# 1.818J/2.65J/3.564J/10.391J/11.371J/22.811J/ESD166J

# SUSTAINABLE ENERGY


Prof. Michael W. Golay

Nuclear Engineering Dept.

# RESOURCE EVALUATION AND DEPLETION ANALYSES

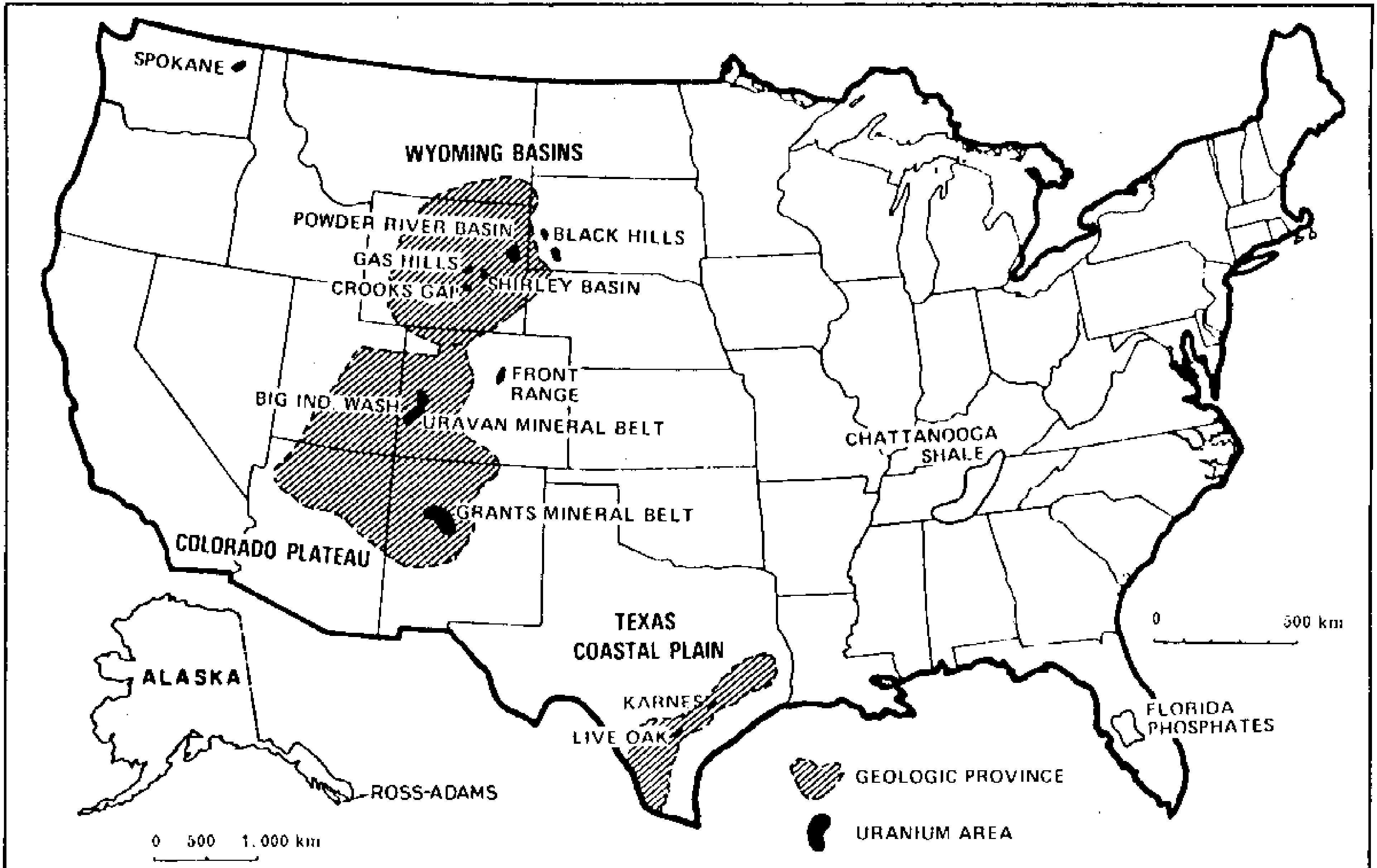
# WAYS OF ESTIMATING ENERGY RESOURCES

- Monte Carlo
- "Hubbert" Method Extrapolation
- Expert Opinion (Delphi)

# FACTORS AFFECTING RESOURCE RECOVERY

- Nature of Deposit
- Fuel Price
- Technological Innovation
  - Deep drilling
  - Sideways drilling
  - Oil and gas field pressurization
  - Hydrofracturing
  - Large scale mechanization

# URANIUM AREAS OF THE U.S.




Courtesy of U.S. Atomic Energy Commission.

# MAJOR SOURCES OF URANIUM

Class 1 – Sandstone Deposits

| | Share | $U^{3}O^{8}$ Concentration (Percent) | Tons $U^{3}O^{8}$ |
|---|---|---|---|
| New Mexico | .49 | 0.25 | Total |
| Wyoming | .36 | 0.20 | 315,000 |
| Utah | .03 | 0.32 | Š $10/lb |
| Colorado | .03 | 0.28 | |
| Texas | .06 | 0.28 | |
| Other | .03 | 0.28 | |
| Class 2 – Vein Deposits | | | 7,100 |
| Class 3 – Lignite Deposits | | 0.01-0.05 | 1,200 |
| Class 4 – Phosphate Rock | | 0.015 | |
| Class 5 – Phosphate Rock Leached Zone (Fla.) | | 0.010 | 54,600 |
| Class 6 – Chattanooga Shale | | 0.006 | 2,557,300 |
| Class 7 – Copper Leach Solution Operations | | 0.0012 | 30,000 |
| Class 8 – Conway Granite | | 0.0012-Uranium<br>0.0050-Thorium | $1x10^{6}$<br>$4x10^{6}$ |
| Class 9 – Sea Water | | $0.33x10^{-6}$ | $4x10^{9}$ |

