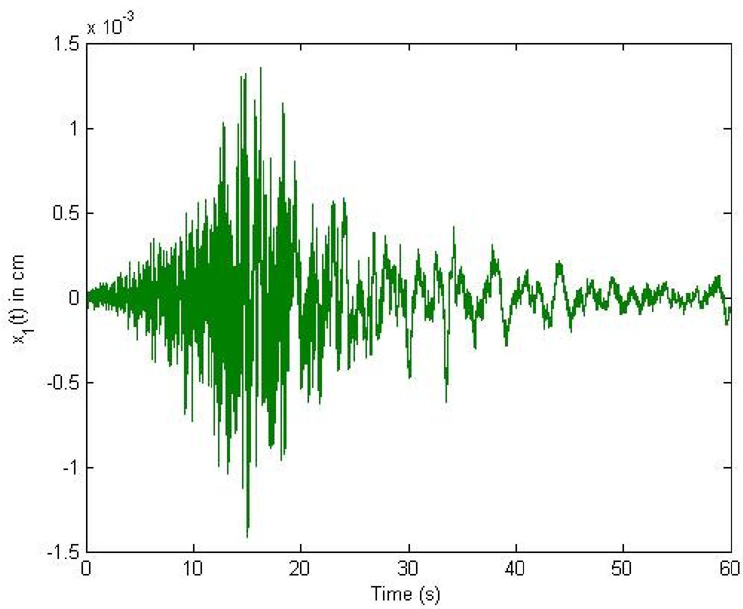

Figure 13: x_1 as a function of t for $\alpha = 0.01$

We also get the maximum value of $x_1(t^*) = 1.3513 \times 10^{-3}$ cm. We can also compute the value of the maximum acceleration of the lumped mass. The maximum acceleration is 65.2882×10^{-3} cm/s^2 and the maximum velocity is 285.28×10^{-3} cm/s.

We can formulate our answer with the maximum displacement vector X_{max} as: $X_{max} = \begin{pmatrix} 1.3513 \\ 0 \end{pmatrix} \times 10^{-3}$ cm

Then $Y = PX$ so $Y_{max} = PX_{max}$. We find $Y_{max} = \begin{pmatrix} 0.9555 \\ 0.9555 \end{pmatrix} \times 10^{-3}$ cm. We also recall that $F = KY$, so $F_{max} = KY_{max}$ and therefore $F_{max} = \begin{pmatrix} 263.66 \\ 263.66 \end{pmatrix}$ N.

Now given the symmetry of the problem, it is clear that the force on the supports are the same. In addition we have just proved that $F_1 = F_2$ so by a trivial force balance on the pipe we get that $F_{support} = 263.66\ N$.

4.2.2 Summary of the results

If we apply the same reasoning and calculations for different values of the damping ratio α we get the results summarized in table

α	$y_{max} \times 10^3$ in cm	F_{max} in N	Amplification ratio $\frac{F_{max}}{Max(M\ddot{u_G})}$
0.01	0.9555	263.66	3.000
0.02	0.9032	249.24	2.284
0.03	0.8920	246.13	1.904
0.04	0.8872	244.82	1.616
0.05	0.8861	244.53	1.437
0.06	0.8847	244.12	1.247
0.07	0.8857	244.40	1.161
0.08	0.8926	246.30	1.070
0.09	0.8971	247.55	1.016
0.10	0.9022	248.94	0.980
0.20	0.9043	249.53	0.740
0.50	0.8177	225.65	0.536
1.00	0.7956	219.53	0.432

Table 2: results for Joshua Tree's earthquake

This results can be summarized on a plot:

