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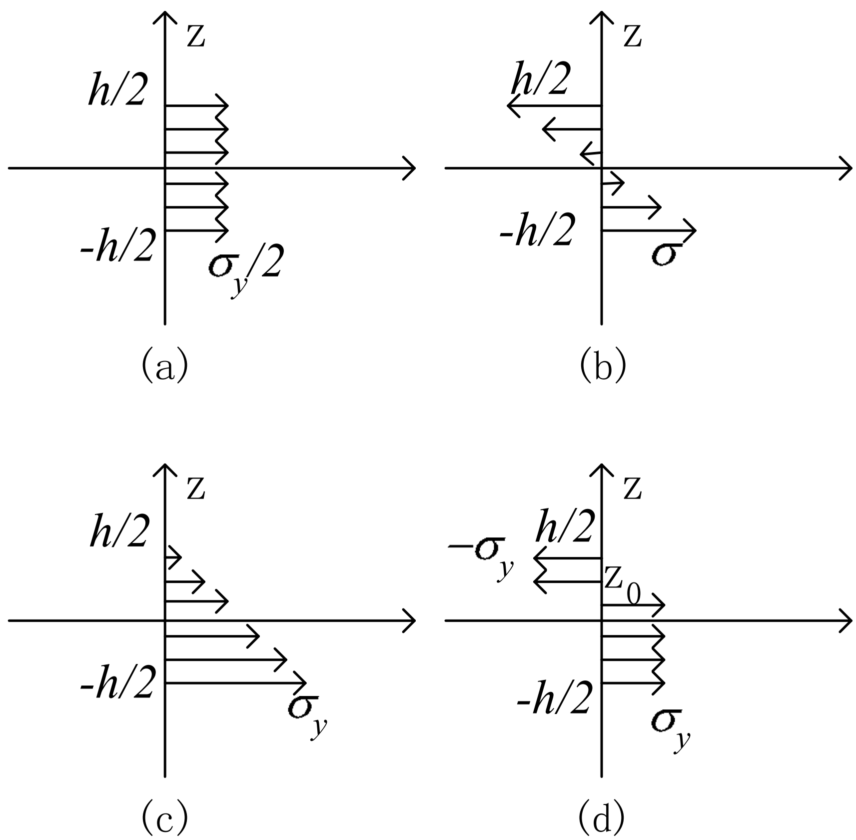
Problem Set IV Solution

1. The stress due to force F is uniform with a value of $\sigma_y/2$ as shown in Figure (a). The stress due to pure bending in elastic regime is linearly symmetric as shown in Figure (b) In elastic regime, combination of force and moment will give a linear stress distribution with a maximum value achieved at $z = -h/2$. Thus when yielding begins, as shown in Figure (c)

$$\sigma = \sigma_y - \sigma_y/2 = \sigma_y/2$$



The moment for reaching the yield at a local space can be calculated. The moment shown in the figure is counter clockwise, therefore:

$$-M_1 = 2 \times \frac{1}{2}(-\frac{\sigma_y}{2}) \times (\frac{h}{2}) \times \frac{2}{3} \times (\frac{h}{2}) \times b$$

$$M_1 = \frac{1}{12}\sigma_y bh^2$$

When further increasing the magnitude of moment, the yielding zone in lower beam will further extend but the maximum stress remains σ_y as elastic perfect plastic material property is assumed. The upper beam become compressed and will also reach the yielding at a certain moment. This situation continues until the beam gets to the yielding at the whole cross section. The stress distribution is shown in Figure (d). To determine the moment, we first need to evaluate the point z_0 which divide the compression and tension. Force balance in axial direction will give:

$$F = \frac{\sigma_y bh}{2} = (\frac{h}{2} - z_0)(-\sigma_y)b + (z_0 + \frac{h}{2})\sigma_y b$$

$$\frac{h}{2} = -(\frac{h}{2} - z_0) + (\frac{h}{2} + z_0)$$

$$z_0 = \frac{h}{4}$$

Therefore, the moment M for actual yielding of the whole beam can be calculated:

$$-M = -\sigma_y \times (\frac{h}{2} - z_0) \times (\frac{h}{2} + z_0)/2 \times b + \sigma_y \times (z_0 - (-\frac{h}{2})) \times (-\frac{h}{2} + z_0)/2 \times b$$