# ESTIMATES OF URANIUM AVAILABILITY FROM GEOLOGICAL FORMATIONS AND OCEANS IN THE U.S.

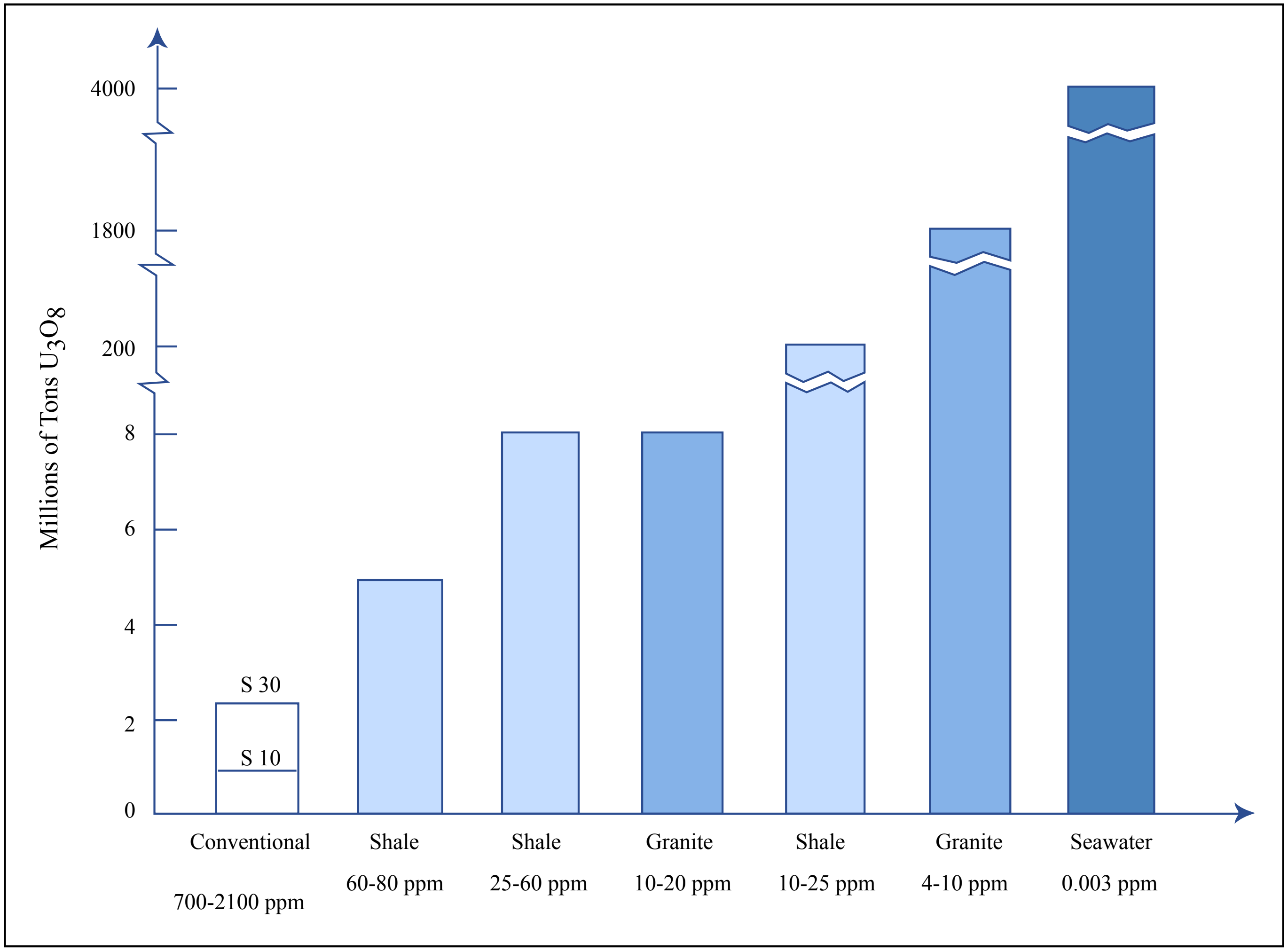



Image by MIT OpenCourseWare.

# DECLINE IN GRADE OF MINED COPPER ORES SINCE 1925

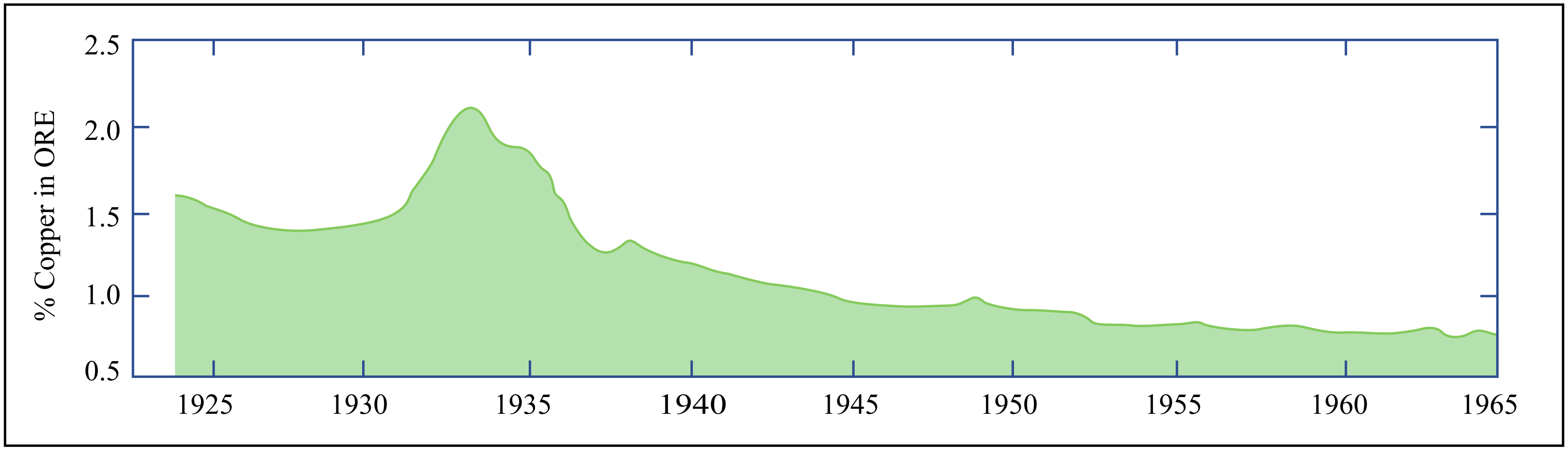
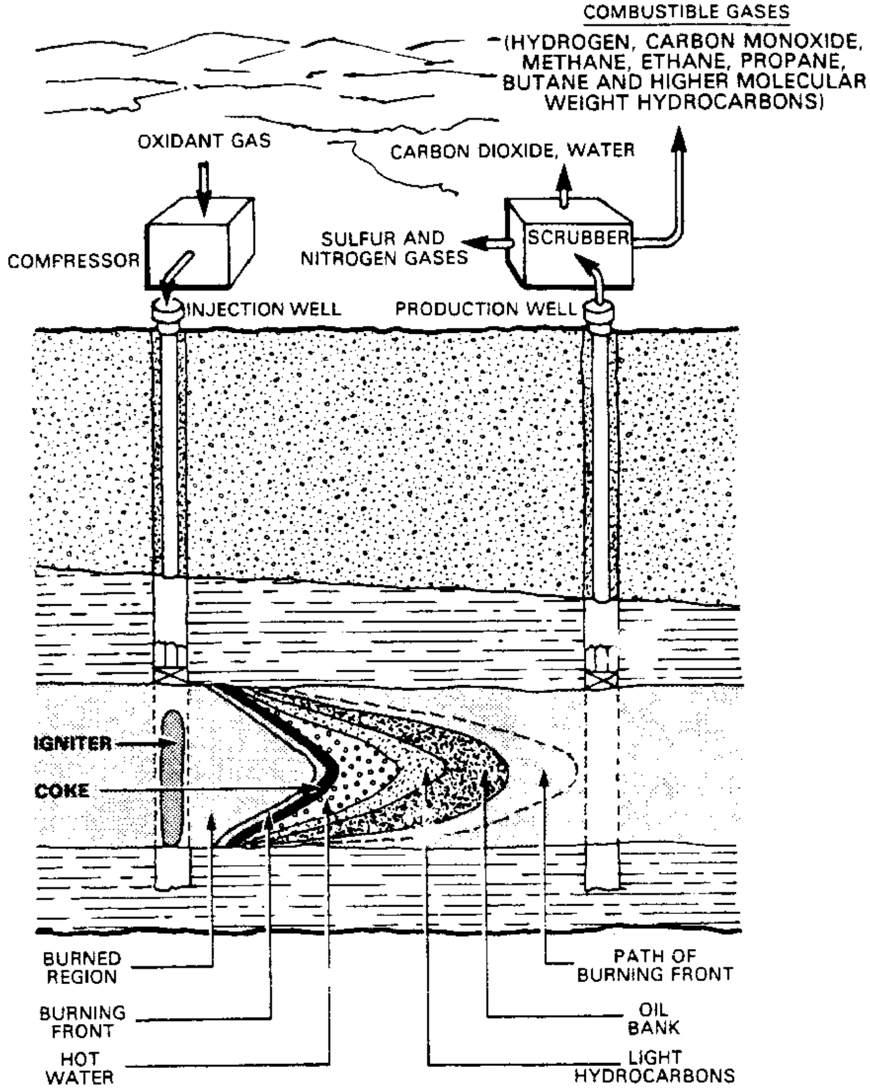


