

## 22.101 Applied Nuclear Physics (Fall 2006)

### Lecture 11 (10/23/06)

#### Nuclear Binding Energy and Stability

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##### References:

W. E. Meyerhof, *Elements of Nuclear Physics* (McGraw-Hill, New York, 1967), Chap.2.

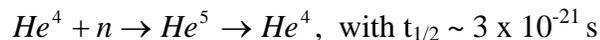
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The stability of nuclei is of great interest because unstable nuclei undergo transitions that result in the emission of particles and/or electromagnetic radiation (gammas). If the transition is spontaneous, it is called a radioactive decay. If the transition is induced by the bombardment of particles or radiation, then it is called a nuclear reaction.

The mass of a nucleus is the decisive factor governing its stability. Knowing the mass of a particular nucleus and those of the neighboring nuclei, one can tell whether or not the nucleus is stable. Yet the relation between mass and stability is complicated. Increasing the mass of a stable nucleus by adding a nucleon can make the resulting nucleus unstable, but this is not always true. Starting with the simplest nucleus, the proton, we can add one neutron after another. This would generate the series,

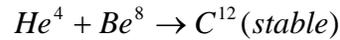


Because  $He^4$  is a double-magic nucleus, it is particularly stable. If we continue to add a nucleon we find the resulting nucleus is unstable,



One may ask: With  $He^4$  so stable how is it possible to build up the heavier elements starting with neutrons and protons? (This question arises in the study of the origin of

elements. See, for example, the Nobel lecture of A. Penzias in *Reviews of Modern Physics*, **51**, 425 (1979).) The answer is that the following reactions can occur



Although  $Be^8$  is unstable, its lifetime of  $\sim 3 \times 10^{-16}$  s is apparently long enough to enable the next reaction to proceed. Once  $C^{12}$  is formed, it can react with another  $He^4$  to give  $O^{16}$ , and in this way the heavy elements can be formed.

Instead of the mass of a nucleus one can use the binding energy to express the same information. The binding energy concept is useful for discussing the calculation of nuclear masses and of energy released or absorbed in nuclear reactions.

### ***Binding Energy and Separation Energy***

We define the binding energy of a nucleus with mass  $M(A,Z)$  as

$$B(A, Z) \equiv [ZM_H + NM_n - M(A, Z)]c^2 \quad (10.1)$$

where  $M_H$  is the hydrogen mass and  $M(A,Z)$  is the atomic mass. Strictly speaking one should subtract out the binding energy of the electrons; however, the error in not doing so is quite small, so we will just ignore it. According to (10.1) the nuclear binding energy  $B(A,Z)$  is the difference between the mass of the constituent nucleons, when they are far separated from each other, and the mass of the nucleus, when they are brought together to form the nucleus. Therefore, one can interpret  $B(A,Z)$  as the *work required* to separate the nucleus into the individual nucleons (separated far from each other), or equivalently, as the *energy released* during the assembly of the nucleus from the constituents.

Taking the actual data on nuclear mass for various  $A$  and  $Z$ , one can calculate  $B(A,Z)$  and plot the results in the form shown in Fig. 10.1.

