Toolbox: Electrical Systems Dynamics

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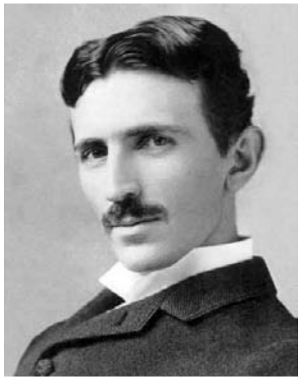
Outline

Image removed due to copyright restrictions. Please see "How Electricity Gets from the Power Station to your Home." PowerWise Teacher's Center, 2007.

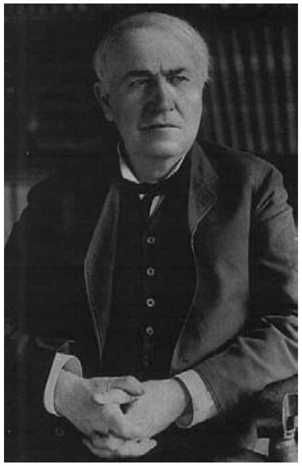
- AC and DC power transmission
- Basic electric circuits
- Electricity and the grid

"The war of the currents"

- Pros and Cons
  - One kills elephants
  - One has simpler infrastructure
- Why do we have AC and not DC?
- Look at a simple transmission circuit to decide.
- Use Voltage=120 VDC and Power=1.2 GW



Tesla

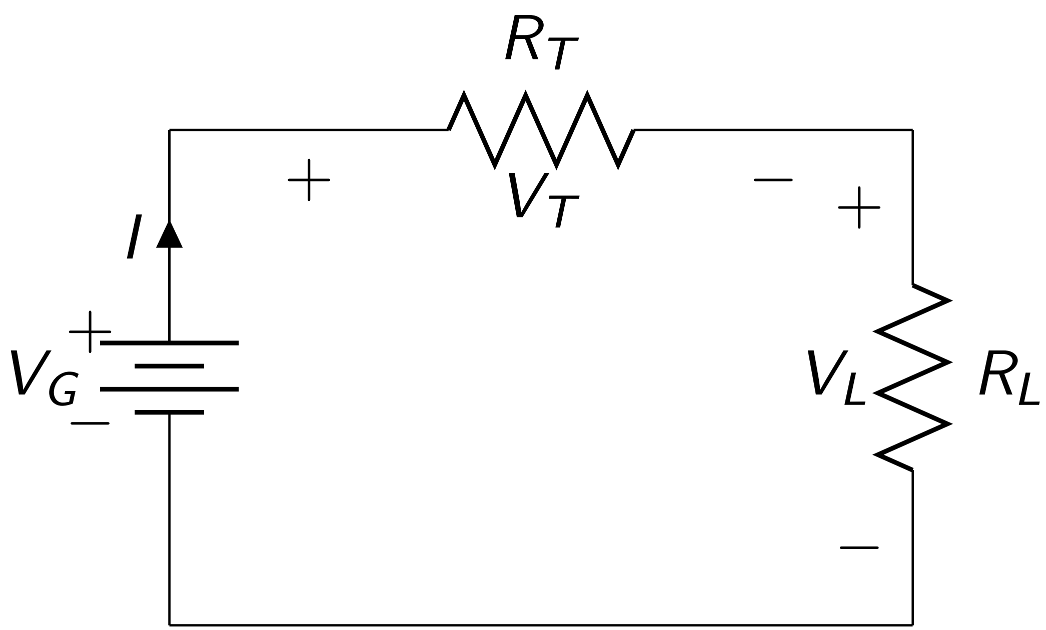


Edison

Efficient transmission requires AC

- Goals of the analysis
  - Find the generator voltage
  - Find the power delivered by the generator
  - Find the power dissipated by the transmission line
  - Find the ratio $P_{\mathrm{Trans}}/P_{\mathrm{Load}}$

