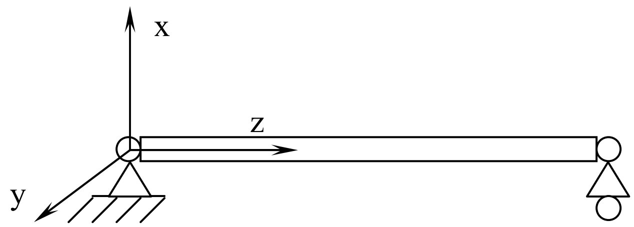
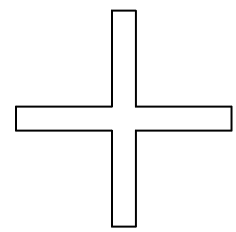
22.314/1.56/2.084/13.14 Fall 2006

Problem Set VII Solution

Solution:



Beam model



Cross section

Geometry and material properties:
L=2.4m; W=200mm; T=7mm
and
E=75 GPa and ν=0.25
Boundary conditions:

At z=0:
u=v=w=0
M_0=0

At z=L:
u=v=0
M_L=0
F_z=0

(a) According to the beam theory:

$$\varepsilon_z = \frac{\sigma_z}{E} + \varepsilon_{zo}; where\ \varepsilon_{zo} = 0.01\varepsilon_{gz}$$

furthermore,

$$\varepsilon_z = \varepsilon_{za} - k_y x; where\ \varepsilon_{za} = \frac{dw}{dz}\ and\ k_y = \frac{d\theta_y}{dz} = \frac{d^2u}{dz^2}$$

therefore,

$$\sigma_z = E(\frac{dw}{dz} - \frac{d^2u}{dz^2}x - 0.01\varepsilon_{gz})$$

At any cross section,

$$M_y = -\int_A \sigma_z x dA = M_0 = 0$$

In the meanwhile,

$$M_y = -\int_A \sigma_z x dA = -\int_A E(\varepsilon_{za} - k_y x - 0.01\varepsilon_{gz}) x dA$$

where *u,v,w,*ε_{za} and k_y are only functions of *z* ,and

$$\varepsilon_{gz} = C_1 N + C_2 N^2 = C_1 N_x(x) N_z(z) + C_2 (N_x(x) N_z(z))^2$$

Therefore,

$$M_y = -E\varepsilon_{za}(z)\int_A dA + Ek_y(z)\int_A x^2 dA + 0.01EC_1N_z(z)\int_A 15\left[1+\frac{0.1x}{0.2}\right]xdA + 0.01EC_2N_z(z)^2\int_A N_x(x)^2 xdA$$

where

$\int_A dA = 0$ for symmetry

$$\int_A x^2 dA = 2\left[\int_0^{\frac{T}{2}} x^2 w dx + \int_{\frac{T}{2}}^{\frac{w}{2}} x^2 T dx\right] = 4.6722\times10^{-6}$$

$$\int_A N_x(x)xdA = \int_A 15xdA + 7.5\int_A x^2 dA = 3.504\times10^{-5}$$

$$\int_A N_x(x)^2 xdA = 225\int_A xdA + 225\int_A x^2 dA + 56.25\int_A x^3 dA = 1.05\times10^{-3}$$

Then,

$$Ek_y(z)\cdot 4.6722\times10^{-6} + EC_1N_z(z)\cdot 3.504\times10^{-7} + EC_2N_z(z)^2\cdot 1.05\times10^{-5} = 0$$

$$k_y = -0.075C_1N_z(z) - 2.25C_2N_z(z)^2 = -0.001453\cos\left[\pi\frac{2z-L}{2L_e}\right] - 0.008991\left(\cos\left[\pi\frac{2z-L}{2L_e}\right]\right)^2$$

$$= -0.001453\cos\left[\pi\frac{2z-L}{2L_e}\right] - 0.004496 - 0.004496\cos\left[\pi\frac{2z-L}{L_e}\right]$$

$$\theta_y(z)-\theta_y(0) = \int_0^z k_y(z')dz' = -\int_0^z\left(0.001453\cos\left[\pi\frac{2z'-L}{2L_e}\right] + 0.004496 + 0.004496\cos\left[\pi\frac{2z'-L}{L_e}\right]\right)dz'$$

