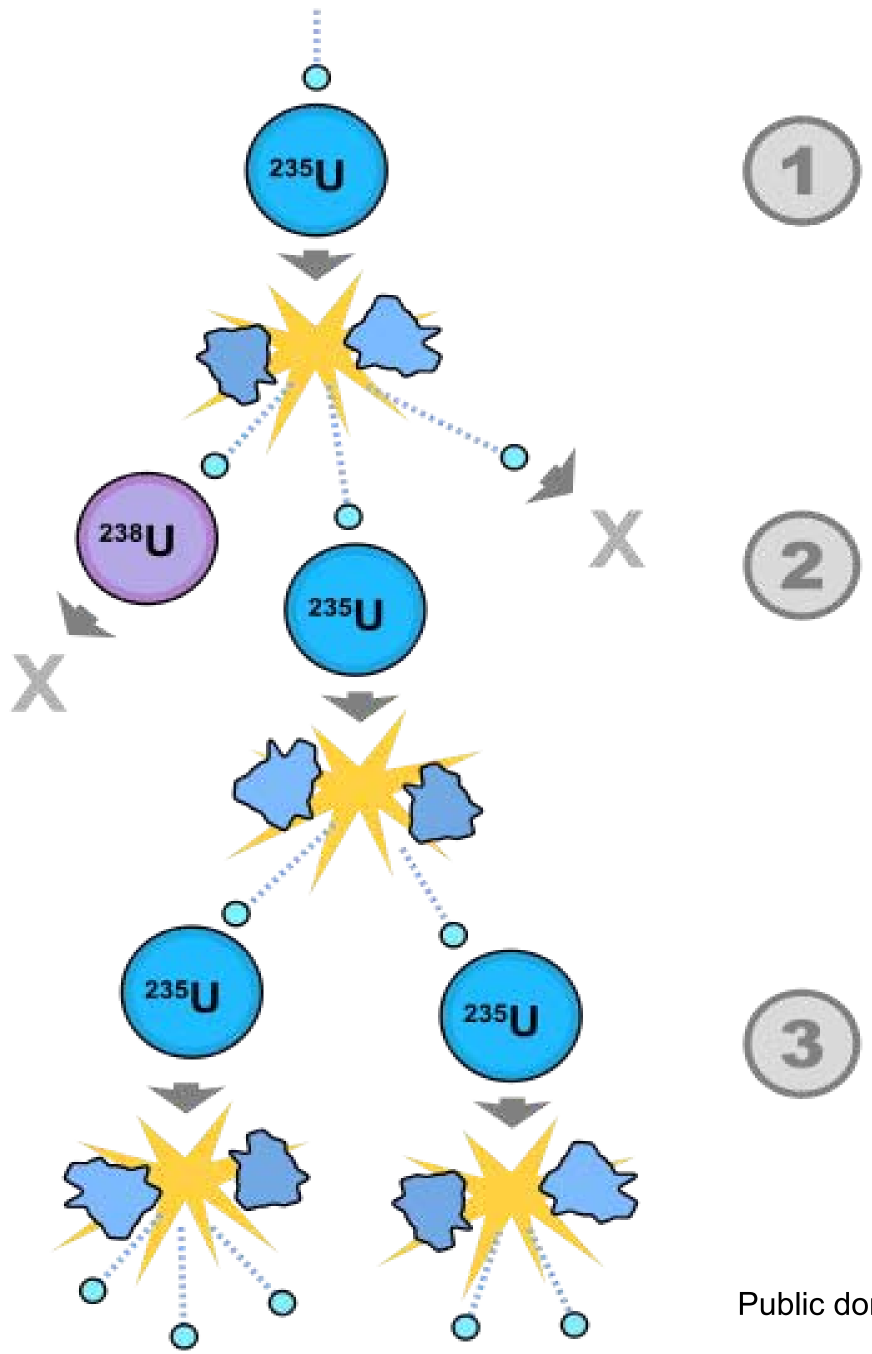
Chapter 5
Kinetics

Fission chain reaction



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Delayed neutrons

Delayed neutrons emitted from the decay of fission products long after the fission event. Delay is caused by half-life of beta decay of delayed neutron pre-cursor nucleus.