A simple electric circuit



- The current $I = \frac{V_L}{R_L}$
- The power to the load $P_L = I^2 R_L = \frac{V_L^2}{R_L}$
- Equating currents (from Kirchhoff's Laws), the transmission line power $P_T = I^2 R_T = \frac{R_T V_L^2}{R_L}$
- The power ratio is then the ratio of resistances: $\frac{P_T}{P_L} = \frac{R_T}{R_L}$
- Generator power $P_G = P_L + P_T = \left(1 + \frac{R_T}{R_L}\frac{V_L^2}{R_L}\right)$
- Generator voltage $V_G = \frac{P_G}{I} = 1 + \left(\frac{R_T}{R_L}\right) V_L$

Efficiency requires most power is dissipated in the load

- If $R_T/R_L \ll 1$, then $P_T \ll P_L$
- Most of the voltage appears across the load.
- So, we have very small transmission losses.

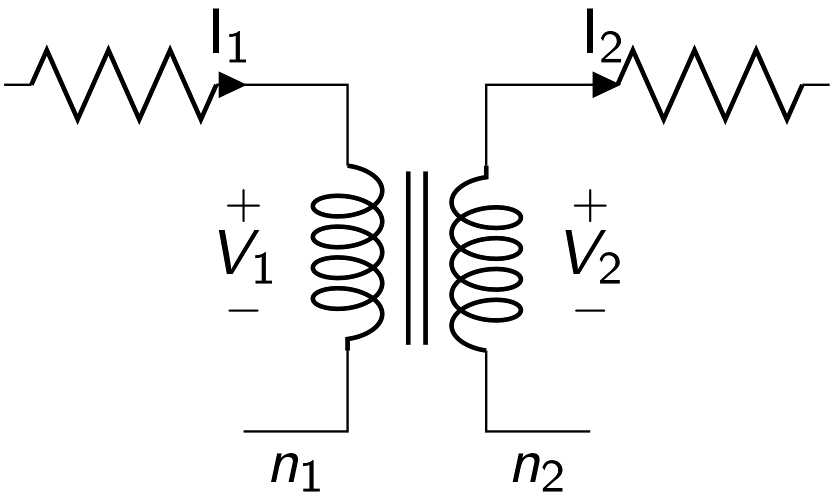
Example for Al and household V

- For $P_L = 1.2$ GW and $V_L = 120$V
- Then $R_L = P_L/V_L^2 = 1.2 \times 10^{-5}\Omega$
- For transmission assume L = 50 km (a short distance)
- An Aluminum cable with A = 5 cm^2 (to minimize sag, Cu not used.)
- Resistivity of Al, $\eta = 2.8 \times 10^{-8}\Omega$-m
- $\therefore \quad R_T = \eta L/A = 2.8\Omega \gg R_L$
- Conclusion: not so good!

AC can be used to increase the voltage

- With AC we can use transformers
- Step up the voltage at the generator
- Transmit power at high voltage, low current
- Step down the voltage at the load
- Transmitting at low current should reduce transmission losses

