

Problem Set III Solution

We will consider the primary stress and secondary stress. Since the primary stresses are defined as external stresses, the only primary stress for this problem is due to system pressure. The secondary stresses are due to constraint. For this problem, the secondary stresses are junction discontinuity stress and thermal stress.

We consider the vessel subjected to static load and cyclic load respectively.

1. Under static load

Given that  $\sigma_y = 320$  MPa and  $\sigma_u = 500$  MPa, from ASME code, we can get that under primary stress:

$$P_m \leq S_m \leq \min\left(\frac{2}{3}\sigma_y, \frac{1}{3}\sigma_u\right) = 166.7 \text{ MPa}$$

For cylinder:

$$P_m = |\sigma_\theta - \sigma_r| = 11P$$

For sphere:

$$P_m = |\sigma_\theta - \sigma_r| = 5.75P$$

Thus we have:

$$11P \leq 166.7 \text{ MPa}$$

$$P \leq 15.6 \text{ MPa}$$

For secondary stresses, we will consider three regions, the cylinder region far from junction, the sphere region far from junction and junction region. The thermal stress is assumed identical in three regions. Define:

the stresses in cylinder region:  $\sigma_{\theta,c}, \sigma_{x,c}, \sigma_{r,c}$

the stresses in sphere region:  $\sigma_{\theta,s}, \sigma_{x,s}, \sigma_{r,s}$

the stresses in junction region:  $\sigma_{\theta,j}, \sigma_{x,j}, \sigma_{r,j}$

It is easy to see that:  $\sigma_{\theta,s} < \sigma_{\theta,j} < \sigma_{\theta,c}$ , since the shear force in junction is in the same direction with the system pressure for sphere, and opposite direction for cylinder. Also,  $\sigma_{x,s} = \sigma_{x,j} = \sigma_{x,c}$  and  $\sigma_{r,s} = \sigma_{r,j} = \sigma_{r,c}$ . Thus the maximum stress intensity will be in the cylinder region. From ASME code:

$$P_m + Q \leq 3S_m \leq 3\min\left(\frac{2}{3}\sigma_y, \frac{1}{3}\sigma_u\right) = 500\text{MPa}$$

$$\begin{aligned} P_m + Q &= \max(|\sigma_{\theta,c} - \sigma_{r,c}|, |\sigma_{x,c} - \sigma_{r,c}|, |\sigma_{\theta,c} - \sigma_{x,c}|) \\ &= \max\left(\frac{PR}{t} + \frac{\alpha E}{1-\nu}(T_{avg} - T(z)) + \frac{P}{2}, \frac{PR}{2t} + \frac{\alpha E}{1-\nu}(T_{avg} - T(z)) + \frac{P}{2}, \frac{PR}{2t}\right) \end{aligned}$$

Average temperature is  $T_{avg}$ :

$$\begin{aligned} T_{avg} &= \frac{\int_{-t/2}^{t/2} T(z)2\pi r dr}{\int_{-t/2}^{t/2} 2\pi r dr} \\ &= T_b - 0.3\Delta T \end{aligned}$$

The thermal stress is:

$$\frac{\alpha E}{1-\nu}(T_{avg} - T(z)) = \frac{\alpha E}{1-\nu}\left(\frac{1}{4}\left(1 - \frac{2z}{t}\right)^2 - 0.3\right)\Delta T$$

We can see that the maximum thermal stress is at the outer surface = 51.4 MPa. Therefore:

$$P_m + Q = 11P + 51.4 \leq 500 \quad \text{MPa}$$

$$P \leq 40.8 \quad \text{MPa}$$

## 2. Under cyclic load

The miners rule is applied.

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} \leq 1$$

Assuming from CZP to HZP, only the pressure oscillations,  $n_1=500$ .

Assuming from HZP to HFP, only the temperature oscillations,  $n_2 = 10^5$ .

From topic D,  $S_{alt} = \frac{1}{2}\Delta\sigma_{max, Tresca}$ . For high cycle, the alternating stress is the thermal stress, the amplitude is:  $S_{alt} = 25.7\text{MPa}$ . Applying a safety factor of 2, and using the most conservative curve 7 on the high cycle fatigue figure, it can be seen that at 51.4 MPa, the cycle number is  $6.5\text{E}5$ , i.e.  $N_2 > 6.5\text{E}5$ . Thus,  $N_1 \geq n_1/(1 - n_2/N_2) = 588$ .

From low cycle fatigue figure, it can be seen that  $S_{alt} \leq 65,000 \text{ psi} = 448 \text{ MPa}$ , at low cycle, the only stress variation is due to pressure variation and:

$S_{alt} = \frac{1}{2}\Delta\sigma_{max, Tresca} = 5.5P$  Again, applying a safety factor of 2:

$$5.5P \leq 448/2 = 224 \quad \text{MPa}$$

$$P \leq 40.7 \quad \text{MPa}$$

From above analysis results, it can be seen that the static primary stress condition in cylinder is the limiting factor. Therefore, the system pressure:

$$P \leq 15.6\text{MPa}$$