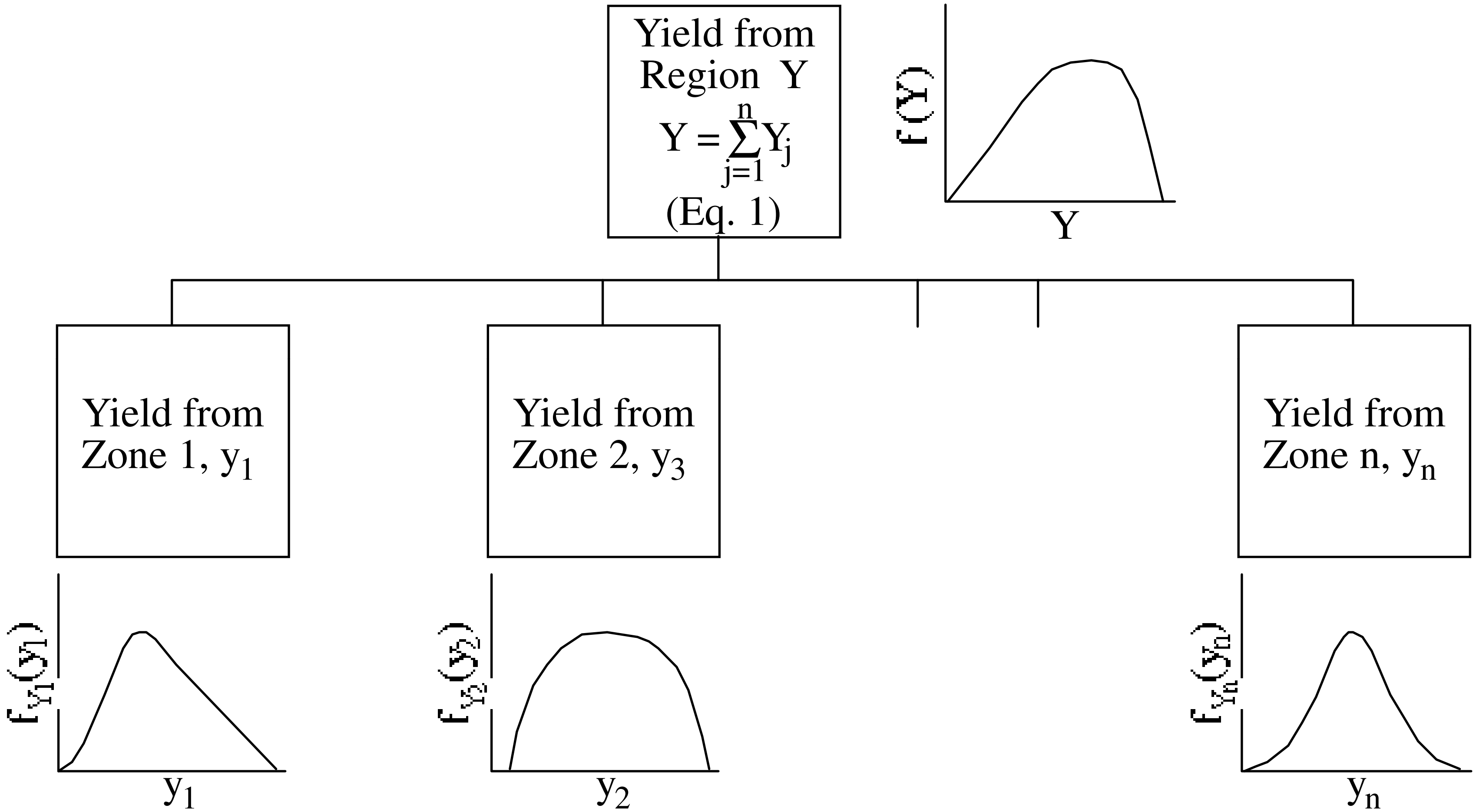

Image by MIT OpenCourseWare.

# RECOVERY BY IN-SITU COMBUSTION




Source: U.S. Department of Energy, "Fossil Energy Research and Development Program of the U.S. Department of Energy, FY 1979," DOE/ET-0013(78), March 1978.

# MONTE CARLO ESTIMATION




Probability density functions are obtained subjectively, using information about deposit characteristics, fuel price, and technology used.

# MONTE CARLO ESTIMATION OF THE PROBABILITY DENSITY FUNCTION OF A FUNCTION OF A SET OF RANDOM VARIABLES, AS

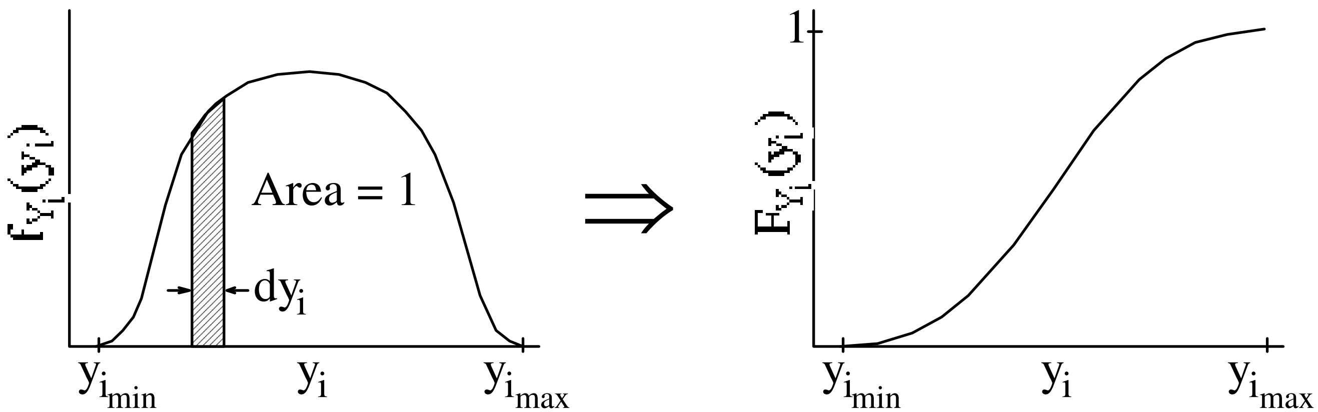
$$G = G(\overline{Z}), \text{ where} \qquad \text{(Eq. 1)}$$

$$\overline{Z} = [y_1, y_2, \ldots, y_n] \text{ and}$$

$Y_i$ is a random variable (i = 1,n)

Note that $\overline{Z}$ and G are also random variables.