Delayed neutrons

Delayed neutrons are grouped into 6 groups of delayed neutron precursors with an average decay constant λ_i defined for each.

$\lambda_i \equiv$ The decay constant for the i^{th} group of delayed neutrons

$\beta_i \equiv$ The fraction of fission neutrons emitted by delayed neutron precursor group i.

$\beta \equiv$ The fraction of fission neutrons that are delayed.

$$= \sum_{i=1}^{6} \beta_i \qquad \beta \sim 0.0075$$

Point Kinetic Equations

For an initially critical system

$$\frac{dp(t)}{dt} = \left(\frac{\rho(t) - \beta}{\Lambda} \right) p(t) + \sum_{k=1}^{K} \lambda_k c_k(t)$$

$$\frac{dc_k(t)}{dt} = \frac{\beta_k}{\Lambda} p(t) - \lambda_k c_k(t)$$

Most important assumption: Assumes that the perturbation introduced in the reactor affects only the amplitude of the flux and not its shape.

Dynamic Reactivity $\rho(t)$

- Most important kinetics parameter
  - Its variations are usually the source of changes in neutronic power
  - Only term that contains the neutron loss operator (M operator)
    - Associated to control mechanisms
    - Also sensitive to temperature
  - No units
    - Expressed in terms of mk (milli-k) or pcm

Delayed-Neutron Fraction *β(t)*

- Effective delayed neutron fraction is linked to the constants of each fissionable isotope which measure the fraction of fission product precursors
  - Called “effective” because it is weighted by the flux in the reactor
- Can vary with burnup
  - Different values exist at BOL and EOL
  - Variations in burnup are on a much larger time scale than usual range of application of point kinetics equations

Prompt-Neutron Lifetime

- Measure of the average time a neutron survives after it appears as either a prompt neutron or a delayed-neutron

Solution with one effective delayed neutron precursor group

$$\frac{dP(t)}{dt} = \left(\frac{\rho_0 - \beta}{\Lambda} \right) P(t) + \lambda C(t)$$

$$\frac{dC(t)}{dt} = \frac{\beta}{\Lambda} P(t) - \lambda C(t), \; t \geq 0$$

Step reactivity change	$t < 0 \Rightarrow \rho(t) = 0, P(t) = P_0$ $t \geq 0 \Rightarrow \rho(t) = \rho_0$

Solution

$$P(0) = P_0, \; C(0) = \frac{\beta}{\lambda\Lambda} P_0$$

Initial conditions

$$P(t) = Pe^{st}, \quad C(t) = Ce^{st}$$

Solutions

$$sP = \left[\frac{\rho_0 - \beta}{\Lambda}\right] P + \lambda C$$

Substitute

$$sC = \frac{\beta}{\Lambda} P - \lambda C$$

Solution

$$\begin{bmatrix} s-\left(\dfrac{\rho_0-\beta}{\Lambda}\right) & -\lambda \\ -\dfrac{\beta}{\Lambda} & s+\lambda \end{bmatrix}\begin{bmatrix} P \\ C \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Linear
Homogeneous
System

$$\left[s-\left(\frac{\rho_0-\beta}{\Lambda}\right)\right][s+\lambda]-\frac{\beta\lambda}{\Lambda}=0$$

$$\Lambda s^2+(\lambda\Lambda+\beta-\rho_0)s-\rho_0\lambda=0$$

Non-trivial solution
if and only if
det A=0

Solution

$$s_{1,2} = \frac{1}{2\Lambda}\left[-\left(\beta - \rho_0 + \lambda\Lambda\right) \pm \sqrt{\left(\beta - \rho_0 + \lambda\Lambda\right)^2 + 4\lambda\Lambda\rho_0}\right]$$

$$P(t) = P_1 e^{s_1 t} + P_2 e^{s_2 t} \text{ and } C(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