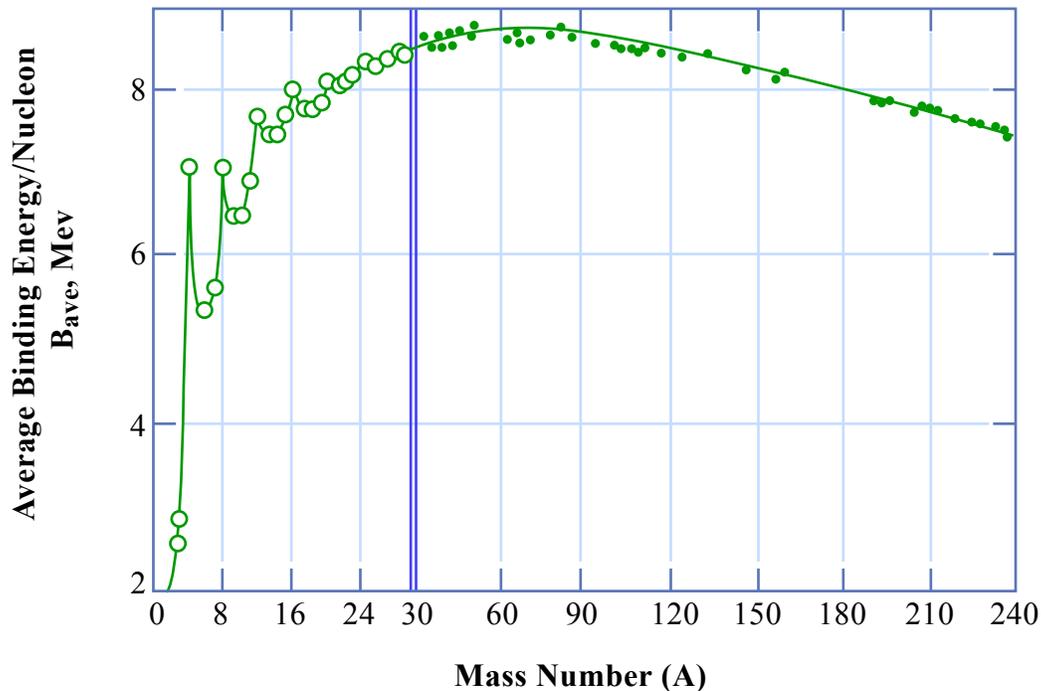


Figure by MIT OCW. Adapted from Meyerhof.

**Fig. 10.1.** Variation of average binding energy per nucleon with mass number for naturally occurring nuclides (and Be8). Note scale change at  $A = 30$ . [from Meyerhof]

The most striking feature of the  $B/A$  curve is the approximate constancy at  $\sim 8$  Mev per nucleon, except for the very light nuclei. It is instructive to see what this behavior implies. If the binding energy of a pair of nucleons is a constant, say  $C$ , then for a nucleus with  $A$  nucleons, in which there are  $A(A-1)/2$  distinct pairs of nucleons, the  $B/A$  would be  $\sim C(A-1)/2$ . Since this is not what one sees in Fig. 10.1, one can surmise that a given nucleon is not bound equally to all the other nucleons; in other words, nuclear forces, being short-ranged, extend over only a few neighbors. The constancy of  $B/A$  implies a saturation effect in nuclear forces, the interaction energy of a nucleon does not increase any further once it has acquired a certain number of neighbors. This number seems to be about 4 or 5.

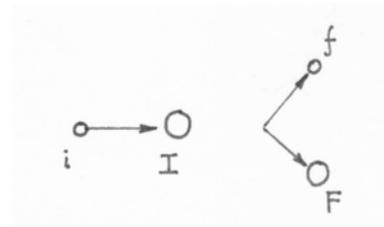
One can understand the initial rapid increase of  $B/A$  for the very light nuclei as the result of the competition between volume effects, which make  $B$  increase with  $A$  like  $A$ , and surface effects, which make  $B$  decrease (in the sense of a correction) with  $A$  like  $A^{2/3}$ . The latter should be less important as  $A$  becomes large, hence  $B/A$  increases (see the discussion of the semi-empirical mass formula in the next chapter). At the other end of the curve, the gradual decrease of  $B/A$  for  $A > 100$  can be understood as the effect of

Coulomb repulsion which becomes more important as the number of protons in the nucleus increases.

As a quick application of the B/A curve we make a rough estimate of the energies release in fission and fusion reactions. Suppose we have symmetric fission of a nucleus with  $A \sim 240$  producing two fragments, each  $A/2$ . The reaction gives a final state with B/A of about 8.5 Mev, which is about 1 Mev greater than the B/A of the initial state. Thus the energy released per fission reaction is about 240 Mev. (A more accurate estimate gives 200 Mev.) For fusion reaction we take  $H^2 + H^2 \rightarrow He^4$ . The B/A values of  $H^2$  and  $He^4$  are 1.1 and 7.1 Mev/nucleon respectively. The gain in B/A is 6 Mev/nucleon, so the energy released per fusion event is  $\sim 24$  Mev.

