22.313 THERMAL-HYDRAULICS IN NUCLEAR POWER TECHNOLOGY

Tuesday, May 17th, 2005, 9 a.m. – 12 p.m.

OPEN BOOK	FINAL (solutions)	3 HOURS

Problem 1 (30%) – Hydraulic analysis of the PWR primary system at cold zero-power conditions

i) The momentum equation for the loop is:

$$\frac{L}{A}\cdot\frac{dm}{dt}=\Delta P_{pump}-(K_{core}+K_{sg})\frac{m^2}{2\rho_\ell A^2} \qquad (1)$$

where m is the mass flow rate, L=40 m is the total length of the loop, A=1.65 m^2 is the flow area, K_{core}=7 and K_{sg}=4 are the form loss coefficients for the core and steam generator, respectively. The acceleration and friction terms were neglected in Equation 1, as per the problem statement. Moreover the gravity term is zero because the fluid is isothermal.

At steady-state $\frac{dm}{dt}=0$ and Equation 1 can be easily solved for the steady-state mass flow rate, m_{ss}:

$$m_{ss}=\sqrt{\frac{2\rho_\ell A^2\Delta P_{pump}}{(K_{core}+K_{sg})}}\approx 9{,}960\ \text{kg/s} \qquad (2)$$

ii) Equation 1 can be re-written as follows:

$$\frac{2\rho_\ell A\cdot L}{(K_{core}+K_{sg})}\cdot\frac{dm}{dt}=m_{ss}^2-m^2 \qquad (3)$$

Equation 3 can be integrated to find m(t) during start-up. Separating the variables, making use of the hint in the problem statement, and setting the initial condition m(0)=0, one gets:

$$m(t)=m_{ss}\frac{1-e^{-t/\tau}}{1+e^{-t/\tau}} \qquad (4)$$

where the time constant, τ, is defined as follows:

$$\tau=\frac{\rho_\ell A\cdot L}{(K_{core}+K_{sg})m_{ss}}=L\sqrt{\frac{\rho_\ell}{2(K_{core}+K_{sg})\Delta P_{pump}}}\approx 0.6\ \text{s} \qquad (5)$$

Equation 4 is plotted in Figure 1. The time it takes to reach 50% of the steady-state value can be calculated by setting $m=0.5 \cdot m_{ss}$ in Equation 4, and solving for t.

$$\tau_{50} = \tau \cdot \ln(3) \approx 0.66 \text{ s} \tag{6}$$

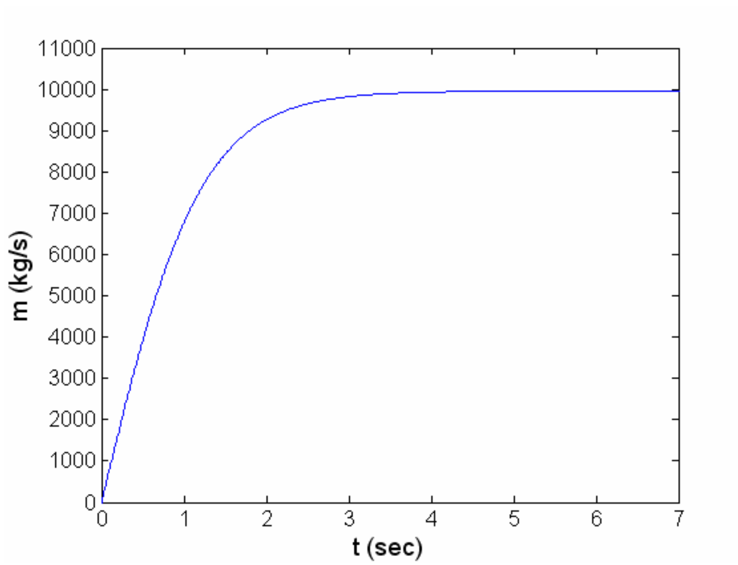

Figure 1. PWR primary system mass flow rate during cold zero-power start-up.

iii) Equation 5 indicates that the time constant is proportional to the loop length and inversely proportional to the square root of the pump head. Thus, it can be concluded that the time constant for the scaled-down loop will be lower than for the actual PWR primary system by a factor of $\sqrt{10} \approx 3.16$.

Problem 2 (25%) – Surface tension effects in borated water draining from a BWR Standby Liquid Control Tank.

i) The water pressure at the bottom of the tank, P_ℓ, can be calculated as follows:

$$P_\ell = P_{atm} + \rho_\ell g L \tag{7}$$

where P_{atm} is the atmospheric pressure, ρ_ℓ is the borated water density and L is the level in the tank. Let us now focus on the liquid/air interface at the hole. For a contact angle >90°, the effect of surface tension is to oppose draining. The condition for static equilibrium (i.e., no draining) is:

$$P_\ell - P_{atm} = \frac{2\sigma}{r} \tag{8}$$

where σ is the surface tension and r is the radius of curvature, which can be derived from simple geometric considerations:

$$r = \frac{d_H}{2\sin\theta} \approx 0.29 \text{ mm} \qquad (9)$$

with d_H=0.5 mm. Combining Equations 7 and 8, one gets the maximum level of borated water that can be held up by the surface tension in the hole, L_{max}:

$$L_{max} = \frac{2\sigma}{\rho_\ell g \cdot r} \approx 49 \text{ cm} \qquad (10)$$

Since the initial level is higher than L_{max}, the borated water will drain until $L=L_{max}$.

ii) If the contact angle is <90°, the tank will drain completely because surface tension no longer opposes draining.

iii) If the tank top is sealed and there is a cover gas, the borated water will drain until the cover gas pressure, P_{cg}, becomes sufficiently low. The condition for static equilibrium is:

$$P_{cg} + \rho_\ell g L = P_{atm} \pm \frac{2\sigma}{r} \qquad (11)$$

where the positive sign on the right-hand side applies to contact angles >90° and the negative sign to contact angles <90°. Thus, the contact angle will affect the equilibrium pressure of the cover gas, but at a certain point draining will stop regardless of the value of the contact angle.