Approximate solution

Assume $\lambda\Lambda / \beta << 1$

$|\rho_0| << \beta$

$$\Rightarrow \quad s_1 \cong \frac{\lambda\rho_0}{\beta - \rho_0}, \quad s_2 \cong -\left(\frac{\beta - \rho_0}{\Lambda}\right)$$

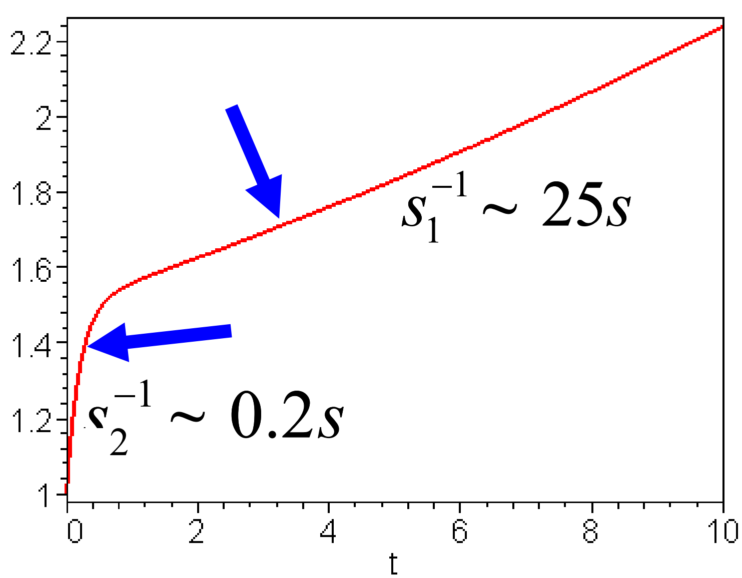
$$P(t) \cong P_0\left[\left(\frac{\beta}{\beta - \rho_0}\right)\exp\left(\frac{\lambda\rho_0}{\beta - \rho_0}\right)t - \left(\frac{\rho_0}{\beta - \rho_0}\right)\exp\left(\frac{\beta - \rho_0}{\Lambda}\right)t\right]$$

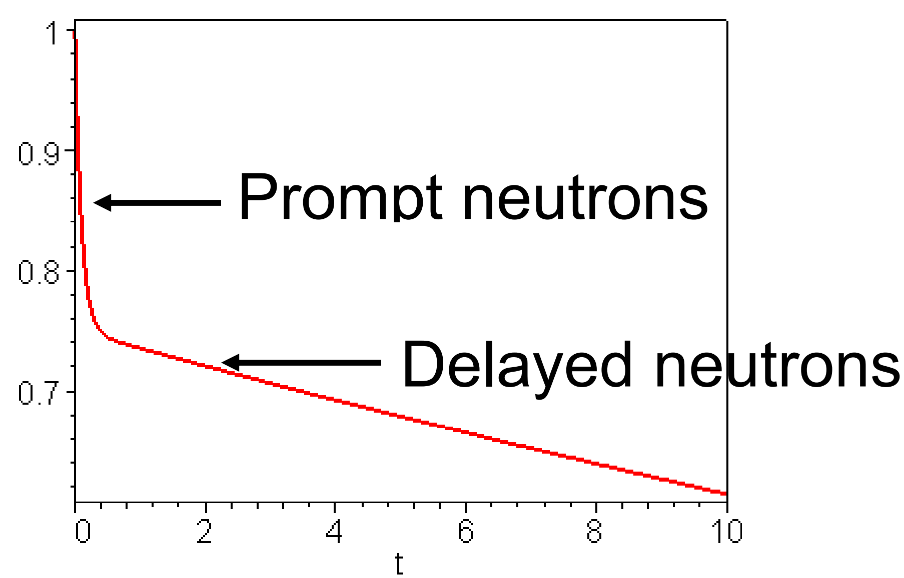
Solution

This solution is not valid for large changes in reactivity!