***Binding Energy in Nuclear Reactions***

The binding energy concept is also applicable to a binary reaction where the initial state consists of a particle  $i$  incident upon a target nucleus  $I$  and the final state consists of an outgoing particle  $f$  and a residual nucleus  $F$ , as indicated in the sketch,



We write the reaction in the form

$$i + I \rightarrow f + F + Q \tag{10.2}$$

where  $Q$  is an energy called the ‘Q-value of the reaction’. Corresponding to (10.2) we have the definition

$$Q \equiv [(M_i + M_I) - (M_f + M_F)]c^2 \tag{10.3}$$

where the masses are understood to be atomic masses. Every nuclear reaction has a characteristic Q-value; the reaction is called exothermic (endothermic) for  $Q > 0$  ( $< 0$ )

where energy is given off (absorbed). Thus an endothermic reaction cannot take place unless additional energy, called the threshold, is supplied. Often the source of this energy is the kinetic energy of the incident particle. One can also express  $Q$  in terms of the kinetic energies of the reactants and products of the reaction by invoking the conservation of *total* energy, which must hold for any reaction,

$$T_i + M_i c^2 + T_I + M_I c^2 \rightarrow T_f + M_f c^2 + T_F + M_F c^2 \quad (10.4)$$

Combining this with (10.3) gives

$$Q = T_f + T_F - (T_i + T_I) \quad (10.5)$$

Usually  $T_I$  is negligible compared to  $T_i$  because the target nucleus follows a Maxwellian distribution at the temperature of the target sample (typically room temperature), while for nuclear reactions the incident particle can have a kinetic energy in the Mev range. Since the rest masses can be expressed in terms of binding energies, another expression for  $Q$  is

$$Q = B(f) + B(F) - B(i) - B(I) \quad (10.6)$$

As an example, the nuclear medicine technique called boron neutron capture therapy (BNCT) is based on the reaction



In this case,  $Q = B(Li) + B(\alpha) - B(B) = 39.245 + 28.296 - 64.750 = 2.791$  Mev. The reaction is exothermic, therefore it can be induced by a thermal neutron. In practice, the simplest way of calculating  $Q$  values is to use the rest masses of the reactants and products. For many reactions of interest  $Q$  values are in the range 1 – 5 Mev. An important exception is fission, where  $Q \sim 170 - 210$  Mev depending on what one considers to be the fission

products. Notice that if one defines  $Q$  in terms of the kinetic energies, as in (10.5), it may appear that the value of  $Q$  would depend on whether one is in laboratory or center-of-mass coordinate system. This is illusory because the equivalent definition, (10.3), is clearly frame independent.

### ***Separation Energy***

Recall the definition of binding energy, (10.1), involves an initial state where all the nucleons are removed far from each other. One can define another binding energy where the initial state is one where only one nucleon is separated off. The energy required to separate particle  $a$  from a nucleus is called the separation energy  $S_a$ . This is also the energy released, or energy available for reaction, when particle  $a$  is captured. This concept is usually applied to a neutron, proton, deuteron, or  $\alpha$ -particle. The energy balance in general is

$$S_a = [M_a(A', Z') + M(A - A', Z - Z') - M(A, Z)]c^2 \quad (10.8)$$

where particle  $a$  is treated as a 'nucleus' with atomic number  $Z'$  and mass number  $A'$ . For a neutron,

$$\begin{aligned} S_n &= [M_n + M(A - 1, Z) - M(A, Z)]c^2 \\ &= B(A, Z) - B(A - 1, Z) \end{aligned} \quad (10.9)$$

$S_n$  is sometimes called the binding energy of the *last neutron*.

Clearly  $S_n$  will vary from one nucleus to another. In the range of  $A$  where  $B/A$  is roughly constant we can estimate from the  $B/A$  curve that  $S_n \sim S_p \sim 8$  Mev. This is a rough figure, for the heavy nuclei  $S_n$  is more like 5 – 6 Mev. It turns out that when a nucleus  $M(A-1, Z)$  absorbs a neutron, there is  $\sim 1$  Mev (or more, can be up to 4 Mev) difference between the neutron absorbed being an even neutron or an odd neutron (see Fig. 10.2). This difference is the reason that  $U^{235}$  can undergo fission with thermal neutrons, whereas  $U^{238}$  can fission only with fast neutrons ( $E > 1$  Mev).

