# 22.314 Problem V Solution Fall 2006

Known

$E := 195000 \quad \nu := 0.3$

$\sigma A := 150 \quad \sigma B := 260 \quad \varepsilon B := 0.54 \cdot 10^{-2}$

Derived properties

$\varepsilon mA := 0$

$\varepsilon mB := \varepsilon B - \frac{\sigma B}{E}$

$\varepsilon mY := 0.002$

$\sigma Y := \frac{\varepsilon mY - \varepsilon mA}{\varepsilon mB - \varepsilon mA} \cdot (\sigma B - \sigma A) + \sigma A$ // Linear interpolation between A anb B to get yield stress $\sigma Y$

$\sigma Y = 204.098$

The 0.2% offset yield stress $\sigma Y$ is 204.098 MPa

From the uniaxial stress-strain curve, when $\sigma p <= \sigma A$, $\varepsilon e = \varepsilon eA = 0$; when $\sigma A < \sigma p <= \sigma B$, $\varepsilon e$ is linear on $\sigma p$; when $\sigma p = \sigma B$, $\varepsilon e = \varepsilon eB = \varepsilon B - \sigma B/E$

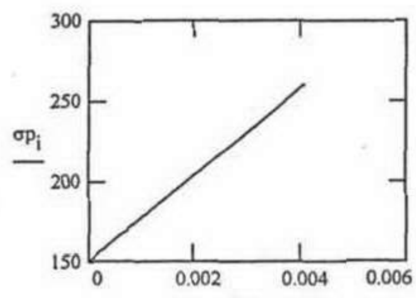
$\varepsilon eA := 0$

$\varepsilon eB := \varepsilon B - \frac{\sigma B}{E}$

$i := 0 .. 100$

$\varepsilon e_i := \varepsilon eB \cdot \frac{i}{100}$

$\sigma p_i := \frac{\varepsilon e_i - \varepsilon eA}{\varepsilon eB - \varepsilon eA} \cdot (\sigma B - \sigma A) + \sigma A$ // $\sigma p$ as a function of $\varepsilon e$



i

Loading sequences and questions

$$\sigma a := \begin{pmatrix} 137.5 & -12.5 & 75 \\ -12.5 & 137.5 & 75 \\ 75 & 75 & 50 \end{pmatrix}$$

1)

$$\sigma kk := \sigma a_{0,0} + \sigma a_{1,1} + \sigma a_{2,2}$$

$$\sigma kk = 325$$

$$i := 0..2$$

$$j := 0..2$$

$$\sigma ae := 0$$

$$Sa_{i,j} := \sigma a_{i,j} - \frac{1}{3}\cdot\delta(i-j,0)\cdot\sigma kk$$

$$Sa = \begin{pmatrix} 29.167 & -12.5 & 75 \\ -12.5 & 29.167 & 75 \\ 75 & 75 & -58.333 \end{pmatrix}$$

$$\sigma ae := \sqrt{\frac{3}{2}\cdot\left[\left(Sa_{0,0}\right)^2 + \left(Sa_{0,1}\right)^2\cdot 2 + \left(Sa_{0,2}\right)^2\cdot 2 + \left(Sa_{1,1}\right)^2 + \left(Sa_{1,2}\right)^2\cdot 2 + \left(Sa_{2,2}\right)^2\right]}$$

$$\sigma ae = 204.634$$

Now calculate VonMises stress for comparision

$$eigena := eigenvals(\sigma a)$$

$$eigena = \begin{pmatrix} 150 \\ 200 \\ -25 \end{pmatrix}$$

$$\sigma VM := \sqrt{\frac{1}{2}\cdot\left[(150-200)^2 + (150+25)^2 + (200+25)^2\right]}$$

$$\sigma VM = 204.634$$

We can see $\sigma ae = \sigma VM$

2) Strain tensor for σa

a) Elastic strain

$i := 0..2$

$j := 0..2$

$\varepsilon ela_{i,j} := \frac{1}{E}\cdot\left[(1+\nu)\cdot\sigma a_{i,j} - \nu\cdot\delta(i-j,0)\cdot\sigma kk\right]$

$$\varepsilon ela = \begin{bmatrix} 4.167\cdot10^{-4} & -8.333\cdot10^{-5} & 5\cdot10^{-4} \\ -8.333\cdot10^{-5} & 4.167\cdot10^{-4} & 5\cdot10^{-4} \\ 5\cdot10^{-4} & 5\cdot10^{-4} & -1.667\cdot10^{-4} \end{bmatrix}$$

b) Mechanical strain

Since σae=204>σa=150, mechanical strain exists and has to be calculated

Since the stress was applied in a proportional manner, Saij/σe is constant, thus we have
εm(i,j)=(3*Sa(i,j)/2*σe)*εe

$\varepsilon ae := 1$

$roota := root\left[\sigma ae - \left[\frac{\varepsilon ae - \varepsilon eA}{\varepsilon eB - \varepsilon eA}\cdot(\sigma B - \sigma A) + \sigma A\right], \varepsilon ae\right]$