$$P(t) \cong P_0\left[\left(\frac{\beta}{\beta-\rho_0}\right)\exp\left(\frac{\lambda\rho_0}{\beta-\rho_0}t\right)-\left(\frac{\rho_0}{\beta-\rho_0}\right)\exp\left(\frac{\rho_0-\beta}{\Lambda}t\right)\right]$$

$$|\rho_0| \quad 0.0025,\ \beta \quad 0.0075,\ \lambda \quad 0.08\mathrm{s}^{-1},\ \Lambda \quad 10^{-3}\mathrm{s}$$





Reactor Period

- Defined as the power level divided by the rate change of power

$$\tau(t) = \frac{p(t)}{dp(t)/dt}$$

  - Period of infinity implies steady-state
  - Small positive period means a rapid increase in power
  - Small negative period means rapid decrease in power
  - If period is constant, power varies according to

$$p(t) = p_0 e^{t/\tau}$$

Reactor Period

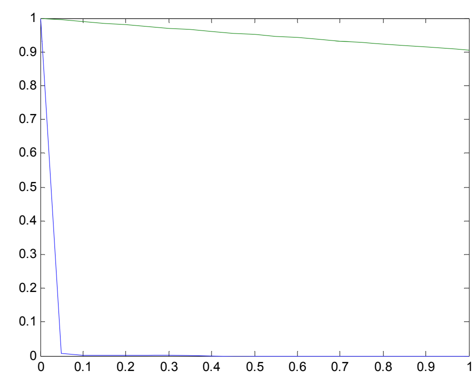
- For the case with one delayed group, the reactor period can be separated in two parts
  - Prompt period
  - Stable period
- The solution has two exponential and they usually have very different coefficients.

Scenario 1

- $\rho_0 < 0$
  - Corresponds to a quick reactor shutdown
  - Both roots are negative
  - s_2 <<< s1 thus the power drops almost instantly to a fraction of its initial power (prompt drop)
  - However, it is impossible to stop a reactor instantaneously

Example

- The second root is so small that in the matter of a fraction of second becomes inconsequential
- Power drops almost instantly to the coefficient of the first exponential term
- Thus the stable period is equal to $1/s_1$
- And the prompt period is equal to $1/s_2$



$$P(t) \cong P_0 \left[\left(\frac{\beta}{\beta - \rho_0} \right) \exp\left(\frac{\lambda \rho_0}{\beta - \rho_0} \right) t - \left(\frac{\rho_0}{\beta - \rho_0} \right) \exp\left(\frac{\rho_0 - \beta}{\Lambda} \right) t \right]$$

Demo

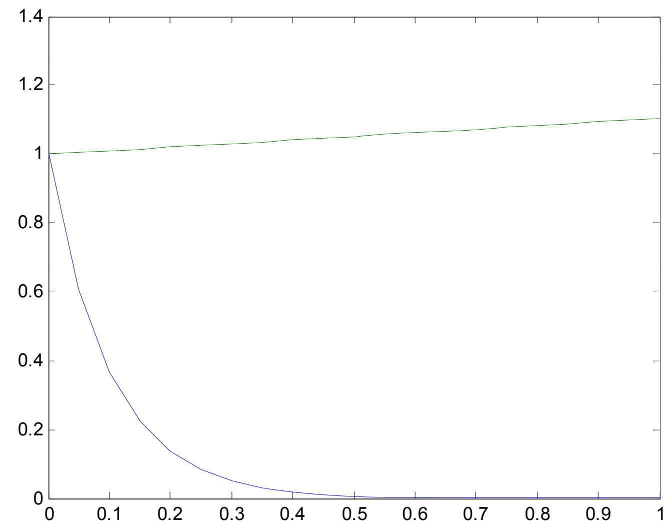
- Negative Step insertion of -12mk
- Parameters
  - Beta = 0.006
  - LAMBDA = 0.001
  - Lambda = 0.1 s^{-1}