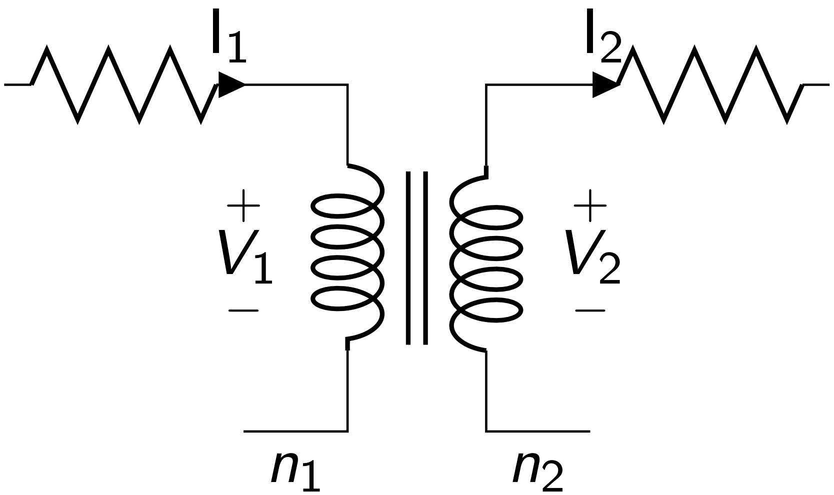
An Ideal Transformer



$$N = n_2/n_1 = \text{turns ratio}$$
$$V_2 = NV_1$$
$$I_2 = I_1/N$$

Physical process is conservation of magnetic flux/energy

An Ideal Transformer



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$$V_2 = NV_1$$
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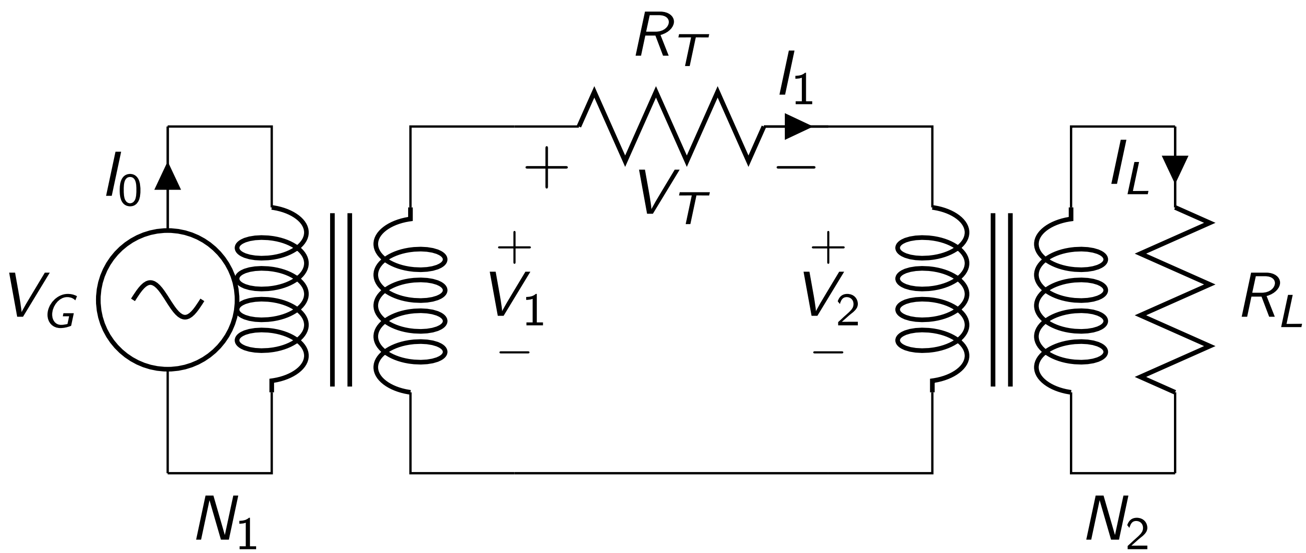
Physical process is conservation of magnetic flux/energy

Common examples of transformers:

Bottom right: Photo by mdverde on Flickr. Bottom left: Image by Tau Zero on Flickr. Top right: Photo by brewbooks on Flickr.

AC transmission reduces losses



- As before, $P_L = 1.2$ GW, $V_L = 120$ V , $R_L = 1.2 \times 10^{-5}\Omega$
- What are transformer and transmission requirements,
- Such that $P_T \ll P_L$?

- From the circuit:

$$P_L = V_L I_L = R_L I_L^2$$
$$P_T = V_T I_1 = R_T I_1^2$$

- From the transformer relation, $I_L = N_2 I_1$, it follows

$$\frac{P_T}{P_L} = \frac{1}{N_2^2}\frac{R_T}{R_L}$$

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$$\frac{P_T}{P_L} = \frac{1}{N_2^2}\frac{R_T}{R_L}$$

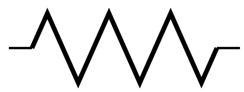
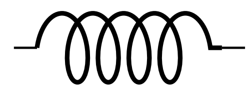
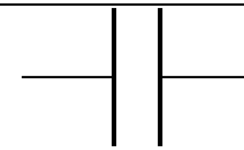
- For $N_2 \gg 1$ a huge reduction in transmission losses
- Practical numbers: $V_0 = 12kV$, $V_1 = 240kV$ (rms)
- This implies that $N_1 = V_1/V_0 = 20$
- Assume small voltage drop across the transmission line. Then $V_2 \approx V_1$
- Second turn ratio becomes $N_2 = V_2/V_L = 2000$
- Our transmission loss formula gives $P_T/P_L \approx 6\%$

