22.314/1.56/2.084/13.14 Fall 2006

Problem Set I Solution

1. Original dimension:
   D_0 = 12.8 mm, $A_0 = \pi D_0^2/4$ = 128.68 mm^2; and L_0 = 50.800 mm.

   (a) Stress at a load F = 22.2 kN:
   $\sigma = F/A_0$ = 172.5 MPa
   Strain:
   $\epsilon = (L - L_0)/L_0$ = (50.848 - 50.8)/50.8 = 0.009448
   Young's Modulus:
   $E = \sigma/\epsilon$ = 182.6 GPa

   (b) Maximum norimal strain is the strain when fracture occurs:
   $(L_{max} - L_0)/L_0$ = (69.8 - 50.8)/50.8 = 0.374

   (c) F_{max} = 51.2 MPa. Tensile strength is:
   F_{max}/A_0 = 397.9 MPa

2. This problem has a stress tensor:

$$\sigma = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & \tau_{yz} \\ 0 & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

   Note that the shear stresses on the plane normal to x direction are zero. Therefore, we can derive the two principal stresses on the yz plane using the solution for a plane stress condition.

$$\sigma_{i,j} = \frac{\sigma_{yy} + \sigma_{zz}}{2} \pm \sqrt{(\frac{\sigma_{yy} - \sigma_{zz}}{2})^2 + \tau_{yz}^2}$$

   σ_i= 464.5 MPa and σ_j= 40.5 MPa.
   Therefore, σ_1 = 464.5 MPa, σ_2 = 440MPa, and σ_3 = 40.5 MPa.
   The maximum normal stress is σ_1 = 464.5 MPa.
   The maximum shear stress is $(\sigma_1 - \sigma_3)/2$ = 212 MPa.

3. (a) The principal stresses are the eigenvalues of the stress tensor. It's solved by using MATLAB (See the code in the end). For

$$\sigma_a = \begin{bmatrix} 55 & -5 & 30 \\ -5 & 55 & 30 \\ 30 & 30 & 20 \end{bmatrix},$$

   the principal stresses are found to be:
   σ_1 = 80 MPa, σ_2 = 60 MPa, and σ_3 = -10 MPa.

For

$$\sigma_b = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix},$$

the principal stresses are found to be:
$\sigma_1 = \sigma_2 = \sigma_3$ = -10 MPa.
For

$$\sigma_c = \sigma_a + \sigma_b,$$

the principal stresses are found to be:
σ_1 = 70 MPa, σ_2 = 50 MPa, and σ_3 = -20 MPa.

(b) See Figure1 and Figure 2. The Mohr's Circle for σ_b is a single point (-10, 0).

(c) The points on the 3-D Mohr's circles that give the stresses on planes normal to each of the original coordinate axes (x, y, z) are:
$(\sigma_x, \sqrt{\tau_{xy}^2 + \tau_{xz}^2})$, $(\sigma_y, \sqrt{\tau_{yx}^2 + \tau_{yz}^2})$, and $(\sigma_z, \sqrt{\tau_{zx}^2 + \tau_{zy}^2})$
as shown in Figure 1 and Figure 2.

