

Short Questions (10% each)

- a) The volumetric heat generation rate can be expressed as the product of the fission energy ($\varepsilon=200$ MeV/fission) times the fission reaction rate (fissions/cm³-second):

$$q''' = \varepsilon \sigma_f N_{235} \phi \quad (1)$$

where $q'''=200$ W/cm³, $\sigma_f=577$ b and the number density of ²³⁵U, can be found as follows:

$$N_{235} = \frac{x_{235} \rho_U}{A_{235}} N_{av} \approx 9.5 \times 10^{20} \text{ nuclei/cm}^3 \quad (2)$$

where $x_{235}=0.04$ is the weight enrichment, $\rho_U=9.3$ g/cm³ is the uranium density in the fuel, $N_{av}=6.022 \times 10^{23}$ is Avogadro's number and $A_{235}=235$ g/cm³ is the atomic weight of ²³⁵U. Thus, from Eq (1), we get $\phi \approx 1.1 \times 10^{13}$ n/cm²-s.

- b)

- i) The conservation of energy equation is:

$$\dot{m} \frac{dh}{dz} = q''(z) \pi D \quad (3)$$

where $\dot{m}=0.5$ kg/s. This equation can be integrated between $z=0$ and $z=z_{sat}$, to give:

$$\dot{m}(h_f - h_{in}) = \pi D \int_0^{z_{sat}} q''(z) dz \Rightarrow \dot{m}(h_f - h_{in}) = DLq''_{\max} \left[1 - \cos\left(\frac{\pi z_{sat}}{L}\right) \right] \quad (4)$$

where h_{in} is the enthalpy at the inlet. The enthalpy difference on the LHS of Eq. (4) is $h_f - h_{in} = c_{p,f}(T_{sat} - T_{in})$, with $T_{in}=260$ °C. Thus, solving for z_{sat} in Eq. (4), we get:

$$z_{sat} = \frac{L}{\pi} \cos^{-1} \left[1 - \frac{\dot{m} c_{p,f} (T_{sat} - T_{in})}{DLq''_{\max}} \right] \approx 0.98 \text{ m}$$

- ii) Integrating Eq. (3) from $z=0$ to $z=L$, we get:

$$\dot{m}(h_{out} - h_{in}) = \pi D \int_0^L q''(z) dz = 2DLq''_{max} \quad (5)$$

where h_{out} is the enthalpy at the outlet, which can be expressed in terms of the outlet quality, x_{out} , as $h_{out} = x_{out}h_g + (1 - x_{out})h_f$. The enthalpy difference on the LHS of Eq. (5) can then be re-written as:

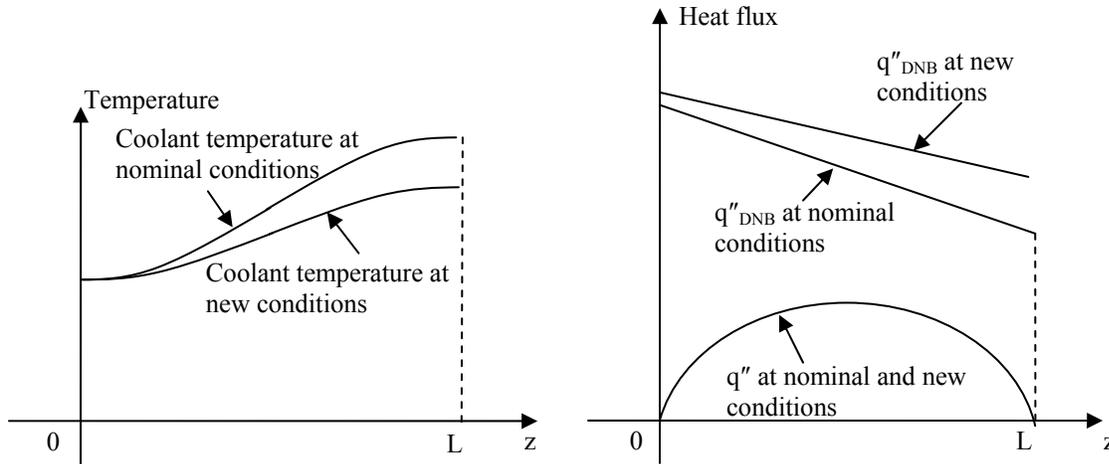
$$h_{out} - h_{in} = (h_{out} - h_f) + (h_f - h_{in}) = x_{out}h_{fg} + c_{p,f}(T_{sat} - T_{in}) \quad (6)$$