The downside to AC: Reactive power

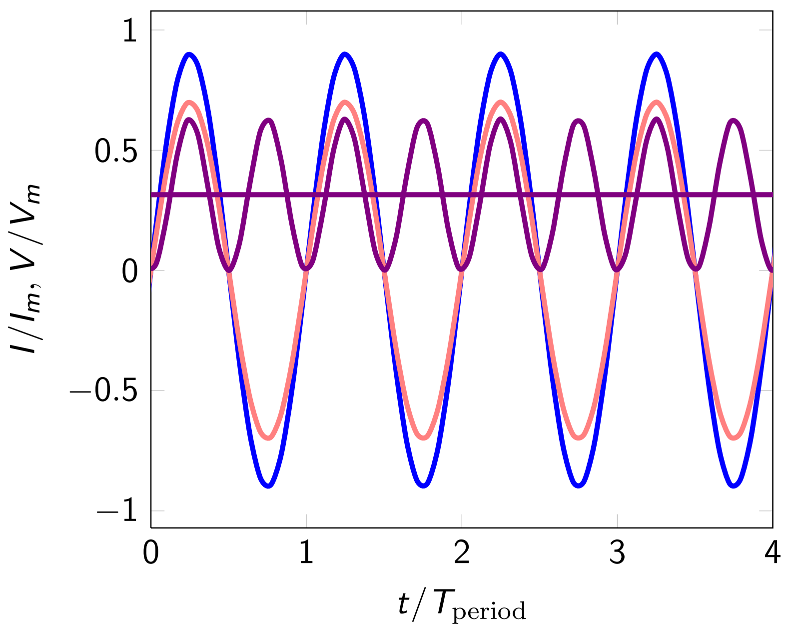
- A down side to AC: Reactive power
- Why? Load is not pure resistive
- Load usually has an inductive component
- Resistance absorbs power
- Inductor circulates power back and forth
- This oscillating power is the reactive power

Resistors, inductors and capacitors, oh my!

There are three basic circuit elements having different Ohm's laws.

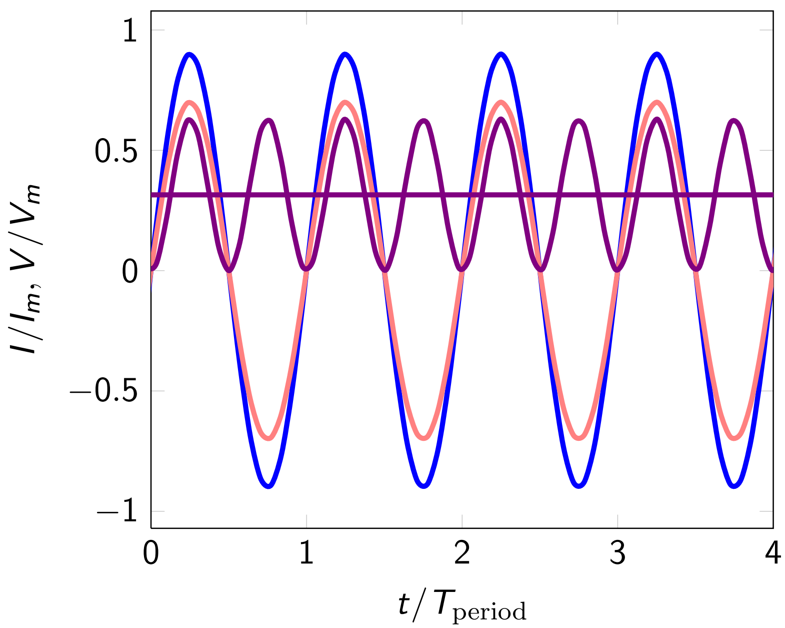
Element	Resistor	Inductor	Capacitor
Symbol	R 	L 	C 
Ohm's Law	$V = RI$	$V = L\frac{dI}{dt}$	$\frac{dV}{dt} = I/C$
$I = \sin \omega t$	$V = R \sin \omega t$	$V = L \cos \omega t$	$V = -\frac{1}{C} \cos \omega t$
Phase shift	0	$\pi/2$	$-\pi/2$
Impedance $Z[\Omega] = V/I$	R	$j\omega L$	$\frac{-j}{\omega C}$