$\varepsilon ae := roota$

$\varepsilon ae = 0.002$

$100\cdot\varepsilon ae = 0.202$ // Mathcad sometimes doesn't show enough significant digits. εae is actually 0.00202 here

$\varepsilon ma_{i,j} := \frac{3\cdot Sa_{i,j}}{2\cdot\sigma ae}\cdot\varepsilon ae$

$$\varepsilon ma = \begin{bmatrix} 4.318\cdot10^{-4} & -1.851\cdot10^{-4} & 0.001 \\ -1.851\cdot10^{-4} & 4.318\cdot10^{-4} & 0.001 \\ 0.001 & 0.001 & -8.637\cdot10^{-4} \end{bmatrix}$$

Total stress

$\varepsilon tola := \varepsilon ela + \varepsilon ma$

$$\varepsilon tola = \begin{pmatrix} 8.485\cdot10^{-4} & -2.684\cdot10^{-4} & 0.002 \\ -2.684\cdot10^{-4} & 8.485\cdot10^{-4} & 0.002 \\ 0.002 & 0.002 & -0.001 \end{pmatrix}$$

3) When stress is proportionally reduced to zero, εela vanishs while εma remains unchanged. εtola=εma when zero stress is reached

$$\sigma b := \begin{pmatrix} 260 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sigma kk := \sigma b_{0,0} + \sigma b_{1,1} + \sigma b_{2,2}$$

$$\sigma kk = 260$$

$$i := 0..2$$

$$j := 0..2$$

$$Sb_{i,j} := \sigma b_{i,j} - \frac{1}{3}\cdot\delta(i-j,0)\cdot\sigma kk$$

$$Sb = \begin{pmatrix} 173.333 & 0 & 0 \\ 0 & -86.667 & 0 \\ 0 & 0 & -86.667 \end{pmatrix}$$

$$\sigma be := \sqrt{\frac{3}{2}\cdot\left[\left(Sb_{0,0}\right)^2 + \left(Sb_{0,1}\right)^2\cdot2 + \left(Sb_{0,2}\right)^2\cdot2 + \left(Sb_{1,1}\right)^2 + \left(Sb_{1,2}\right)^2\cdot2 + \left(Sb_{2,2}\right)^2\right]}$$

$$\sigma be = 260$$

4) Strain tensors for σb

a) Elastic strain

$$i := 0..2$$

$$j := 0..2$$

$$\varepsilon elb_{i,j} := \frac{1}{E}\cdot\left[(1+\nu)\cdot\sigma b_{i,j} - \nu\cdot\delta(i-j,0)\cdot\sigma kk\right]$$

$$\varepsilon elb = \begin{pmatrix} 0.001 & 0 & 0 \\ 0 & -4 \cdot 10^{-4} & 0 \\ 0 & 0 & -4 \cdot 10^{-4} \end{pmatrix}$$

b) Mechanical strain

During the second loading, when σe increases from 0 to σae, no mechanical strain is produced. Since σbe=260 > σae=204, new mechanical strain will be produced when σe increases from σae to σbe.

$$\varepsilon be := 1$$

$$rootb := root\left[\sigma be - \left[\frac{\varepsilon be - \varepsilon eA}{\varepsilon eB - \varepsilon eA} \cdot (\sigma B - \sigma A) + \sigma A\right], \varepsilon be\right]$$

$$\varepsilon be := rootb$$

$$\varepsilon be = 0.004$$

$$100 \cdot \varepsilon be = 0.407$$

$$\varepsilon madd_{i,j} := \frac{3 \cdot Sb_{i,j}}{2 \cdot \sigma be} \cdot (\varepsilon be - \varepsilon ae)$$

$$\varepsilon madd = \begin{pmatrix} 0.002 & 0 & 0 \\ 0 & -0.001 & 0 \\ 0 & 0 & -0.001 \end{pmatrix}$$

εmadd is the addtional mechanical strain tensor that is produced when σe increases from σae to σbe. Adding εmadd to εma we can get the total mechanical strain when σb is reached.

$$\varepsilon mb := \varepsilon madd + \varepsilon ma$$

$$\varepsilon mb = \begin{pmatrix} 0.002 & -1.851 \cdot 10^{-4} & 0.001 \\ -1.851 \cdot 10^{-4} & -5.916 \cdot 10^{-4} & 0.001 \\ 0.001 & 0.001 & -0.002 \end{pmatrix}$$

c) Total stress

$$\varepsilon tolb := \varepsilon elb + \varepsilon mb$$

$$\varepsilon tolb = \begin{pmatrix} 0.004 & -1.851 \cdot 10^{-4} & 0.001 \\ -1.851 \cdot 10^{-4} & -9.916 \cdot 10^{-4} & 0.001 \\ 0.001 & 0.001 & -0.002 \end{pmatrix}$$

5) When stress is proportionally reduced to zero, $\varepsilon elb$ vanishs while $\varepsilon mb$ remains unchanged. $\varepsilon tolb = \varepsilon mb$ when zero stress is reached

Note: Some of the elements in the matrices may not be accurate because Mathcad sometimes doesn't show enough significant digits.