- Power drops by 33% almost instantly, and then decays slowly

Scenario 2

- $0 < \rho_0 < \beta$
- One root is positive and the other is negative
- Power increases rapidly and the grows exponentially

- Power increases rapidly by beta/(beta-rho)
  - Positive prompt jump
- Stable period is equal to $1/s_1$
- Prompt period is equal to $1/s_2$



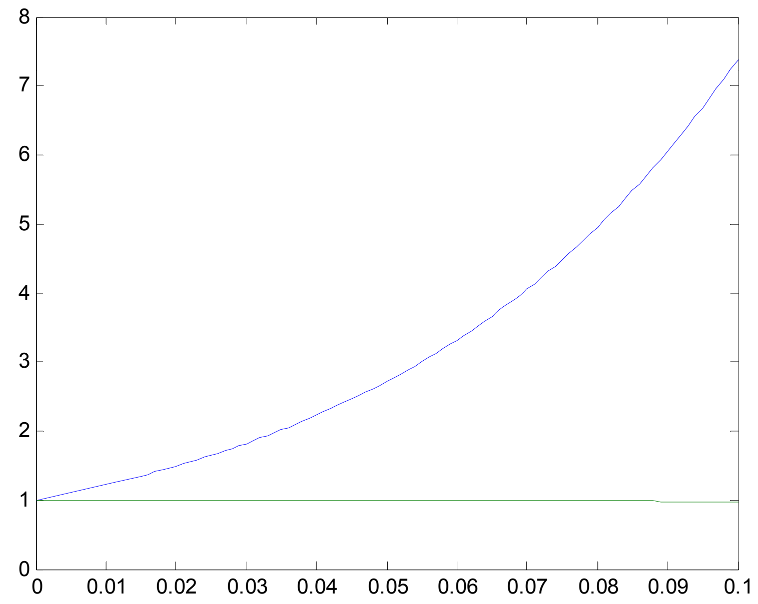
Demo

- Positive Step insertion of 1mk
- Parameters
  - Beta = 0.006
  - LAMBDA = 0.001
  - Lambda = 0.1 s^{-1}
- If rho approaches beta, the stable period becomes very short

Scenario 3

- $\rho_0 > \beta$ (prompt super-critical)
- Reactor is critical without the need of the delayed neutrons
- One root is positive and one is negative
- Reactor period becomes less than 1s

- Power increases at a very rapid rate
- Disastrous consequences
  - Unless a feedback mechanism can cancel out the reactivity



Demo

- Positive Step insertion of 7mk
- Parameters
  - Beta = 0.006
  - LAMBDA = 0.001
  - Lambda = 0.1 s^{-1}
- Reactor is critical (or supercritical) without the presence of delayed neutrons
  - Prompt jump dominates

Limiting cases- Small reactivity insertions

$$\rho_0 \ll \beta,\ \text{thus } |s_1| \ll \lambda_1 < \lambda_2 ... < l^{-1}$$

$$T = \frac{1}{s_1} = \frac{1}{\rho_0}\left[l + \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i} \right] \cong \frac{\langle l \rangle}{\rho_0} \cong \frac{\langle l \rangle}{k-1}$$