Phase lags increase reactive power



- Current, $I = I_m \sin(\frac{2\pi t}{T_{\text{period}}})$
- Voltage, $V = V_m \sin(\frac{2\pi t}{T_{\text{period}}})$

Phase lags increase reactive power

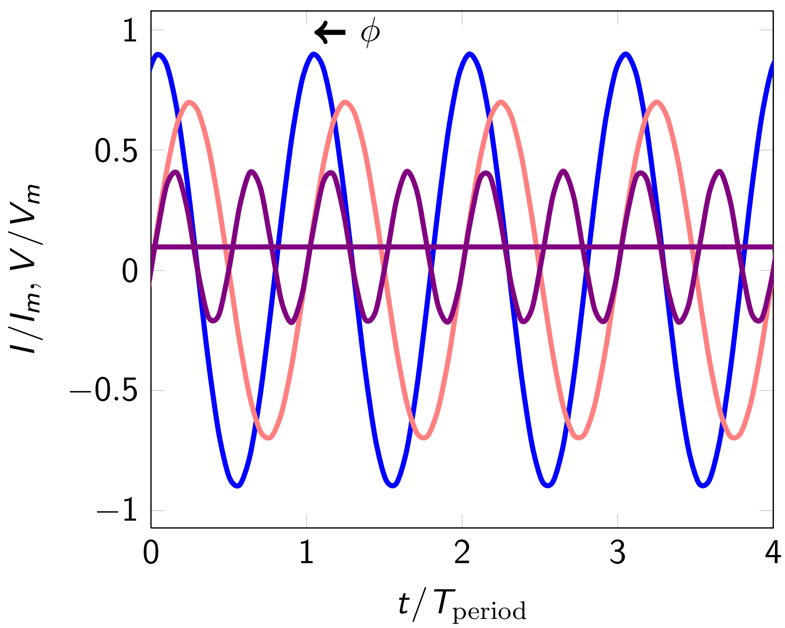


- Current, $I = I_m \sin(\frac{2\pi t}{T_{\text{period}}})$
- Voltage, $V = V_m \sin(\frac{2\pi t}{T_{\text{period}}})$
- Power,

$$P = I \cdot V = I_m V_m \sin^2(\frac{2\pi t}{T_{\text{period}}})$$

$$= \frac{1}{2} I_m V_m \left(1 - \cos(2\frac{2\pi t}{T_{\text{period}}})\right)$$

Phase lags increase reactive power



- Current, $I = I_m \sin(\frac{2\pi t}{T_{\text{period}}} + \phi)$
- Voltage, $V = V_m \sin(\frac{2\pi t}{T_{\text{period}}})$
- Power,

$$P = I \cdot V = I_m V_m \sin(\frac{2\pi t}{T_{\text{period}}}) \sin(\frac{2\pi t}{T_{\text{period}}} + \phi)$$

$$= \frac{1}{2} I_m V_m \left(\cos(\phi) - \cos(2\frac{2\pi t}{T_{\text{period}}} + \phi)\right)$$

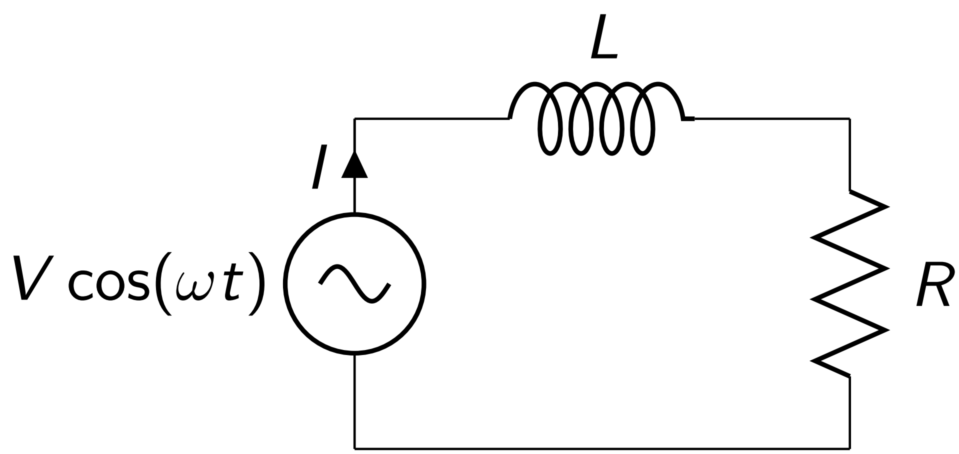
$\cos(\phi)$ is known as the power factor