Problem 3 (25%) – Flow split between a heated and an adiabatic channel.

i) The mass equation for the system is:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_{tot} \qquad (12)$$

where $\dot{m}_1$ and $\dot{m}_2$ are the mass flow rate in channels 1 and 2, respectively. The energy equations are:

$$\dot{Q} = \dot{m}_1 c_p (T_{1L} - T_o) \qquad \text{(channel 1)} \qquad (13)$$

$$0 = \dot{m}_2 c_p (T_{2L} - T_o) \qquad \text{(channel 2)} \qquad (14)$$

where T_{1L} and T_{2L} are the temperature at the outlet of channels 1 and 2, respectively.
The momentum equations are:

$$P_o - P_L = K\frac{m_1^2}{2\rho_o A^2} + \rho_o\left[1 - \frac{\beta}{2}\left(T_{1L} - T_o\right)\right]gL \quad \text{(channel 1)} \qquad (15)$$

$$P_o - P_L = \rho_o gL \quad \text{(channel 2)} \qquad (16)$$

where P_o is the inlet plenum pressure. Equations 12 through 16 are 5 equations in the 5 unknowns $\dot{m}_1$, $\dot{m}_2$, T_{1L}, T_{2L} and P_o. Substituting Equation 13 into Equation 15, eliminating P_o from Equations 15 and 16, and solving for $\dot{m}_1$, one gets:

$$\dot{m}_1 = \left[\frac{\beta A^2 \dot{Q}\rho_o^2 gL}{c_p K}\right]^{1/3} \qquad (17)$$

ii) If $\dot{m}_2 = 0$, $\dot{m}_1 = \dot{m}_{tot}$ from Equation 12. Solving Equation 17 for $\dot{Q}$, one gets:

$$\dot{Q}_o = \frac{c_p K \dot{m}_{tot}^3}{\beta A^2 \rho_o^2 gL} \qquad (18)$$

iii) If the heat rate in channel 1 is increased beyond $\dot{Q}_o$, the flow in channel 2 actually reverses. Explanation: in this system the column weight in channel 2 sets the pressure drop for both channels (see Equation 16). Focus now on channel 1. Because of the heating, the column weight in channel 1 is lower than the total pressure drop (Equation 15). So in general, channel 1 will have higher flow rate than channel 2. When the heating is so high that the flow rate in channel 1 is higher than the total flow rate m_{tot}, the flow in channel 2 has to reverse to satisfy continuity (Equation 12).

Problem 4 (20%) – Quenching experiments to simulate boiling heat transfer during a LB-LOCA.

i) The main differences are geometry (spherical vs. cylindrical) and materials (copper vs. zirconium). Geometry differences will have an effect mostly on film boiling and DNB. Materials differences will have an effect mostly on nucleate boiling. Because of geometry, size and materials differences, the experiment and reactor situation will also have different thermal capacities, and thus different time scales.

ii) The energy balance for the sphere is:

$$\rho C_p V\frac{dT}{dt} = -q''S = -h(T - T_{sat})S \qquad (19)$$

where ρ and C_p are the copper density and specific heat, respectively, T, V and S are the sphere temperature, volume and surface, respectively, q" is the heat flux at the surface, h is the heat transfer coefficient, and T_{sat} is the saturation temperature of water.

iii) The qualitative sketch of the sphere temperature history for an initial temperature of 1,500°C is shown in Figure 2. The sphere goes through all heat transfer regimes, including transition boiling, because the situation is temperature controlled, not heat-flux controlled. Note that the film boiling region has the longest duration because of its large temperature width. The concavity of the T-t curve can be determined by differentiating Equation 19:

$$\rho C_P V \frac{d^2T}{dt^2} = S\frac{dq''}{dT}\cdot\left(-\frac{dT}{dt}\right) \qquad (20)$$

Since the term $\left(-\frac{dT}{dt}\right)$ is obviously positive, the concavity depends only on the derivative of the heat flux with respect to temperature. Thus, the concavity is positive for film boiling, nucleate boiling and natural convection, but is negative for transition boiling.

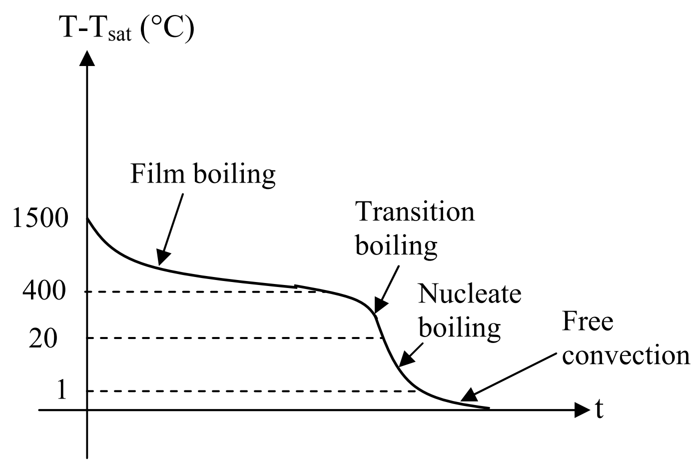

Figure 2. Temperature history during quenching.