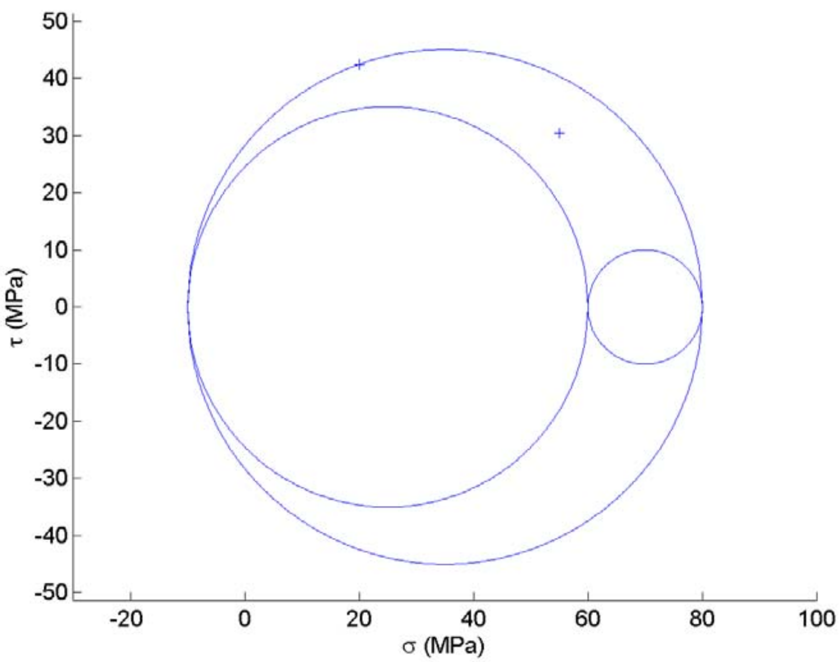

Figure 1: 3-D Mohr's Circle for stress tensor σ_a

(d) The unit vectors are corresponding eigenvector of each eigenvalue (principal stress) of the stress tensor. For σ_a, the unit vectors are as follows:
$$u_1 = \begin{bmatrix} 0.5774 \\ 0.5774 \\ 0.5474 \end{bmatrix}; u_2 = \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix}; u_3 = \begin{bmatrix} 0.4082 \\ 0.4082 \\ -0.8165 \end{bmatrix}.$$

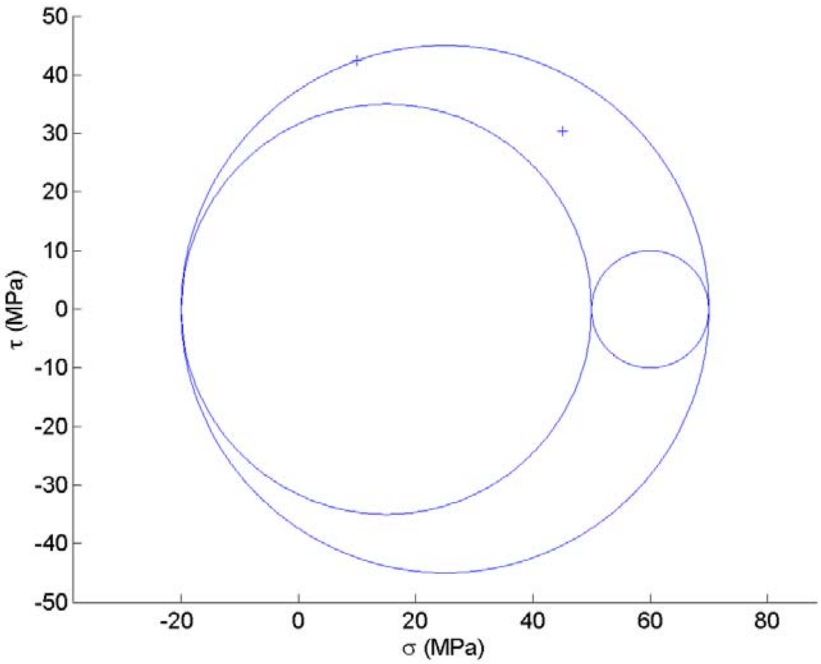

Figure 2: 3-D Mohr's Circle for stress tensor σ_c

For σ_b:

$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$

Since σ_c is the sum of σ_a and hydrostatic pressure tensor σ_b, the unit vectors should be the same as those of σ_a.

$u_1 = \begin{bmatrix} 0.5774 \\ 0.5774 \\ 0.5474 \end{bmatrix}; u_2 = \begin{bmatrix} 0.7071 \\ -0.7071 \\ 0 \end{bmatrix}; u_3 = \begin{bmatrix} 0.4082 \\ 0.4082 \\ -0.8165 \end{bmatrix}.$

(e)

$$\sigma_{Tresca} = max\{|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\}$$

$$\sigma_{vonMises} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

Therefore:

	σ_a	σ_b	σ_c
σ_{Tresca}	90 (MPa)	0 (MPa)	90 (MPa)
$\sigma_{vonMises}$	81.85 (MPa)	0 (MPa)	81.85 (MPa)

