

Exercise 7. Fluids and Hyperbolic Equations.

1. Prove equation 7.29, the amplification factor for the Lax Friedrichs scheme.
2. Consider a gas in one spatial dimension that obeys the equations

$$\begin{aligned}
 \text{Continuity:} \quad & \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \\
 \text{Momentum:} \quad & \frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v^2) = -\frac{\partial}{\partial x} p \\
 \text{State:} \quad & p = p(\rho) \quad \text{with} \quad \frac{dp}{d\rho} = K\rho
 \end{aligned} \tag{1}$$

Here, the equation of state is expressed in differential form. The parameter K is simply a constant.

- (a) Convert this into the form of a state and flux vector equation

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{\partial \mathbf{f}}{\partial x}. \tag{2}$$

where

$$\mathbf{u} = \begin{pmatrix} \rho \\ \Gamma \end{pmatrix} \tag{3}$$

is the state vector ($\Gamma = \rho v$) and you should give the flux vector \mathbf{f} .

- (b) Calculate the Jacobian matrix $\mathbf{J} = \partial \mathbf{f} / \partial \mathbf{u}$.
- (c) Find its eigenvalues.

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