# MONTE CARLO ESTIMATION OF PROBABILITY DENSITY AND CUMULATIVE DISTRIBUTION FUNCTIONS



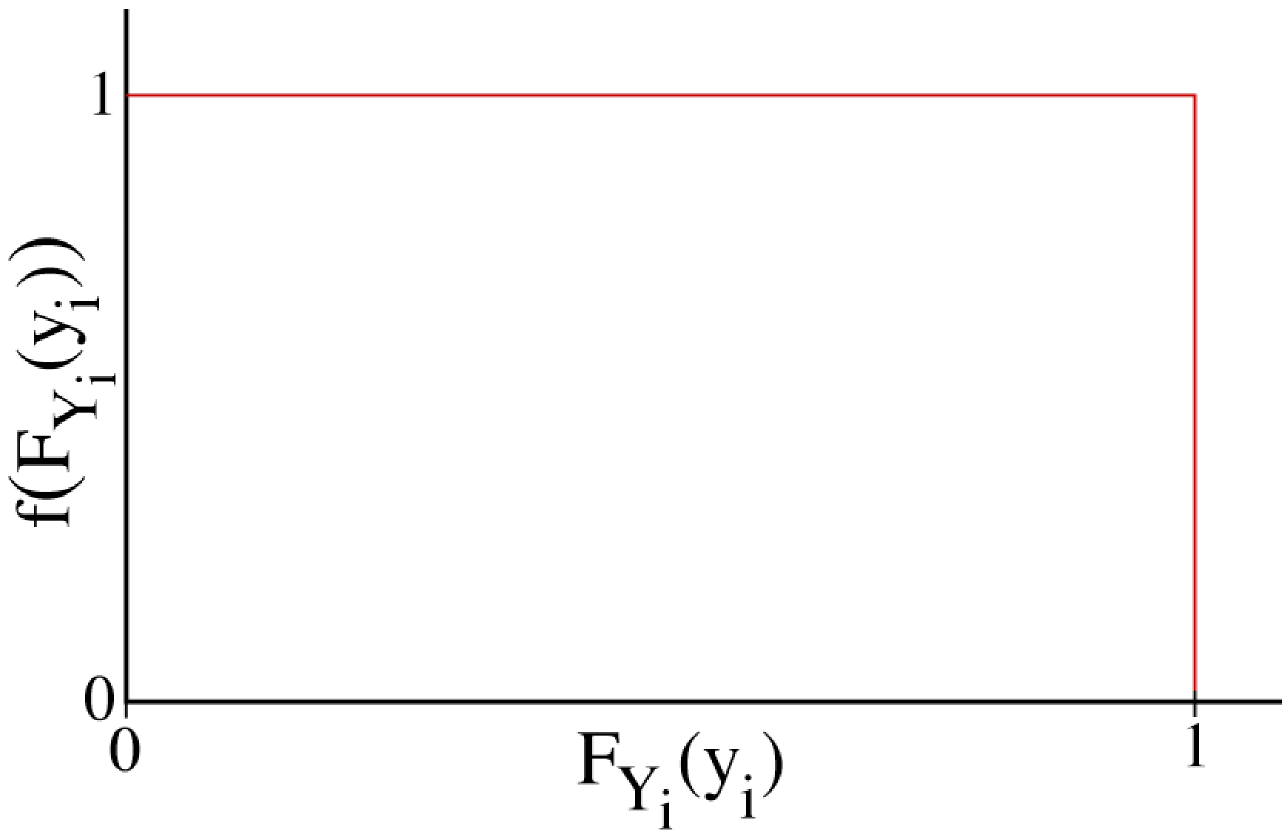

$$\text{Prob.}(y_i < Y_i < y_i + \delta y_i) = f_{Y_i}(y_i)\,dy_i \quad \text{(Eq. 2)}$$

$$\text{Prob.}(Y_i < y_i) = F_{Y_i}(y_i) = \int_{y_{i_{min}}}^{y_i} f_{Y_i}(y_i')\,dy_i' \quad \text{(Eq. 3)}$$

Consider $Y_i$ to be a random variable within $[y_{i_{min}},\ y_{i_{max}}]$

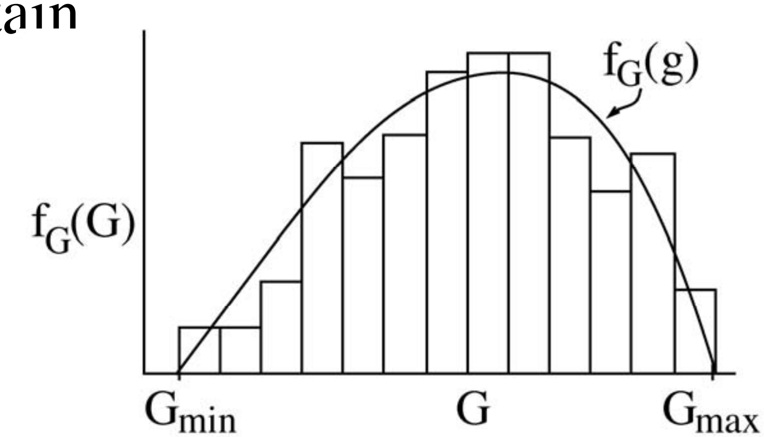
# MONTE CARLO ESTIMATION, Continued

Note: $F_{Y_i}(y_i)$ is uniformly distributed within [0, 1]



# MONTE CARLO ESTIMATION, Continued

1. Utilize a random number generator to select a value of $F(y_i)$ within range [0, 1] $\Rightarrow$ corresponding value of $y_i$ (Eq. 3).
2. Repeat step 1 for all values of i and utilize selected values of $\bar{Z}_1 = \left[y_{1_1}, \quad y_{2_1}, \quad \cdots, \quad y_{n_1}\right]$ to calculate a value of $\bar{Z}_1$ (Eq. 1), (note $\bar{Z}$ is also a random variable).
3. For the k-th set of selected values of $\bar{Z}_K = \left[y_{1_K}, y_{2_K}, \cdots, y_{n_K}\right]$ can obtain the corresponding value of $G_K = G\left(K/\bar{Z}_K\right)$
4. Repeat step 2 many times and obtain a set of values of vector $\bar{Z}$, and corresponding value of $G_k$.
5. Their abundance distributions will approximate those of the pdfs of the variables $\bar{Z}$ and $G\left(\bar{Z}\right)$ as



# M. KING HUBBERT'S MINERAL RESOURCE ESTIMATION METHOD

ASSUMED CHARACTERISTICS OF MINERAL RESOURCE EXTRACTION

- As More Resource Is Extracted The Grade Of The Marginally Most Attractive Resources Decreases, Causing
  - Need for improved extraction technologies
  - Search for alternative deposits, minerals
  - Price increases (actually, rarely observed)

# M. KING HUBBERT'S MINERAL RESOURCE ESTIMATION METHOD, Continued

POSTULATED PHASES OF MINERAL RESOURCE EXTRACTION

- Early: Low Demand, Low Production Costs, Low Innovation
- Growing: Increasing Demand And Discovering Rate, Production Growing With Demand, Start of Innovation
- Mature: Decreasing Demand And Discovery Rate, Production Struggling To Meet Demand, Shift To Alternatives
- Late: Low Demand, Production Difficulties, Strong Shift To Alternatives (rarely observed)