Reactive power must be supplied.

- For parts of the AC cycle the instantaneous power is greater than the average power
- Generator must be able to deliver this higher power even though it is returned later
- Bottom line: generator must have a higher volt-amp rating than average power delivered: VARs and Watts.
- Higher rating $\rightarrow$ bigger size $\rightarrow$ higher cost

Phase shifts are introduced by inductance

- A motor will have an inductance, L.



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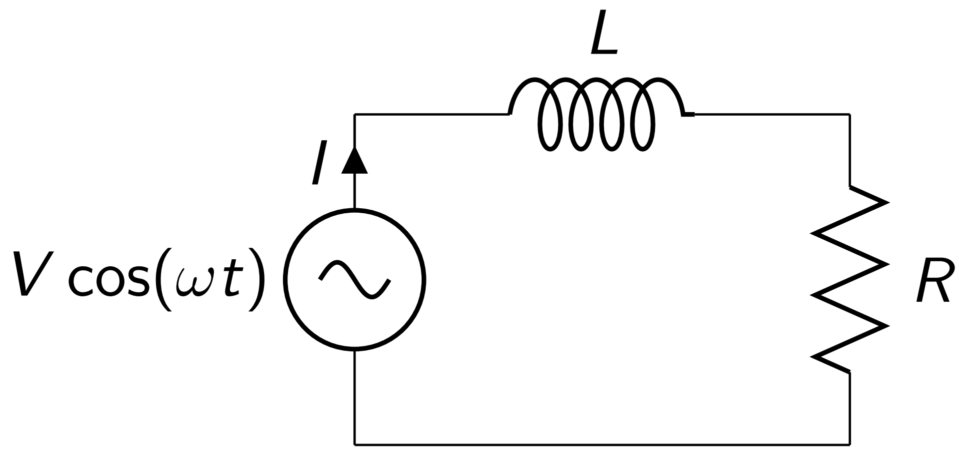

Phase shifts are introduced by inductance

- A motor will have an inductance, L.
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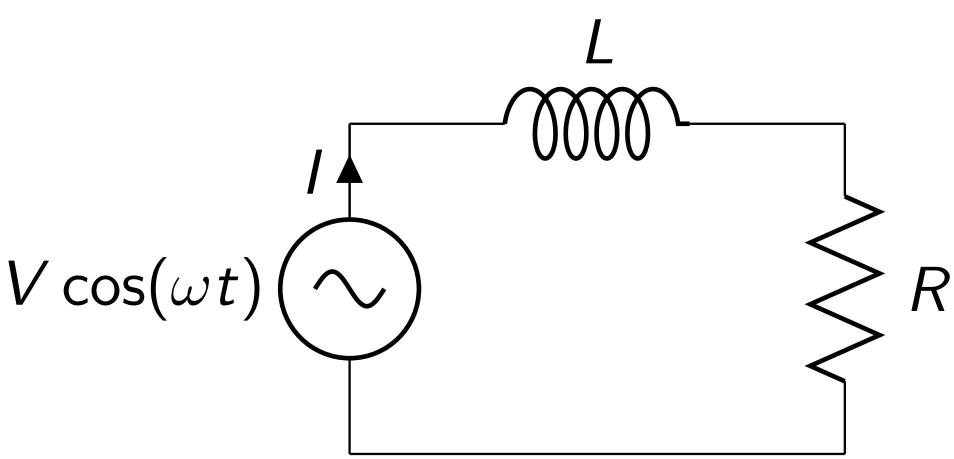
- Amplitude of current will also be reduced.