Substituting Eq. (6) into Eq. (5) and solving for x_{out} , we get:

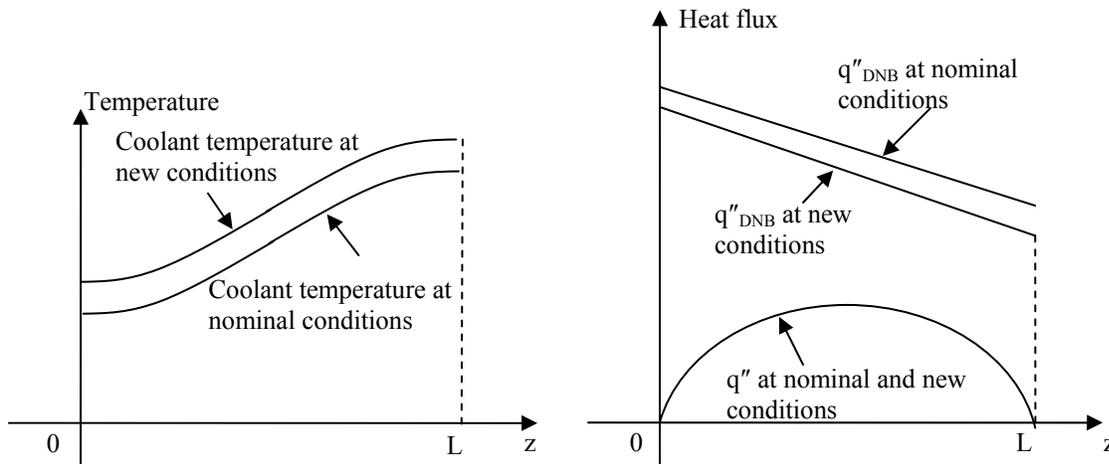
$$x_{out} = \frac{2DLq''_{max} - c_{p,f}(T_{sat} - T_{in})}{h_{fg}} \approx 0.024$$

c)

- i) Higher flow rate, nominal power and inlet temperature. The coolant temperature is lower, thus the DNB heat flux is higher. The operating heat flux does not change. Therefore, the MDNBR is higher.



- ii) Higher inlet temperature, nominal power and flow rate. The coolant temperature is higher, thus the DNB heat flux is lower. The operating heat flux does not change. Therefore, the MDNBR is lower.



Problem 1 (50%) - Calculating the Mass Flow Rate for a Prescribed Pressure Drop

i)

The mass flow rate, \dot{m} , is:

$$\dot{m} = GA$$

$A = \frac{\pi}{4}D^2 = 0.785 \text{ cm}^2$ is the flow area, and $D=1 \text{ cm}$. The mass flux, G , can be found from the imposed friction pressure drop:

$$\Delta P_{fric} = f \cdot \frac{L}{D} \cdot \frac{G^2}{2\rho} \tag{7}$$

where $L=10 \text{ m}$, ρ is the coolant density and f is the friction factor. The problem statement suggests to (i) assume $Re > 30000$, (ii) neglect roughness and (iii) assume fully-developed flow. Under these assumptions, the friction factor can be calculated from the following correlation:

$$f = \frac{0.184}{Re^{0.2}} = \frac{0.184}{(GD/\mu)^{0.2}} \tag{8}$$

Substituting Eq. (8) into Eq. (7) and solving for G , one gets:

$$G = \left[\frac{2\rho\Delta P_{fric} D^{1.2}}{0.184L\mu^{0.2}} \right]^{1/1.8} \tag{9}$$

