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22.313J, 2.59J, 10.536J THERMAL-HYDRAULICS IN POWER TECHNOLOGY

OPEN BOOK | MID-TERM QUIZ (solutions) | 1.5 HOURS

Problem 1 (50%) – Bubbly flow of air in a vinegar fermentation tank

i) Note that since the vinegar velocity is zero (i.e., the vinegar is stagnant), the air bubble rise velocity coincides with the bubble-liquid relative velocity, v_b. For air bubbles of 1-mm equivalent diameter in vinegar we have $M \sim 3\times10^{-11}$, Eö~0.1 and, thus, from the Re-Eö-M diagram, $Re \sim 10^2$. Therefore, from the definition of Re, we get a bubble rise velocity $v_b \sim 0.1$ m/s.

ii) The volume of vinegar in the tank is $V_{vin}=1.5$ m^3. The tank cross sectional area is $A_{tank}=\pi/4 \cdot D^2 \sim 1.13$ m^2, where D=1.2 m is the tank diameter. Therefore, the vinegar level prior to air injection is $L_o=V_{vin}/A_{tank} \sim 1.33$ m. Upon air injection the level rises to accommodate the air volume. The total volume of the air-vinegar mixture is $V_{tot}=V_{air}+V_{vin}$. Since $V_{air}=\alpha V_{tot}$, one gets $V_{tot}=V_{vin}/(1-\alpha)$, and thus the new level, L, is:

$$L=V_{tot}/A_{tank}=V_{vin}/[(1-\alpha)A_{tank}] \qquad (1)$$

where α is the void fraction. According to the drift flux model, the void fraction can be calculated as:

$$\alpha = \frac{j_v}{C_o j + V_{vj}} \qquad (2)$$

where $C_o=1$ and $V_{vj}=v_b=0.1$ m/s, as per the hint in the problem statement. j_v and $j=j_v+j_\ell$ are the air and total superficial velocities, respectively. However, in our case it is $j=j_v$ because the vinegar is stagnant and thus its superficial velocity (j_ℓ) is zero. The air superficial velocity can be calculated as $j_v=xG/\rho_v$. Now, x=1 because the vinegar does not flow; $G=\dot{m}_{air}/A_{tank} \sim 0.018$ kg/m^{2}s, thus $j_v \sim 0.015$ m/s. Equation (2) gives $\alpha \sim 0.13$, and finally Eq. (1) gives L~1.53 m. So the level increase due to air injection is about 20 cm.

iii) If the injector holes were larger, the size of the bubbles would be higher, thus their velocity would be higher, which would result in a lower void fraction, and finally a lower level in the tank. The design with smaller holes is better, because the smaller bubbles have higher surface-to-volume ratio and longer residence time in the vinegar, thus delivering oxygen at a higher rate, which increases the rate of fermentation.

Problem 2 (45%) – Droplets generation and removal in a steam turbine

i) The maximum stable diameter of the droplets that are entrained at the tip of the blades can be readily estimated from the critical Weber number.

$$D_{d,max}=\frac{We_{cr}\sigma}{\rho_g V_g^2} \sim 14.4\ \mu m \tag{3}$$

where We_{cr}=22 and V_g=100 m/s is the steam velocity.

ii) The desired separation efficiency is 75% (i.e., reduce the amount of droplets by a factor 4). Since the efficiency of wire separators increases with the operating steam velocity, the minimum number of screens will be attained by using the maximum allowable velocity, V_g=10 m/s. The separation efficiency of a single wire, η_w, is:

$$\eta_w = 1 - \exp(-0.2 \cdot Stk) \sim 0.933 \tag{4}$$

where $Stk = \frac{\rho_f D_d^2 V_g}{9\mu_g D} \sim 13.5$ is the Stokes number, D_d=14.4 μm and D=1 mm.

The efficiency of multi-screen wire separators, η_{mN}, is:

$$\eta_{mN} = 1 - \exp\left[-\frac{8}{3} \cdot \frac{(1-\varepsilon)\eta_w}{\pi D} NL \right] \tag{5}$$

where L=5 mm and $\varepsilon = 1 - \frac{3\pi}{4}\left(\frac{D}{L}\right)^2 \sim 0.906$. Solving Eq. (5) for the number of screens N, one finds:

$$N = -\frac{3}{8} \cdot \frac{\pi D}{(1-\varepsilon)\eta_w L} \ln(1-\eta_{mN}) \sim 3.7 \tag{6}$$

where η_{mN} was set equal to 0.75. Thus, the minimum number to obtain at least 75% separation efficiency is 4.

iii) Wire separators are simple and reasonably efficient. However, they are delicate and susceptible to failure by erosion/corrosion, because the wire is thin and the surface-to-volume ratio is very high. Chevrons are more expensive, but also more rugged and generally have higher separation efficiencies, as they can operate at higher velocity thanks to the scoops, which increase the breakthrough velocity. Since in a large power plant the capital cost of the moisture separator is usually a small fraction of the total cost, chevrons should be preferred as they are more reliable and efficient.

Problem 3 (5%) – Effect of droplet entrainment on void fraction and pressure drop in annular flow

i) Droplet entrainment reduces the slip ratio (because more liquid is moving at a speed close to the speed of the vapor) and thus increases the void fraction ($\alpha = \dfrac{1}{1+\dfrac{\rho_v}{\rho_\ell}\cdot S\cdot\dfrac{1-x}{x}}$).

Therefore, the sign of $\dfrac{d\alpha}{dz}$ is positive. Note that the flow quality (x) and the mass flux (G) are constant along this channel.

ii) Physically, the momentum increase due to the acceleration of the liquid is higher than the momentum decrease due to the de-acceleration of the vapor. Therefore, there is a net acceleration of the mixture $\left(\dfrac{dP}{dz}\right)_{acc}$ >0.

Mathematically,

$$\left(\frac{dP}{dz}\right)_{acc} = \frac{d}{dz}\left[\frac{G^2}{\rho_m^+}\right] = G^2\frac{d}{dz}\left[\frac{1}{\rho_m^+}\right] = G^2\frac{d}{dz}\left[\frac{x^2}{\alpha\rho_v}+\frac{(1-x)^2}{(1-\alpha)\rho_\ell}\right] \qquad (7)$$

Since we know from 'i' that the void fraction is increasing, the two-phase density, ρ_m^+, ought to be decreasing. More rigorously,

$$\frac{d}{dz}\left[\frac{1}{\rho_m^+}\right] = \frac{d}{dz}\left[\frac{x^2}{\alpha\rho_v}+\frac{(1-x)^2}{(1-\alpha)\rho_\ell}\right] \qquad (8)$$

At high α (typical of annular flow) the first term of the derivative ($\dfrac{x^2}{\alpha\rho_v}$) is more sensitive than the second term ($\dfrac{(1-x)^2}{(1-\alpha)\rho_\ell}$) to a change of α (again note that x is constant here). Therefore, the derivative is dominated by the second term, suggesting that $\left(\dfrac{dP}{dz}\right)_{acc}$ >0 if α increases, as is the case here.