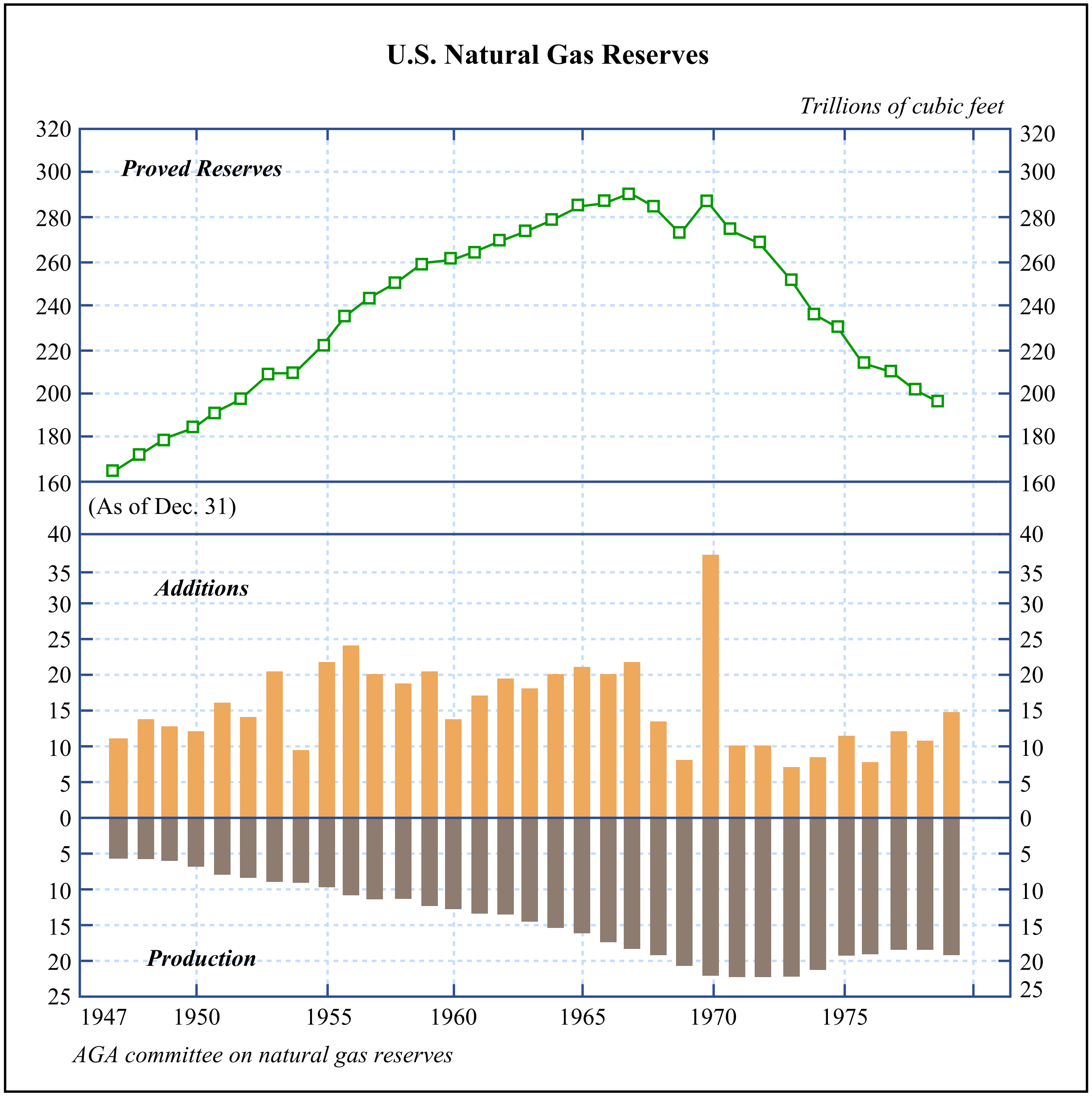
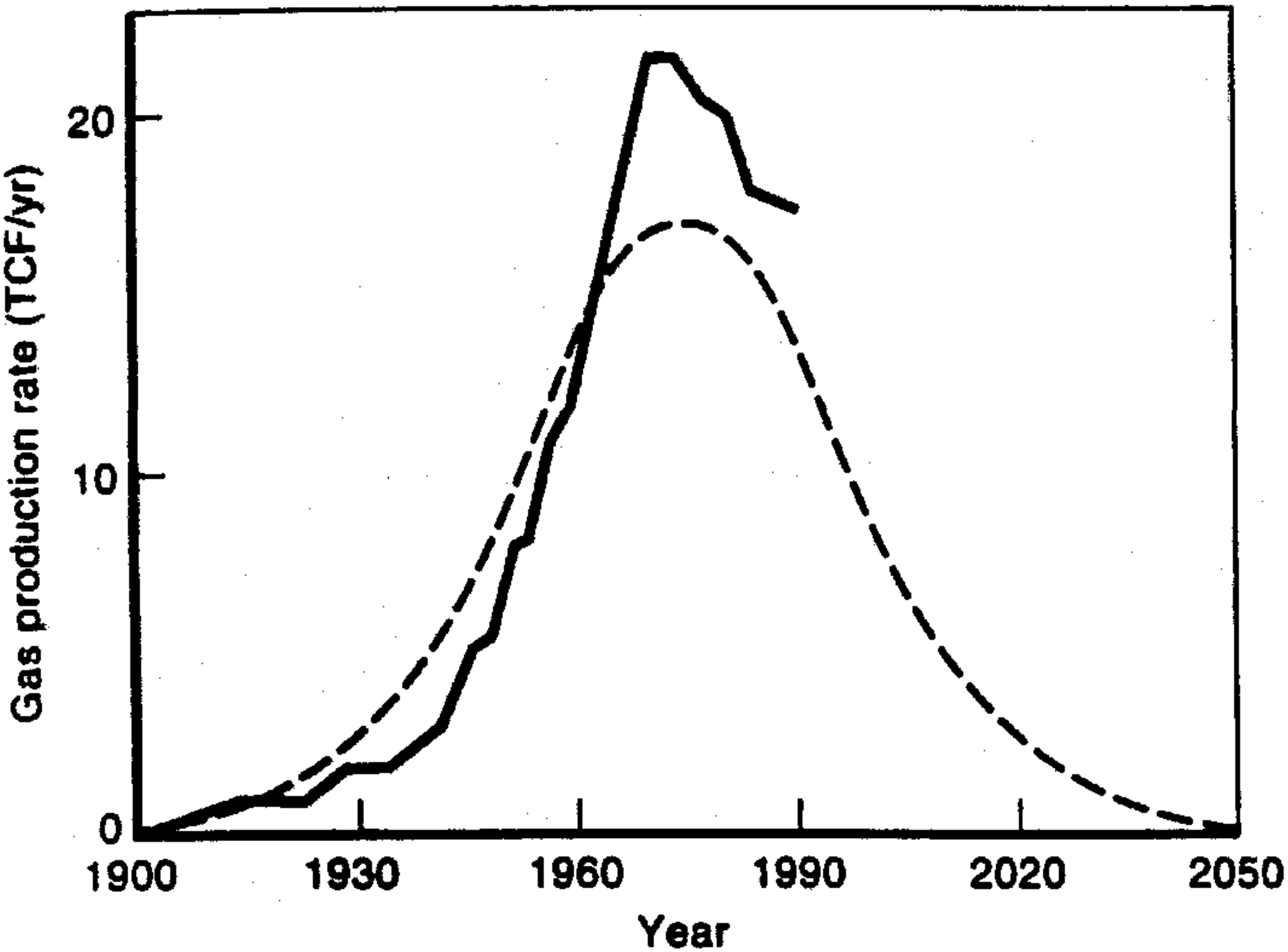


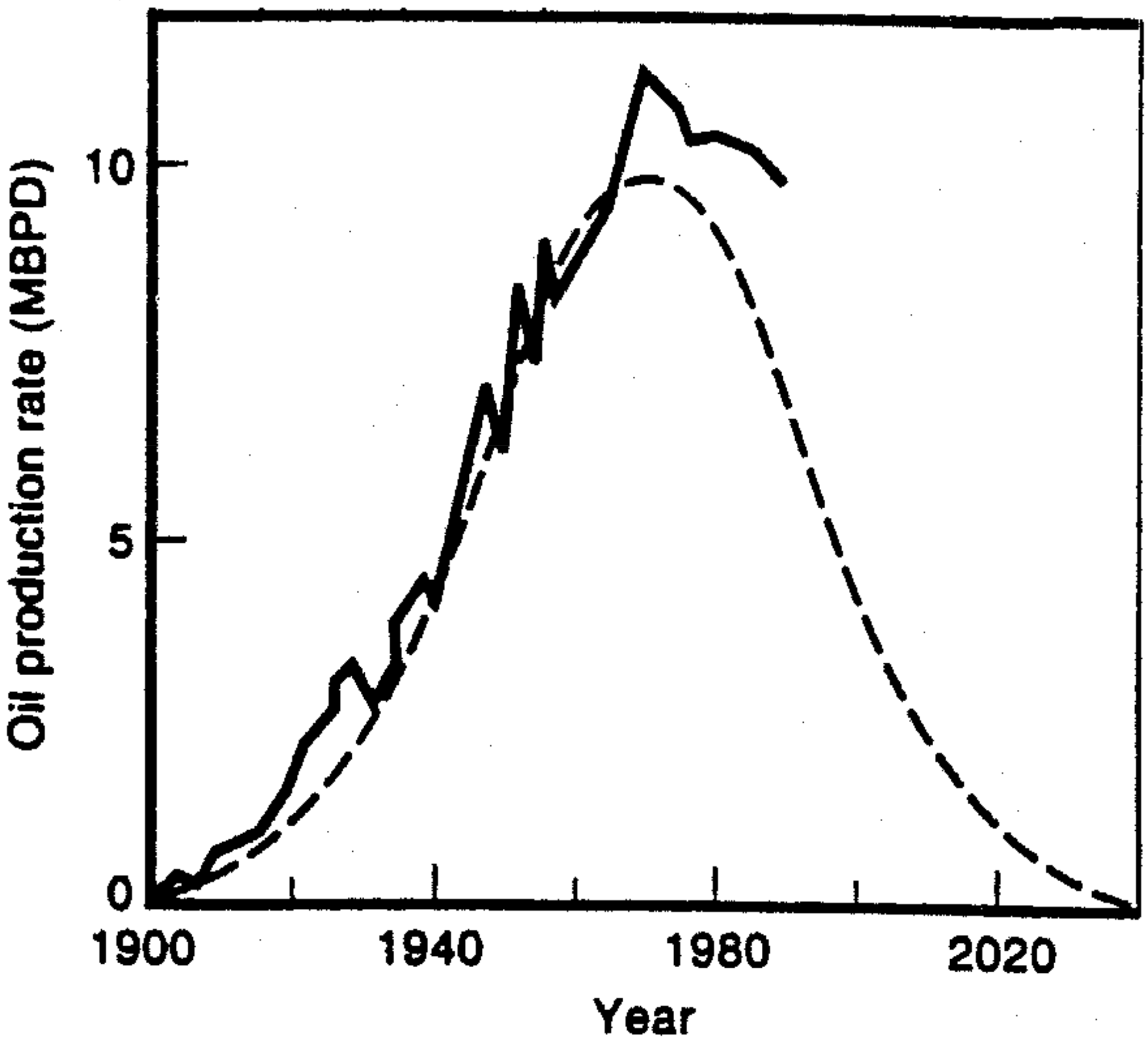

Image by MIT OpenCourseWare. Adapted from the American Gas Association.