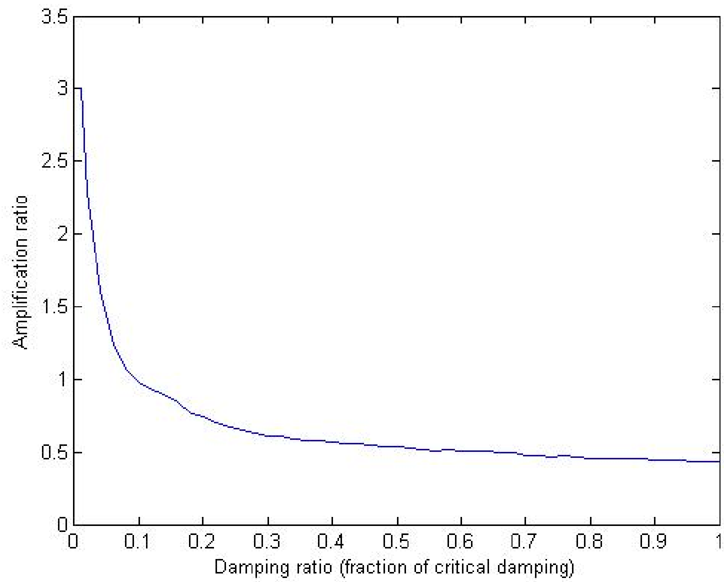

Figure 14: Variation of the force damping with damping ratio

4.3 Calculations for the New Hampshire's earthquake

α	$y_{max} \times 10^3$ in cm	F_{max} in N	Amplification ratio $\frac{F_{max}}{Max(M\ddot{u_G})}$
0.01	1.2641	348.81	1.680
0.02	1.1089	305.99	1.475
0.03	1.0134	279.64	1.264
0.04	0.9909	273.43	1.196
0.05	0.9703	267.74	1.104
0.06	0.9179	253.28	1.100
0.07	0.9190	253.58	0.994
0.08	0.8908	245.80	0.960
0.09	0.8611	237.62	0.927
0.10	0.8368	230.91	0.971
0.20	0.6889	190.08	0.874
0.50	0.5876	162.14	0.524
1.00	0.5713	157.63	0.340

Table 3: Results for NH's earthquake

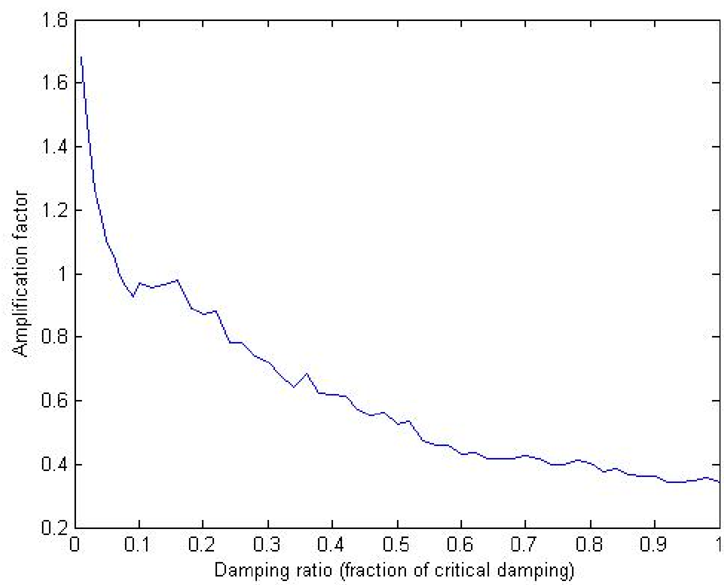

Figure 15: Variation of the force damping with damping ratio

4.4 Interpreting the results

The general idea here is to be able to determine the force that applies on the support during an earthquake. From the results and the sketches showed before one concludes that apart from a puzzling rise around 0.07 % for the Joshua-Tree earthquake, the amplification of the ground motion is decreasing with the damping ratio.

We also note that even though the Joshua Tree earthquake was much stronger in terms of magnitude than the earthquake in New Hampshire, the effects are relatively close. See figure 16 for a plot.