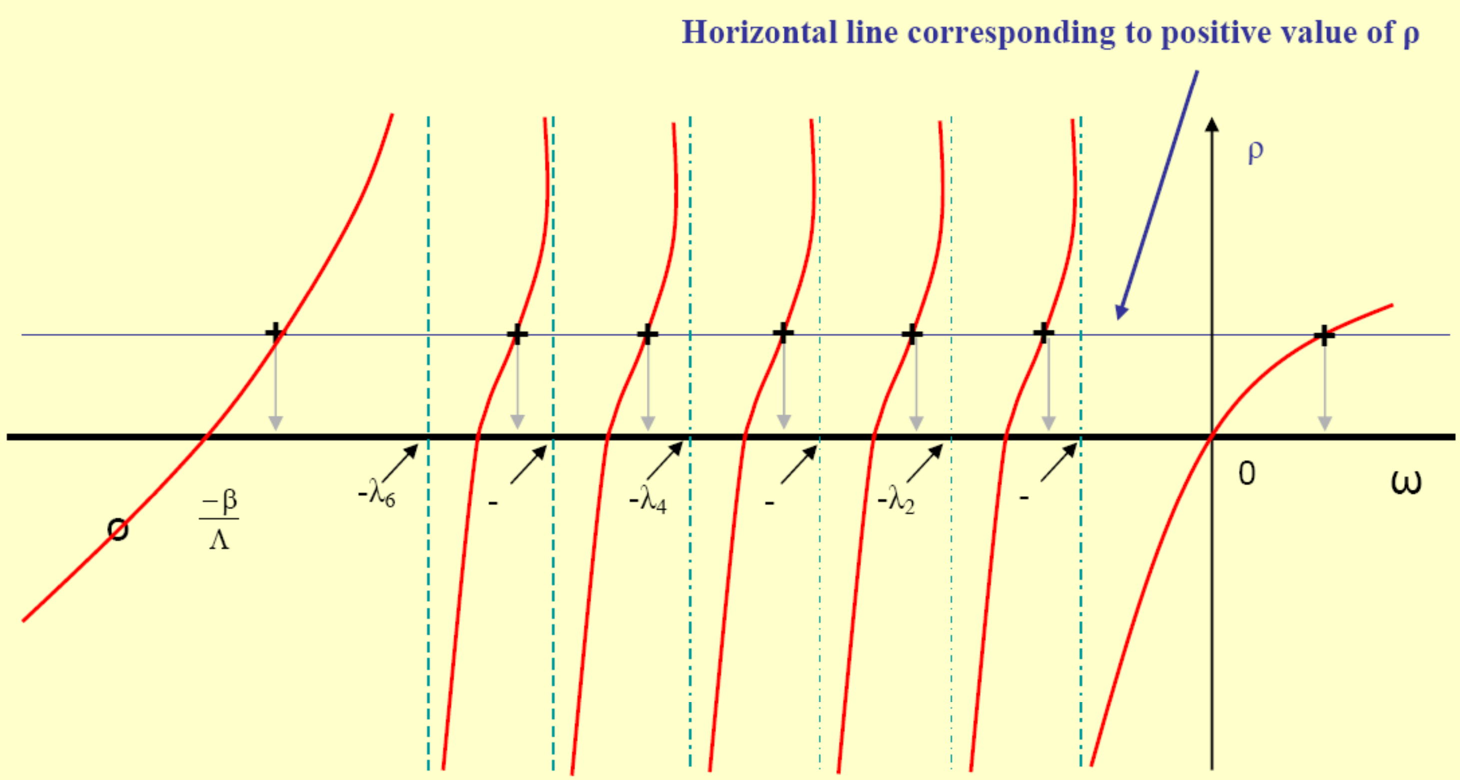
$$\rho_0 \gg \beta,\ \text{thus } s_1 \gg \lambda_i$$

Limiting cases-Large reactivity insertions

$$\rho_0 \cong \frac{s_1}{s_1 + l^{-1}} + \frac{l^{-1}}{s_1 + l^{-1}} \sum_{i=1}^{6} \beta_i = \frac{s_1 + \beta l^{-1}}{s_1 + l^{-1}}$$

