Exercise 8. Boltzmann's Equation.

1. Prove that particles of charge q moving in a magnetic field $\mathbf{B}$ and hence subject to a force that depends on velocity: $q\mathbf{v} \times \mathbf{B}$, nevertheless have $\nabla_v.\mathbf{a} = 0$.

2. (a) Write down Boltzmann's equation in one space (x) and velocity (v) dimension (so that $f = f(x, v)$) for particles that have no acceleration, which are constantly sourced at a uniform rate S (particles per unit volume, per unit time). The source of particles has a Maxwellian velocity distribution function, $S(v) = S_0\sqrt{\frac{m}{2\pi kT}}\exp(-mv^2/kT)$, and is independent of x.

(b) If the particles are perfectly absorbed at $x = \pm L$, and therefore no particles enter the region $x = [-L, L]$ from outside, solve Boltzmann's equation within the region (analytically) to find $f(v, x)$ in steady state. [Hint. The resulting velocity distribution function is not Maxwellian.]

(c) Sketch the distribution as a function of v at the positions

(i) $x = 0$,

(ii) $x = L/2$.

3. Particles move without collisions in one-dimension under the influence of an acceleration a that is constant, independent of x or v.

(a) Find the characteristics of the Vlasov equation (Boltzmann's equation without collisions) for the distribution function, and sketch them in phase space (i.e. on a v versus x plot).

(b) Consider the region $x > 0$, for a case when a is negative. Suppose that particles enter the region at $x = 0$ from below ($v > 0$) with a known velocity distribution

$$f(v) = \frac{f_0}{1 + v^2/v_t^2}$$

where f_0 and v_t are simply constants.

(i) Solve the Vlasov equation to find $f(v)$ for $v < 0$, at $x = 0$.

(ii) Solve to find $f(v)$ for all v at $x = 1$.