$$I = \frac{V}{(\omega^2 L^2 + R^2)^{1/2}}$$



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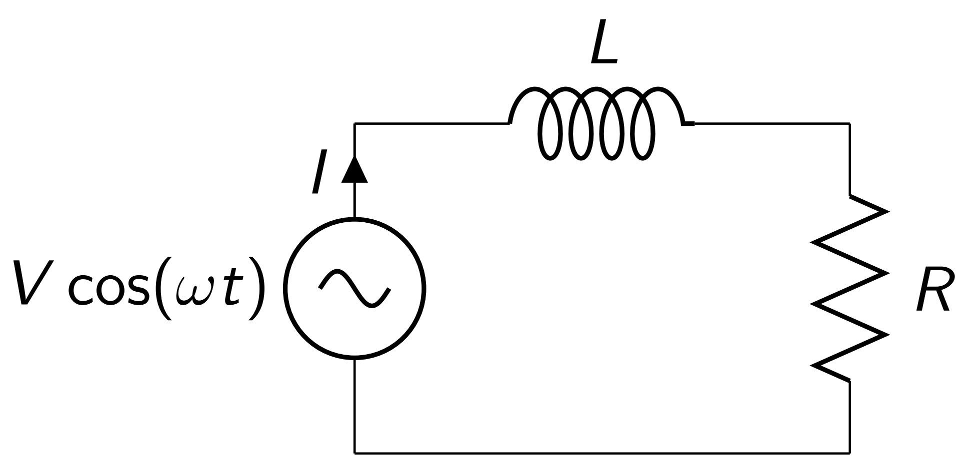
- This all follows from adding up the voltages for a simple circuit:

$$L\frac{dI}{dt} + RI = V\cos(\omega t)$$



How to minimize the VA requirement?

- To minimize the VA requirements on the generator we want $\phi \to 0$
- Assume the average power absorbed by the load is $\langle P_L \rangle$
- Calculate the peak generator power $[P_G(t)]_{\max}$ as a function of $\langle P_L \rangle$
- Note: The peak is $2\times$ (rms volt-amp rating)
- Less generator power is cheaper.

Peak power from inductance

- The power dissipated in the load:

$$\langle P_L \rangle = RI^2$$
$$= \frac{RV^2}{\omega^2 L^2 + R^2}$$

- The peak power delivered by the generator

$$P_{\text{peak}} = VI\,(1+\cos\phi) = \frac{V^2}{(\omega^2 L^2 + R^2)^{1/2}} \left(1 + \frac{R}{(\omega^2 L^2 + R^2)^{1/2}}\right)$$

- Using the expression for $\langle P_L \rangle$, we get:

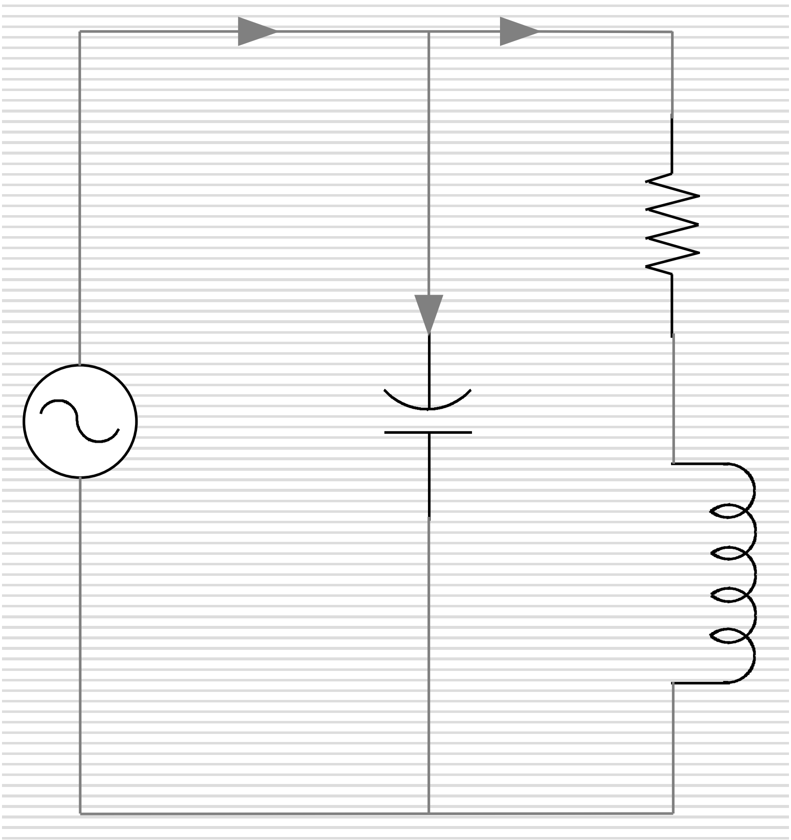
$$\frac{P_{\text{peak}}}{2\,\langle P_L \rangle} = \frac{\left(\omega^2 L^2 + R^2\right)^{1/2} + R}{2R} \geq 1$$