# U.S. NATURAL GAS PRODUCTION



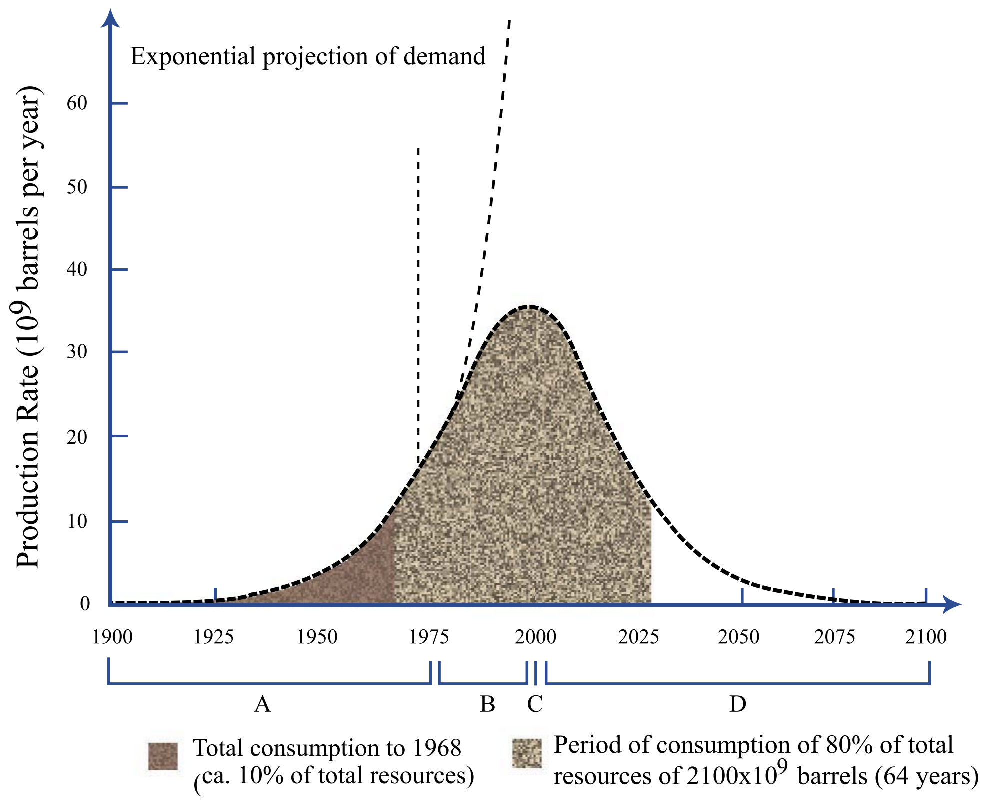

Comparison of estimated (Hubbert) production curve and actual production (solid line).

# U.S. CRUDE OIL PRODUCTION




Comparison of estimated (Hubbert) production curve and actual production (solid line).

# COMPLETE CYCLE OF WORLD CRUDE-OIL PRODUCTION




A: Exponentially growing demands are supplied.

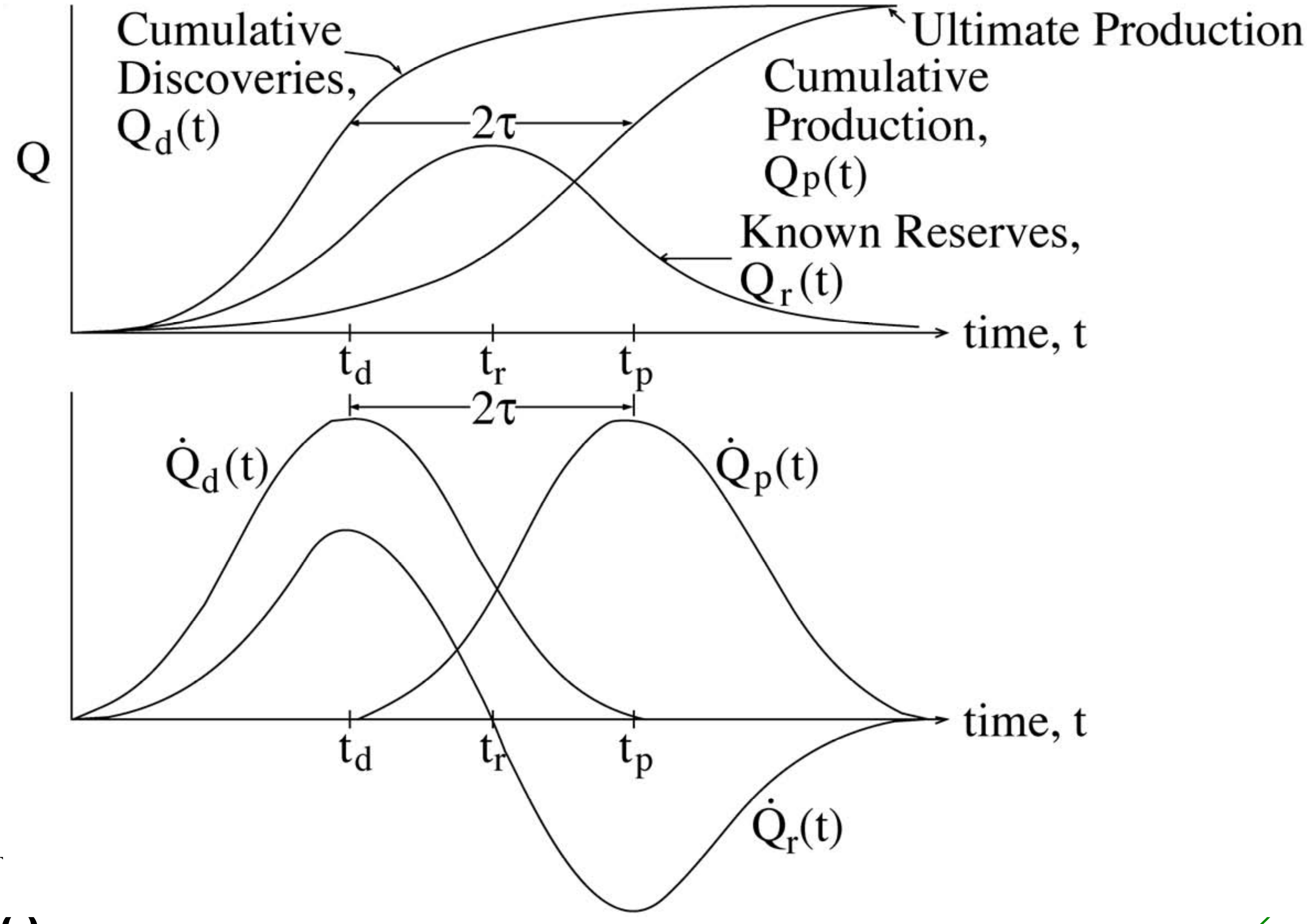
B: Discovery and extraction become increasingly difficult; production is unable to meet demands.