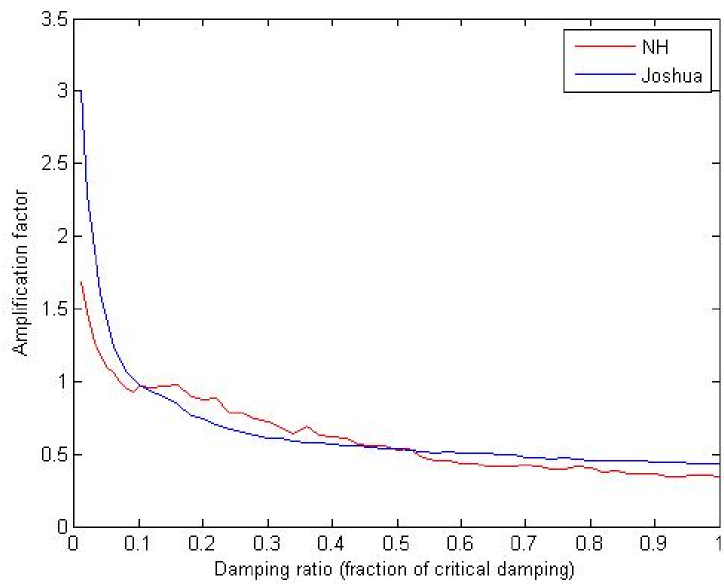

Figure 16: Maximum amplifications on the support for the two earthquakes

We can make the following comments on the results:

- The two earthquake have the same order of effect on the support even though the Joshua Tree Earthquake is a 7.1 Richter's scale earthquake whereas the NH's one is only 4.5. But actually they were recorded at different distance from the epicenter and on different types of rock. The NH was recorded at 10.8 km, whereas the Joshua Tree at a distance of 48.4 km from the epicenter. If we consider that the seismic wave propagate only at the surface, than the effect of the seismic wave are divided by $(\frac{48.4}{10.8})^2 \approx 20$!
- The NH has a much larger dependance in damping ratio than the Joshua Tree. This can be explained by looking at the Fourier spectrum on figures

10 and 12 (One should not pay attention to the absolute values of the coefficients which have not been normalized). We can see that the NH spectra has most of its components around the first natural frequency. Recalling the simple HO on figure 4 we see that around the natural frequency the amplification is very sensitive to the damping ratio. On the other hand, the Joshua Tree spectra is mostly located well below the natural frequency, where the amplification is less sensitive to the damping.

4.5 Conclusions and comments

One can draw the following conclusion from the results obtained:

- This kind of calculation enables the engineer to choose the right parameter in order to avoid failure of the component over the life time of the plant. Given the fact that we do not have data of the cost of damping, it is not possible to go further and to assess the cost of the damping necessary to meet the design criteria.
- The choice of the appropriate damping ratio is really dependant on the site where the plant is to sit.
- This calculation does not introduce any probabilistic concepts, and is thus conservative in the sense that a relatively larger than expected earthquake has to be assumed in order to have a security margin

The following probabilistic model, still theoretical and hard to apply, introduces the concepts of probability of failure, cost of failure *etc.* which can allow a more coherent and reliable cost-effectiveness assessment.

5 Overview of cost-effectiveness design method

The lack of data, and also the lack of calculation power leads us to switch here from a numerical case of a modeled pipe with real earthquake pipe to a more theoretical approach where the purpose is to capture the cost of the deterioration of the pipe as a function of the damping ratio, and to find the best value of the damping ratio that would come from a cost-benefit analysis.

This concept of cost-effectiveness design of a nuclear device is a new field of research. I am presenting here some results from some recent articles ([2] and [3]).

5.1 Evaluating the probability of occurrence of failure

The first step is to get an idea of the probability at which the component that we are studying (the pipe), will fail if an earthquake occurs.

Defining the component failure In our case the failure means breaking the pipe. Most of the stresses are concentrated at the edge of the pipe (where the junction is). At this place, the moment shall not exceed a critical moment M_{lim} that is computed given the properties of the material. The condition of failure is thus:

$$M(z=0)\ or M(z=L) \geq M_{lim} \tag{16}$$

Power Density Spectrum The power density is defined as
$\frac{|F(\omega)|^2}{2\pi}$

It represents the content in power of the signal (*i.e.* the accelerograph) with the frequency. From this PSD, one can easily determine the response spectra by making a point by point calculation.