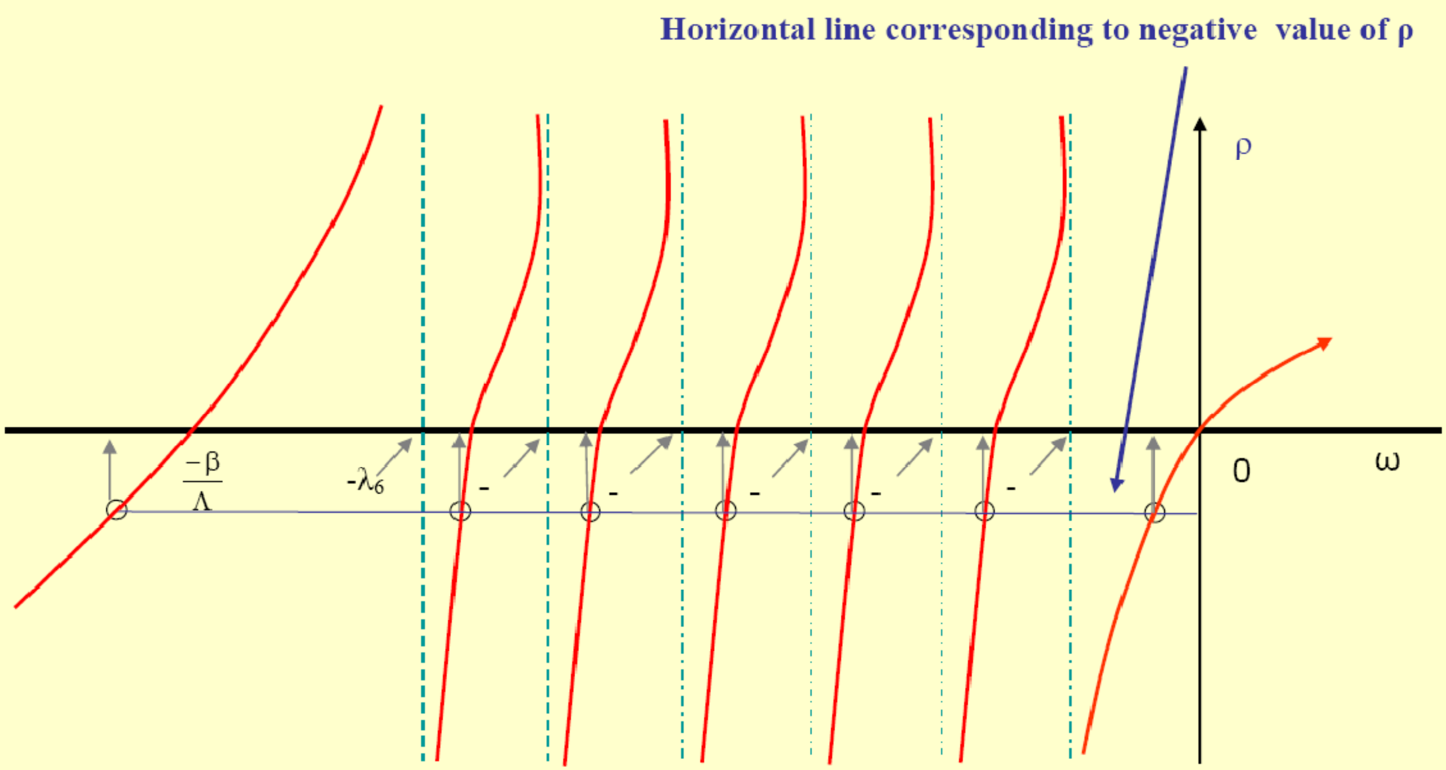
$$T = \frac{1}{s_1} \cong \frac{l}{k(\rho_0 - \beta)} \cong \frac{l}{k-1}$$

Positive Reactivity



Graphical Solution of Inhour Equation: Positive Reactivity

Negative Reactivity



Graphical Solution of Inhour Equation: Negative Reactivity

Typical parameters

	LWR	CANDU	Fast Reactor
Λ	5×10^{-5}	1×10^{-3}	1×10^{-6}
β	0.0075	0.006	0.0035
λ	0.1	0.1	0.1

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