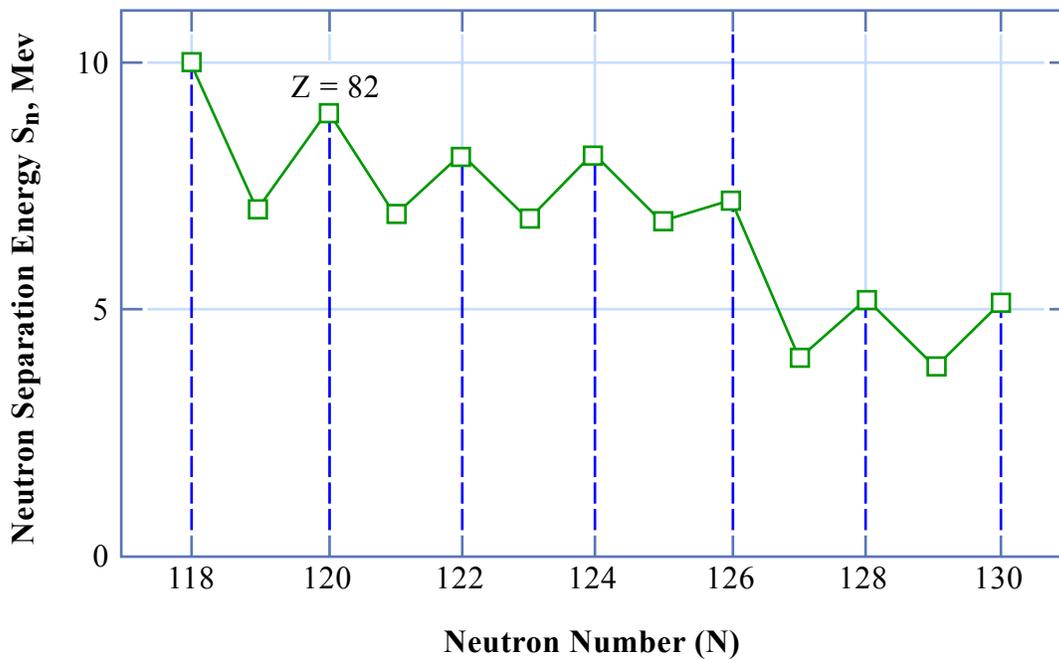


Figure by MIT OCW. Adapted from Meyerhof.

**Fig. 10.2.** Variation of the neutron separation energies of lead isotopes with neutron number of the absorbing nucleus. [from Meyerhof]

Generally speaking the following systematic behavior is observed in neutron and proton separation energies,

$$S_n(\text{even } N) > S_n(\text{odd } N) \quad \text{for a given } Z$$

$$S_p(\text{even } Z) > S_p(\text{odd } Z) \quad \text{for a given } N$$

This effect is attributed to the pairing property of nuclear forces – the existence of extra binding between pair of identical nucleons in the same state which have total angular momenta pointing in opposite directions. This is also the reason for the exceptional stability of the  $\alpha$ -particle. Because of pairing the even-even (even  $Z$ , even  $N$ ) nuclei are more stable than the even-odd and odd-even nuclei, which in turn are more stable than the odd-odd nuclei.

### ***Abundance Systematics of Stable Nuclides***

One can construct a stability chart by plotting the neutron number  $N$  versus the atomic number  $Z$  of all the stable nuclides. The results, shown in Fig. 10.3, show that  $N \sim Z$  for low  $A$ , but  $N > Z$  at high  $A$ .