Probability of failure knowing an earthquake occurred The next step is to determine, by using the appropriate PSD (and that is where I start to lack some data), what is the probability that the ground motion will cause our component to enter condition (16) (*i.e.* to fail). In [2], the authors assume that $\ddot{u_G}$ is a zero-mean stationary Gaussian process. $S_M(\omega)$ is defined as the PSD of the bending moment in the support. It can be obtained by using the transfer vector function of the moment $h_M(\omega)$ that transform the acceleration of the ground into the moments on the supports (this function is easy to find using the differential equation governing the motion of the beam). We finally get $S_M(\omega) = |h_M(\omega)|^2 S_g(\omega)$. As long as the ground acceleration has zero-mean, than the moment is a Gaussian with zero-mean, and thus its standard deviation is $\sigma_M^2 = \int_{-\infty}^{+\infty} S_M(\omega) d\omega$.

What is of interest for us is to determine at which rate the moment will cross M_{lim}. To obtain this value the authors are using the Rice Formula to compute the cross rate. A classical result for a Gaussian process is the following:

$$\nu_M = \frac{1}{2\pi} \frac{\dot{\sigma_M}}{\sigma_M} exp(-\frac{M_{lim}^2}{2\sigma_M^2}) \tag{17}$$

Where ν_M is the crossing rate of M with M_{lim} and $\dot{\sigma_M}$ is the time rate of the standard deviation σ_M.

$$\dot{\sigma_M} = \int_{-\infty}^{\infty} \omega^2 S_M(\omega) d\omega$$

The bottom line of this calculation is that ν_M is a quantity that is relatively easy to calculate when one has the PSD.

From the crossing rate, we can assume that the failure event follows a Poisson distribution. This assumption is very common in hazard events modeling. Thus, if we assume that an earthquake had occurred, then the probability that we have r failure during t seconds of an earthquake is:

$$P_{f_M(X_t=r)} = \frac{(\nu_M t)^r}{r!} exp(-\nu_M t) \tag{18}$$

With this distribution, we verify that $E(X_t) = \nu_M t$ which is what we had expected.

Now, we know that one failure is enough to fail the component, so the probability of failure is 1 minus the probability of no failure over t *i.e.*

$$P_{f_M \ |eq} = 1 - e^{-\nu_M t} \tag{19}$$

This probability is evaluated assuming a typical value of 6 to 7 seconds for t (strong motion duration time). One should note here that this probability is a function of the damping ratio α through S_M.

Expected life-cycle cost The notion of expected life-cycle cost is at the very heart of the cost-effectiveness analysis of our component. The occurrence of an earthquake is satisfyingly modeled by a Poisson distribution where the mean value of the random variable ν describes the average rate of occurrence of an earthquake. Then the mean of the real cost over the life time of the plant, or the expected life-cycle cost $E[C(\alpha)]$ is defined as

$$E[C(\alpha)] = C_\alpha \alpha + C_f P_f(\alpha) \frac{\nu}{\lambda}(1 - exp(-\lambda t_{life})) \tag{20}$$

Where t_{life} is the total life-length of the plant, λ is the annual discount rate, C_α the marginal cost of α and C_f the cost of failure.

Conclusions on the model This very simple model enables to get a simple formula of the expected cost that can then be maximized in order to minimize the expected cost of the pipe. Nevertheless, data such as PSD, and even C_α or C_f are very hard to obtain and cannot be estimated by a back-of-the-envelope calculation.

6 Conclusion

As we have seen, some methods exist to assess the cost-effectiveness of nuclear device system. The major method is a probabilistic method, but it is still a field of research and is not yet applicable in the Nuclear Industry.

My feeling about this, is that the model is very interesting for minor components of the nuclear plant where a failure can be tolerated. But when dealing with major components, components that are responsible for the safety of the plant, I think it will be difficult if not impossible to convince the utility managers to go for probabilistic cost-effectiveness design. And in this case, the old dynamic analysis of the piping system with significant margin is still the major option.

References

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