C: Maximum attainable production rate

D: Declining production, ultimately decaying exponentially

Image by MIT OpenCourseWare.

# RESOURCE BEHAVIOR UNDER "HUBBERT" ASSUMPTIONS




Timing:

$t_d$, $t_r$, $t_p$ respecti

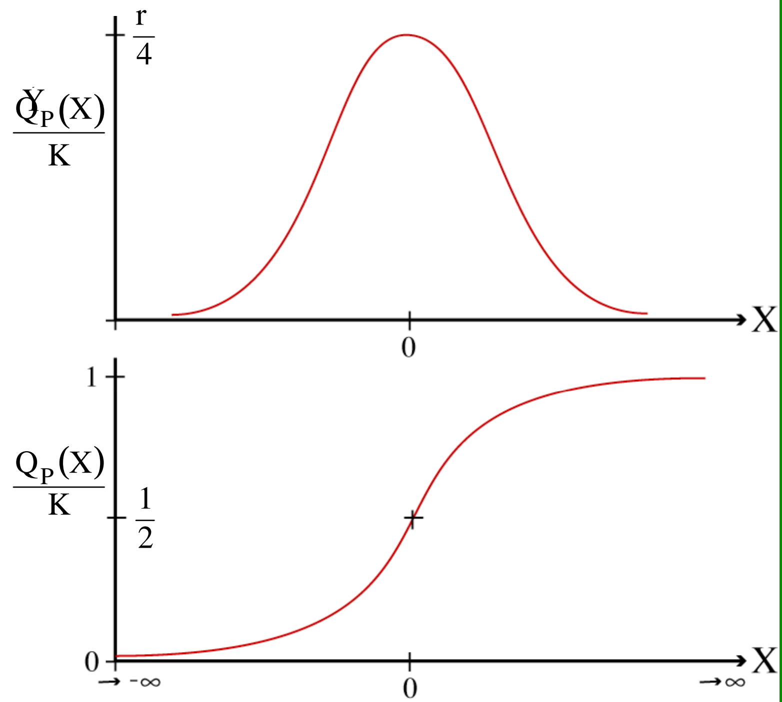
$Q_d$, $Q_r$, $Q_p$.

# LOGISTIC FUNCTION

Hubbert's assumed "logistical' relationships

Rate of Production

$$\dot{Q}_P = \frac{d\left[Q_P(X)\right]}{dt} = rQ_P(X)\left[1 - \frac{Q_P(X)}{K}\right],$$




Cumulative Production

$$Q_P(X) = \frac{KQ_{P_o}(X)}{Q_{P_o}(X) - \left(Q_{P_o}(X) - K\right)e^{-rX}}$$

where r = relative rate of change
K = carrying capacity or
ultimate production value

$$Q_{P_o} \equiv Q_P(X = 0)$$

$Q_P(X)$ = cumulative production at time, $(t - t_o) = X$

# LOGISTIC FUNCTION, continued

$$Q_P(-\infty) = 0 \qquad Q_P(0) = \frac{K}{2} = Q_{P_O} \qquad P(\infty) = K$$

$$\dot{Q}(x) \equiv \left.\frac{d\,Q_P(X)}{d\,t}\right|_{max} = \left.\frac{d\,Q_P(X)}{d\,X}\right|_{X=0} = \frac{r\,K}{4}.$$

$$Q_P(X) \approx \int_{-\infty}^{X} \frac{K}{\sqrt{2\pi\sigma}}\, e^{-1/2\left(\frac{X'}{\sigma}\right)^2} d\,X', \text{ where } \sigma = \sqrt{\frac{8}{\pi}\frac{1}{r}}.$$

Let: $t - t_o \equiv X.$

# EQUATIONS

Conservation of Resource:

$$Q_d(t) = Q_r(t) + Q_p(t) \qquad \text{(Eq. 4)}$$

Rate Conservation:

$$\dot{Q}_d(t) = \dot{Q}_r(t) + \dot{Q}_p(t) \qquad \text{(Eq. 5)}$$

Approximate Results:

$$t\left(\dot{Q}_d = 0\right) - t\left(\dot{Q}_r = 0\right) = 2\tau \qquad \text{(Eq. 6)}$$

$$\tau \approx \begin{cases} \left(t_r - t_p\right) \\ \left(t_d - t_r\right) \end{cases} \qquad \text{(Eq. 7)}$$

or

$$t_r \approx \frac{1}{2}\left(t_d + t_p\right) \qquad \text{(Eq. 8)}$$

$$Q_{p\ \text{ultimate}} \approx 2Q_d\left(t_d\right) \qquad \text{(Eq. 9)}$$

# EQUATIONS, Continued

If we assume Gaussian distributions for $Q_r(t)$, $\dot{Q}_d(t)$ and $\dot{Q}_p(t)$ with each having the same standard deviation, $\sigma$, obtain

$$Q_r(t) = \frac{Q_{r_o}}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{t-t_r}{\sigma}\right)^2\right] \quad \text{(Eq. 10)}$$

$$\dot{Q}_d(t) = \frac{Q_{d_o}}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{t-t_d}{\sigma}\right)^2\right] \quad \text{(Eq. 11)}$$

$$\dot{Q}_p(t) = \frac{Q_{p_o}}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{t-t_p}{\sigma}\right)^2\right], \quad \text{(Eq. 12)}$$

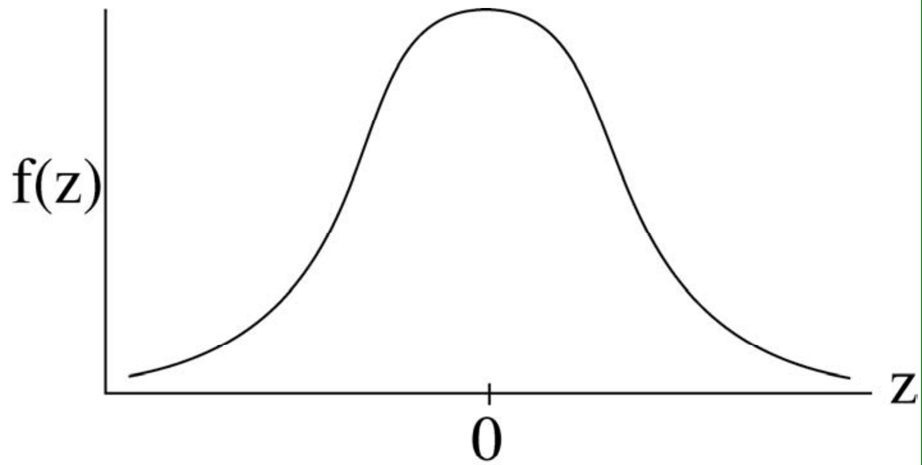
Then, when $Q_r$ is at a maximum $t = t_r$ and $\dot{Q}_r = 0$

$$\ddot{Q}_r(t_r) = \frac{-1xQ_{r_o}}{\sigma^2} \Rightarrow \boxed{\sigma^2 = \frac{-1xQ_r(t_r)}{\ddot{Q}_r(t_r)}}, \text{ or} \quad \text{(Eq. 13)}$$