Capacitance can balance out reactive power

- Recall from our table that the phase lags are opposite for inductance and capacitance
- $\tan\phi_L = \frac{\omega L}{R}$, $\tan\phi_C = \frac{-1}{\omega C}$
- Short answer: there is a capacitance that will keep the current and voltage in phase (but not eliminate the power factor)

$$C = \frac{L}{R^2 + \omega^2 L^2}$$

- Long answer follows.



Analysis

- I – V relation for a capacitor

$$\hat{I}_1(t) = C\frac{dV_G}{dt}$$

- I – V relation for the load

$$V_G = R\hat{I}_2 + L\frac{d\hat{I}_2}{dt}$$

- Conservation of current

$$\hat{I}(t) = \hat{I}_1(t) + \hat{I}_2(t)$$

Solution

- Assume $V_G = V\cos(\omega t)$ (all voltages now rms)
- Current in the capacitor branch

$$\hat{I}_1(t) = -\omega C V \sin(\omega t)$$

- Current in the load branch (from before)

$$\hat{I}_2(t) = \frac{V}{\left(R^2 + \omega^2 L^2\right)^{1/2}} \cos(\omega t - \phi)$$

The total current

- The total current flowing from the generator

$$\hat{I}(t) = \hat{I}_1(t) + \hat{I}_2(t)$$

$$= V\left[\frac{\cos(\omega t - \phi)}{\left(R^2 + \omega^2 L^2\right)^{1/2}} - \omega C \sin(\omega t)\right]$$

$$= V\left\{\frac{\cos(\omega t)\cos\phi}{\left(R^2 + \omega^2 L^2\right)^{1/2}} + \left[\frac{\sin\phi}{\left(R^2 + \omega^2 L^2\right)^{1/2}} - \omega C\right]\sin(\omega t)\right\}$$

The value of C

- Choose C for zero reactive power
- Set sin(t) coefficient to zero

$$\omega C = \frac{\sin \phi}{\left(R^2 + \omega^2 L^2\right)^{1/2}}$$

- Simplify by eliminating the power factor

$$C = \frac{L}{\left(R^2 + \omega^2 L^2\right)}$$

Calculate the peak power

- Calculate the peak power to learn what has happened to the VA rating

$$
\begin{aligned}
P_{peak} &= \left[P_G(t)\right]_{\max} = 2\left[VI\right]_{\max} \\
&= 2V^2 \frac{\cos\phi}{\left(R^2 + \omega^2 L^2\right)^{1/2}} \\
&= 2\frac{RV^2}{\left(R^2 + \omega^2 L^2\right)} \\
&= 2\left\langle P_L \right\rangle
\end{aligned}
$$

The Result

- It worked!!
- The VA requirement has been reduced

$$VA = \frac{P_{peak}}{2} = \langle P_L \rangle$$

Discussion

- AC is good for transmission
- Have to manage reactive power

Other aspects:

- HVDC transmission lines
- AC losses from corona discharge
- Voltage and frequency tolerances
- Stability of the grid to perturbations, eg a power plant going offline or a transmission line going down.

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