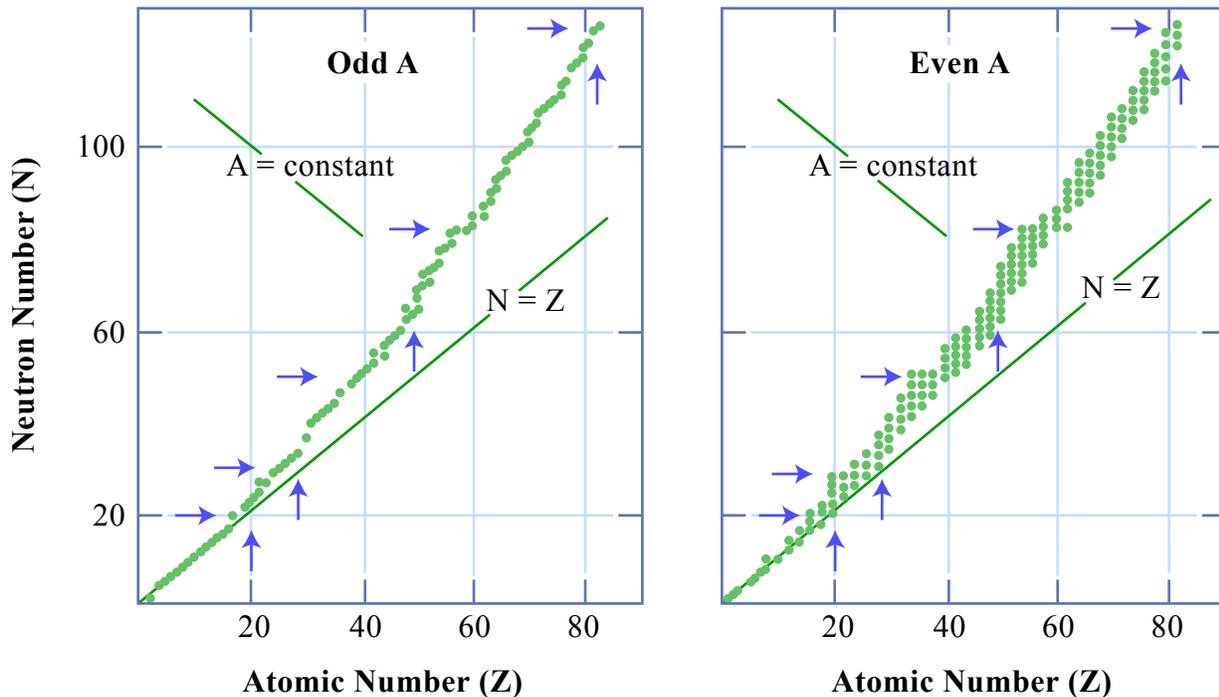


Figure by MIT OCW. Adapted from Meyerhof.

**Fig. 10.3.** Neutron and proton numbers of stable nuclides which are odd (left) and even isobars (right). Arrows indicate the magic numbers of 20, 28, 50, 82, and 126. Also shown are odd-odd isobars with  $A = 2, 6, 10,$  and  $14.$  [from Meyerhof]

Again, one can readily understand that in heavy nuclei the Coulomb repulsion will favor a neutron-proton distribution with more neutrons than protons. It is a little more involved to explain why there should be an equal distribution for the light nuclides (see the following discussion on the semi-empirical mass formula). We will simply note that to have more neutrons than protons means that the nucleus has to be in a higher energy state, and is therefore less stable. This symmetry effect is most pronounced at low  $A$  and becomes less important at high  $A$ . In connection with Fig. 10.3 we note:

- (i) In the case of odd  $A$ , only one stable isobar exists, except  $A = 113, 123.$
- (ii) In the case of even  $A$ , only even-even nuclides exist, except  $A = 2, 6, 10,$  and  $14.$

Still another way to summarize the trend of stable nuclides is shown in the following table [from Meyerhof]

### Systematics of Stability Trends in Nuclei

A	Z	N	Type	Alternative Designation	Number of Stable + Long-Lived Nuclides	Degree of Stability	Usual Number of Stable Isotopes /element
Even	Even	Even	e-e	Even mass, even N	$166 + 11 = 177$	Very pronounced	Several (2 and 3)
Odd	Even	Odd	e-o	Odd mass, odd N	$55 + 3 = 58$	Fair	1
Odd	Odd	Even	o-e	Odd mass, even N	$51 + 3 = 54$	Fair	1
Even	Odd	Odd	o-o	Even mass, odd N	$6 + 4 = 10$	Low	0
					$278 + 21 = 299$		

Figure by MIT OCW. Adapted from Meyerhof.