$M = \frac{3}{16}\sigma_y bh^2$ is larger than M_1

2. Total strain energy is the sum of distortion energy and dilation energy:

$$U_T = U_D + U_S$$

$$U_D = U_T - U_S$$

$$U_T = \frac{1}{2}\sum_{i=1}^{3}\sigma_i\epsilon_i$$

$$U_S = \frac{1}{2}(-P\epsilon_v)$$

Therefore:

$$\begin{aligned} U_D &= \frac{1}{2}\sum_{i=1}^{3}\sigma_i\epsilon_i - \frac{1}{2}(-P\epsilon_v) \\ &= \frac{1}{2}\sum_{i=1}^{3}(S_i - P)(\epsilon_i' + \epsilon_v/3) - \frac{1}{2}(-P\epsilon_v) \qquad (1) \\ &= \frac{1}{2}\sum_{i=1}^{3}S_i\epsilon_i' + \frac{1}{2}\sum_{i=1}^{3}S_i\epsilon_v/3 + \frac{1}{2}(-P)\sum_{i=1}^{3}\epsilon_i' + \frac{1}{2}(-P)\sum_{i=1}^{3}\epsilon_v/3 - \frac{1}{2}(-P)\epsilon_v \end{aligned}$$

Since

$$\sum_{i=1}^{3}S_i = \sum_{i=1}^{3}(\sigma_i + P) = \sigma_1 + \sigma_2 + \sigma_3 + 3P = 0 \qquad (2)$$

$$\sum_{i=1}^{3}\epsilon_i' = \sum_{i=1}^{3}(\epsilon_i - \epsilon_v/3) = \epsilon_1 + \epsilon_2 + \epsilon_3 - \epsilon_v = 0 \qquad (3)$$

Plug Eq 2 and Eq 3 into Eq 1, we can get:

$$U_D = \frac{1}{2}\sum_{i=1}^{3} S_i \epsilon_i'$$

3. (a) From the class note, assuming a constant value of thermal conductivity the linear power is proportional to the the temperature difference between the center line and fuel surface.

$$q' = 4\pi k \Delta T$$

The maximum linear power is limited by the melting of fuel, i.e. when melting firstly occurs at the center line of the fuel. Thus assume $T_{fo} = 700^{o}\mathrm{C}$ and plug melting points and average thermal conductivities of UO_2, UC and UN into above equations, we can get an estimation of the maximum achievable linear heat generation rates:

$$q'_{max,UO_2} = 95 \quad \mathrm{kW/m}$$

$$q'_{max,UC} = 489 \quad \mathrm{kW/m}$$

$$q'_{max,UN} = 554 \quad \mathrm{kW/m}$$

(b) Refer to note M12, thermal stress due to a parabolic temperature distribution would give a maximum stress intensity at the outer surface of the fuel in elastic regime. Using Tresca's yield condition, when fracture firstly occurs at the outer surface of the fuel, the linear heat generation rate can be related to the the fracture stress as follows:

$$q' = \frac{\sigma_T 16\pi k(1-\nu)}{E\alpha}$$

From the class notes, E = 175 GPa, $\nu = 0.3$ for UO_2, assume the same properties for UC. Plug material properties into the above equation, we can get:

$$q'_{UO_2} = 7.7 \quad \mathrm{kW/m}$$

$$q'_{UC} = 24.3 \quad \mathrm{kW/m}$$

Thus the q' to initiate cracking is typically less than 10% of that to initiate melting of the fuel. Both UC and UN provide an opportunity to increase q' significantly without violating the limits.