$$= -\frac{0.001453L_e}{\pi}\left[\sin\left(\pi\frac{2z-L}{2L_e}\right)+\sin\frac{\pi L}{2L_e}\right] - 0.004496z - \frac{0.004496L_e}{2\pi}\left[\sin\left(\pi\frac{2z-L}{L_e}\right)+\sin\frac{L\pi}{L_e}\right]$$

$$= -0.001175\sin(1.2368z-1.4842) - 0.004496z - 0.001818\sin(2.4736z-2.9684) - 0.001484$$

and let $\theta_y(0) = \mathrm{C}$

Again,

$$u(z)-u(0) = \int_0^z \theta_y(z')dz'$$

$$= -0.01\int_0^z\left[0.1175\sin(1.2368z'-1.4842) + 0.4496z' + 0.182\sin(2.4736z'-2.9684) + 0.1484 - C\right]dz'$$

$$= -0.00095[\cos 1.4842 - \cos(1.2368z-1.4842)] - 0.002248z^2$$

$$- 0.000736[\cos 2.9684 - \cos(2.4736z-2.9684)] - 0.001484z + Cz$$

$$= 0.000736\cos(2.4736z-2.9684) + 0.00095\cos(1.2368z-1.4842) - 0.002248z^2$$

$$- 0.001484z + 0.0006428 + Cz$$

and the boundary conditions: $u(0)=0$ and $u(L)=0$

we get $C=0.006879$

therefore,

$$u(z) = 0.000736\cos(2.4736z-2.9684) + 0.00095\cos(1.2368z-1.4842) - 0.002248z^2$$

$$+ 0.0054z + 0.0006428$$

(b) To obtain *w(z)*, we first calculate the axial strain $\varepsilon_{za} = \dfrac{dw(z)}{dz}$

We already know

$$F_z = \int_A \sigma_z dA = F_{zL} = 0$$

In the meanwhile,

$$F_z = \int_A \sigma_z dA = \int_A E(\varepsilon_{za} - k_y x - 0.01\varepsilon_{gz})dA$$

$$= E\varepsilon_{za}\int_A dA - Ek_y(z)\int_A xdA - 0.01EC_1N_z(z)\int_A N_x(x)dA - 0.01EC_2N_z(z)^2\int_A N_x(x)^2 dA$$

where

$$\int_A dA = A = 2TW - T^2 = 0.0028$$

$$\int_A xdA = 0$$

$$\int_A N_x(x)dA = \int_A 15dA + 7.5\int_A xdA = 0.042$$

$$\int_A N_x(x)^2 dA = 225\int_A dA + 225\int_A xdA + 56.25\int_A x^2 dA = 0.6303$$

Therefore,

$$\varepsilon_{za} = 0.15C_1N_z(z) + 2.25107C_2N_z(z)^2$$

$$= 0.002905\cos\left[\pi\frac{2z-L}{2L_e}\right] + 0.008996\left(\cos\left[\pi\frac{2z-L}{2L_e}\right]\right)^2$$

$$= 0.002905\cos\left[\pi\frac{2z-L}{2L_e}\right] + 0.004498 + 0.004498\cos\left[\pi\frac{2z-L}{L_e}\right]$$

Then, we can obtain w(z) by integrating ε_{za} over z

$$w(z) - w(0) = \int_0^z \varepsilon_{za}(z')dz'$$

$$= \frac{0.002905L_e}{\pi}\left[\sin\left(\pi\frac{2z-L}{2L_e}\right) + \sin\frac{\pi L}{2L_e}\right] + \frac{0.004498L_e}{2\pi}\left[\sin\left(\pi\frac{2z-L}{L_e}\right) + \sin\frac{L\pi}{L_e}\right] + 0.004498z$$

$$= 0.002349\sin(1.2368z - 1.4842) + 0.001818\sin(2.4736z - 2.9684) + 0.004498z + 0.002653$$

Therefore:

$$\boxed{w(L) = w(2.4) = 0.016102m}$$