The MATLAB code to solve Problem 3:

```
%=================================================================
%     22.314 (Fall 06) Problem Set I-3.
%     09/19/2006
%
%=================================================================

Sigma_a  = [ 55 -5 30
             -5 55 30
             30 30 20];
Sigma_b  = [-10 0 0
            0 -10 0
            0  0 -10];
Sigma_c  = Sigma_a  +  Sigma_b;

% eig returns eigenvalues and eigenvectors of a matrix
[Eigvec_a Eigval_a] = eig(Sigma_a);
[Eigvec_b Eigval_b] = eig(Sigma_b);
[Eigvec_c Eigval_c] = eig(Sigma_c);

% Mohr's Circle for Sigma_a
s1       = Eigval_a(3,3);
s2       = Eigval_a(2,2);
s3       = Eigval_a(1,1);
c1       = (s1 + s2)/2; r1       = (s1 - s2)/2;
c2       = (s2 + s3)/2; r2       = (s2 - s3)/2;
c3       = (s1 + s3)/2; r3       = (s1 - s3)/2;

x1       = linspace(s2, s1, 200);
y1p      = sqrt( r1^2 - (x1 - c1).^2); y1n      = -y1p;
x2       = linspace(s3, s2, 200);
y2p      = sqrt( r2^2 - (x2 - c2).^2); y2n      = -y2p;
x3       = linspace(s3, s1, 500);
y3p      = sqrt( r3^2 - (x3 - c3).^2); y3n      = -y3p;

figure; hold on;
axis([-30 100 -50 50]);
axis equal;
plot(x1, y1p);
plot(x1, y1n);
plot(x2, y2p);
plot(x2, y2n);
plot(x3, y3p);
plot(x3, y3n);
sigma_x  = Sigma_a(1,1);
```

```
tau_x     = sqrt(Sigma_a(1,2)^2 + Sigma_a(1,3)^2);
sigma_y   = Sigma_a(2,2);
tau_y     = sqrt(Sigma_a(2,1)^2 + Sigma_a(2,3)^2);
sigma_z   = Sigma_a(3,3);
tau_z     = sqrt(Sigma_a(3,1)^2 + Sigma_a(3,2)^2);

plot(sigma_x, tau_x, '+');
plot(sigma_y, tau_y, '+');
plot(sigma_z, tau_z, '+');
xlabel('\sigma (MPa)')
ylabel('\tau (MPa)')

%Mohr's Circle for Sigma_c;
s1        = Eigval_c(3,3);
s2        = Eigval_c(2,2);
s3        = Eigval_c(1,1);

c1        = (s1 + s2)/2; r1        = (s1 - s2)/2;
c2        = (s2 + s3)/2; r2        = (s2 - s3)/2;
c3        = (s1 + s3)/2; r3        = (s1 - s3)/2;

x1        = linspace(s2, s1, 200);
y1p       = sqrt( r1^2 - (x1 - c1).^2); y1n       = -y1p;
x2        = linspace(s3, s2, 200);
y2p       = sqrt( r2^2 - (x2 - c2).^2); y2n       = -y2p;
x3        = linspace(s3, s1, 500);
y3p       = sqrt( r3^2 - (x3 - c3).^2); y3n       = -y3p;

figure; hold on;
axis([-20 100 -50 50]);
axis equal;
plot(x1, y1p);
plot(x1, y1n);
plot(x2, y2p);
plot(x2, y2n);
plot(x3, y3p);
plot(x3, y3n);

sigma_x   = Sigma_c(1,1);
tau_x     = sqrt(Sigma_c(1,2)^2 + Sigma_c(1,3)^2);
sigma_y   = Sigma_c(2,2);
tau_y     = sqrt(Sigma_c(2,1)^2 + Sigma_c(2,3)^2);
sigma_z   = Sigma_c(3,3);
tau_z     = sqrt(Sigma_c(3,1)^2 + Sigma_c(3,2)^2);
```

```
plot(sigma_x, tau_x, '+');
plot(sigma_y, tau_y, '+');
plot(sigma_z, tau_z, '+');
xlabel('\sigma (MPa)')
ylabel('\tau (MPa)')
```