Eq. (9) gives $G \approx 5040 \text{ kg/m}^2\text{-s}$ and $G \approx 6490 \text{ kg/m}^2\text{-s}$, for sodium and liquid salt, respectively. Thus, the mass flow rates are 0.404 kg/s and 0.509 kg/s , respectively. Note that in both cases, the value of the Reynolds number is above 30000, thus that assumption is verified.

ii)

Since the heat flux is uniform, the maximum surface temperature will occur at the channel outlet and will be equal to (from Newton's law of cooling):

$$T_{surf} = T_{out} + \frac{q''}{h} \quad (10)$$

where T_{out} is the bulk coolant temperature at the channel outlet, $q'' = 200 \text{ kW/m}^2$ and h is the heat transfer coefficient. From the conservation of energy equation (integrated between $z=0$ and $z=L$), we have:

$$T_{out} = T_{in} + \frac{q'' \pi D L}{\dot{m} c_p} \quad (11)$$

where $T_{in} = 600^\circ\text{C}$ and c_p is the coolant specific heat. The outlet temperature is calculated from Eq. (11) to be ~ 720 and $\sim 651^\circ\text{C}$ for sodium and the liquid salt, respectively. As for the heat transfer coefficient, in the case of sodium ($Pr = 0.037 \ll 1$, turbulent, fully-developed flow, constant heat flux in round channel) the correct correlation is $Nu = 7 + 0.025 Pe^{0.8}$ (with $Pe \equiv Re \cdot Pr \sim 1114$), from which we get $h = 83.1 \text{ kW/m}^2\text{-K}$, whereas in the case of liquid salt ($Pr = 4.82 > 1$, turbulent, fully-developed flow, constant heat flux in round channel) the Dittus-Boelter correlation is suitable, from which we get $h \sim 17.5 \text{ kW/m}^2\text{-K}$. Substituting these values into Eq. (10), we get $T_{surf} \approx 722$ and 663°C for sodium and liquid salt, respectively. Note that, in spite of a much higher heat transfer coefficient, the surface temperature in the sodium case is higher than in the liquid salt case. This is due to the lower specific heat of sodium which results in a higher T_{out} .

Problem 2 (20%) - Sizing the Silicon Carbide Layer in a TRISO Fuel Particle

The stresses for a thin spherical shell of radius R_s can be calculated as follows:

$$\begin{aligned} \sigma_r &= -(p_i + p_o)/2 \\ \sigma_\theta &= \sigma_\phi = (p_i - p_o)R_s / (2t) \end{aligned} \quad (12)$$

Where $R_s = 280 \text{ }\mu\text{m}$, $p_o = 8 \text{ MPa}$, t is the (unknown) thickness of the shell, and p_i is the internal pressure due to the fission gases. The fission gas pressure can be calculated from the perfect gas equation as follows:

$$p_i = NRT/V_{FG} = 36.8 \text{ MPa} \quad (13)$$

Where $N=10^{-7}$ mol, $R=8.31$ J/mol-K, $T=1223$ K (950°C) and $V_{FG} = 0.3 \frac{4}{3} \pi R_s^3 = 2.8 \times 10^{-11}$ m³. The Von Mises failure criterion is expressed by the following inequality:

$$\sqrt{\frac{1}{2}[(\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_\varphi)^2 + (\sigma_\theta - \sigma_\varphi)^2]} < S_y \quad (14)$$

where $S_y=200$ MPa for SiC at 950°C. Noting that $\sigma_\theta = \sigma_\varphi$, Eq. (14) becomes:

$$(\sigma_\theta - \sigma_r) < S_y \quad (15)$$

Substituting Eqs. (12) into Eq. (15), one gets:

$$(p_i - p_o)R_s / (2t) + (p_i + p_o) / 2 < S_y \quad (16)$$

Solving Eq. (16) for t , one gets the minimum required value of the shell thickness to prevent failure:

$$t_{\min} = R_s \frac{p_i - p_o}{2S_y - (p_i + p_o)} = 22.7 \mu\text{m}$$

Note that the use of the thin-shell theory is justified because $R_s/t_{\min} > 10$.

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