Exercise 7. Fluids and Hyperbolic Equations.

1. Prove equation 7.29, the amplification factor for the Lax Friedrichs scheme.

2. Consider a gas in one spatial dimension that obeys the equations

$$\begin{aligned} \text{Continuity:} \quad & \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) &=& \quad 0 \\ \text{Momentum:} \quad & \frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho v^2) &=& \quad -\frac{\partial}{\partial x} p \\ \text{State:} \quad & p = p(\rho) \quad \text{with } \frac{dp}{d\rho} &=& \quad K\rho \end{aligned} \tag{1}$$

Here, the equation of state is expressed in differential form. The parameter K is simply a constant.

(a) Convert this into the form of a state and flux vector equation

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{\partial \mathbf{f}}{\partial x}. \tag{2}$$

where

$$\mathbf{u} = \begin{pmatrix} \rho \\ \Gamma \end{pmatrix} \tag{3}$$

is the state vector ($\Gamma = \rho v$) and you should give the flux vector $\mathbf{f}$.

(b) Calculate the Jacobian matrix $\mathbf{J} = \partial \mathbf{f}/\partial \mathbf{u}$.

(c) Find its eigenvalues.