# EQUATIONS, Continued

For the normal distribution

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$




$$f(1) = 0.67, \quad z \equiv \frac{t - t_o}{\sigma} = 1$$

$\Rightarrow \sigma \cong 25$ yr. for U.S. petroleum and natural gas

$$F(z) = \int_{-\infty}^{z} f(z')dz', \text{ Cumulative distribution function}$$

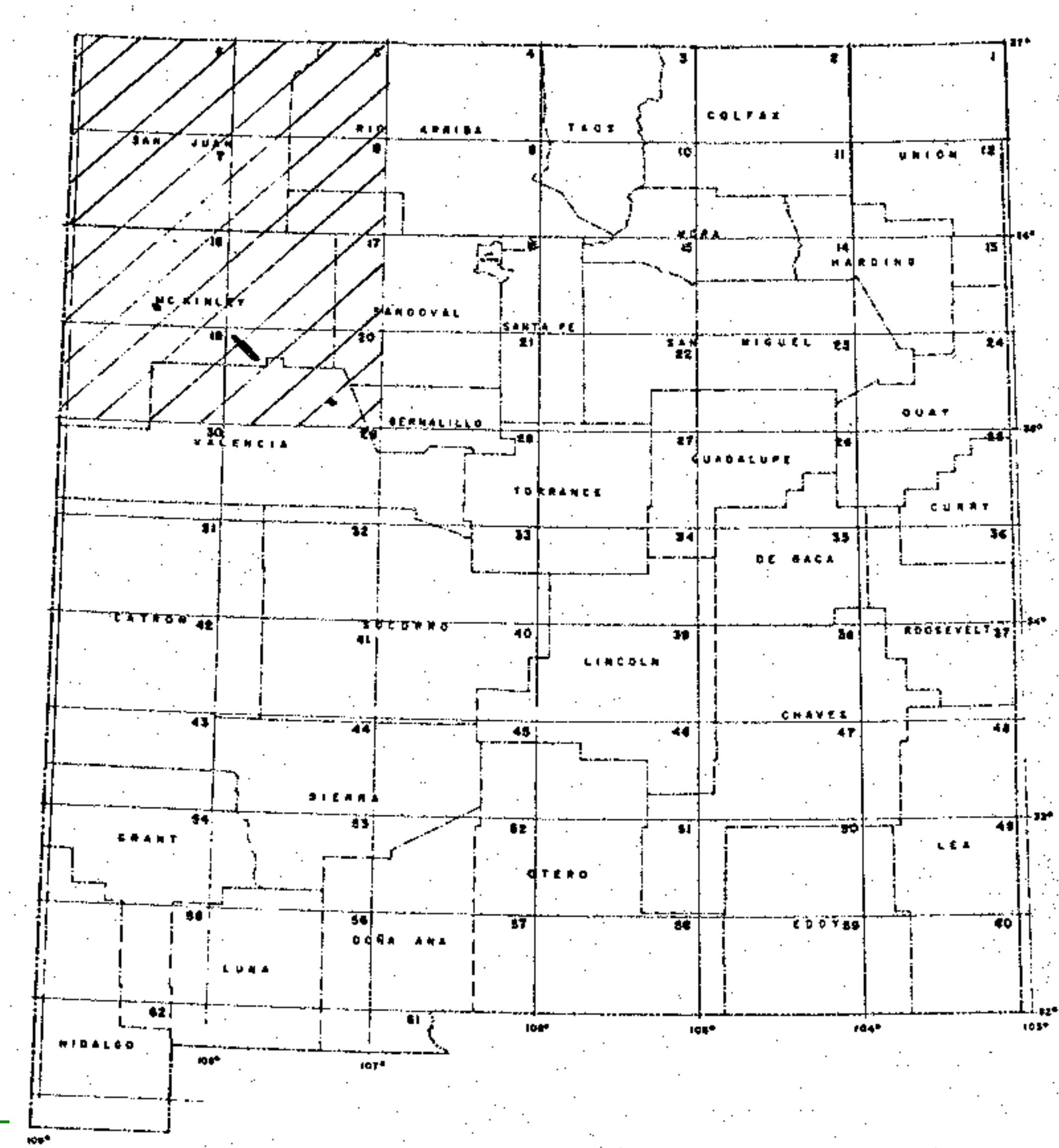
F(3) = 0.99, Approximately the state of full depletion

$\Rightarrow$ Time of exploitation $\cong 6\sigma = 150$ yrs

$\Rightarrow$ End date of major U.S. oil, natural gas production

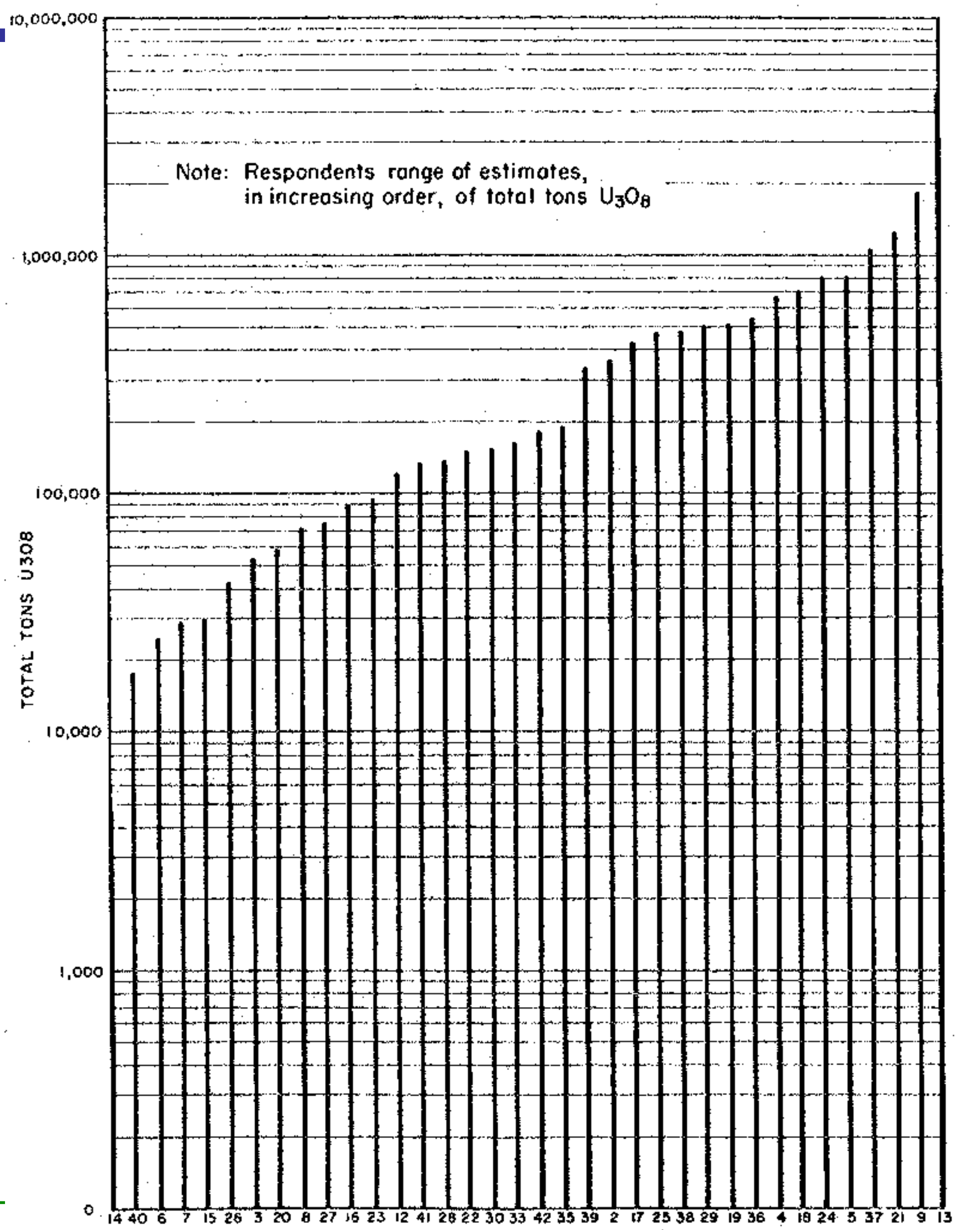
= 1900 + 150 = **<u>2050</u>**

# SUBJECTIVE PROBABILITY STUDY – STATE OF NEW MEXICO




Courtesy of U.S. Atomic Energy Commission.

# NEW MEXICO SUBJECTIVE PROBABILITY STUDY (AFTER DELPHI)




Courtesy of U.S. Atomic Energy Commission.

22.081J / 2.650J / 10.291J / 1.818J / 2.65J / 10.391J / 11.371J / 22.811J / ESD.166J
Introduction to Sustainable Energy
Fall 2010