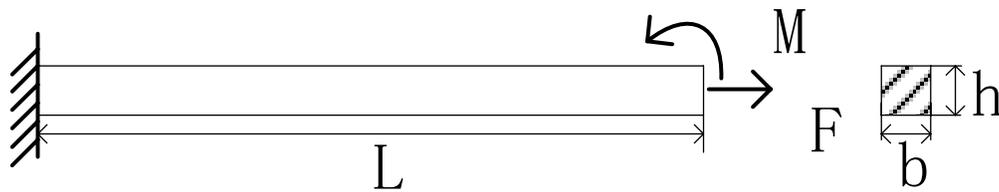


Problem Set IV

Due 10/12/06

1. Consider a cantilever beam with one end fixed into a wall. The dimensions are shown in the figure below. A force $F = bh\sigma_y/2$ is applied to the other end. σ_y is the yield strength. Now a bending moment is applied as shown in the figure. Determine the magnitude of the moment M at which the the beam fails. Assume idealized material law: elastic perfect plastic.



2. The stress tensor can be written as the sum of a deviatoric stress tensor and a dilation stress tensor and the strain can be written as the sum of volumetric strain and deviatoric strain as follows:

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & S_3 \end{bmatrix} + \begin{bmatrix} -P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -P \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} = \begin{bmatrix} \epsilon'_1 & 0 & 0 \\ 0 & \epsilon'_2 & 0 \\ 0 & 0 & \epsilon'_3 \end{bmatrix} + \begin{bmatrix} \epsilon_v/3 & 0 & 0 \\ 0 & \epsilon_v/3 & 0 \\ 0 & 0 & \epsilon_v/3 \end{bmatrix}$$

Where $P = -(\sigma_1 + \sigma_2 + \sigma_3)/3$ is the hydrostatic pressure and $\epsilon_v = \epsilon_1 + \epsilon_2 + \epsilon_3$ is the volumetric strain.

Please show that the distortion energy U_D is equal to:

$$U_D = \frac{1}{2} \sum_{i=1}^3 S_i \epsilon'_i$$

3. Consider several ceramic fuel materials with properties listed in the table below:

Property	UO ₂	UC	UN
Thermal conductivity average (W/m°C)	3.6	23	21
Melting point (°C)	2800	2390	2800
Linear coefficient of expansion (/°C)	10.1E-6	11.1E-6	9.4E-6
Fracture strength (MPa)	110	60	-

- (a) Calculate the theoretical maximum linear heat generation rate.
- (b) Compare the the minimum linear heat generation rates at which fracture would occur for UO_2 and UC.