22.01 Fall 2015, Problem Set 5 (Analytical Version)

Due: November 3rd, 11:59PM on Stellar

October 27, 2015

Complete all the assigned problems, and do make sure to show your intermediate work. Please upload your full problem set in PDF form on the Stellar site. Make sure to upload your work at least 15 minutes early, to account for computer/network issues.

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1 Conceptual Questions

Consider the three methods by which photons can interact with electrons in matter: Compton scattering, the photoelectric effect, and pair production.

1. At which photon energies are each of these effects the most prominent? In other words, which of these effects can be neglected at which energies?
2. In figure 10.5, why is the energy of the Compton edge of Compton electrons not equal to the energy of the incoming photon, and at what photon scattering angle is this Compton edge produced? Explain the origin of this energy difference.
3. In figure 10.17, which electron energy shell transitions (give the numbers of the levels involved) are responsible for the discontinuities in the attenuation coefficient? Which of the three photon interaction methods is responsible for these discontinuities?
4. In figure 10.13, why does the pair production cross section become non-zero abruptly at energies above $\frac{2\hbar\omega}{m_e c^2}$?
5. Explain why it is more likely to see a single-escape pair production peak from a NaI detector (figure 10.18), while it is more likely to see a double-escape pair production peak from a semiconductor detector (figure 10.19).

2 Analytical Questions

1. By treating the photon in figure 10.2 as a particle with energy $\hbar ck$ and momentum $\hbar k$, and by conserving x-momentum, y-momentum, and total energy, show the origins (re-derive) equations 10.6 through 10.9.
2. Derive a relation between the minimum and maximum values of the Klein-Nishina cross section as a function of incoming photon energy. Graph the angle of the angle of minimum scattering probability as a function of incoming photon energy. The angle of maximum scattering probability is always at zero degrees... with one exception. What is it?
3. For these problems, refer to the NIST table of x-ray attenuation coefficients.
    (a) Explain the qualitative differences in the attenuation coefficients of beryllium and lead in a quantitative manner, at the following energies: $E_\gamma < 100$keV, $E_\gamma = 1$MeV, $E_\gamma = 100$MeV. By this, we mean compare relative values of the relevant scattering cross sections, and explain any discrepancies between these and the relative values of the attenuation coefficients.

(b) What is the origin of the discontinuities in the attenuation coefficient for lead? Why is there more than one step change within close proximity at some places?

(c) For which energies is the *attenuation coefficient* in water higher than that in air? What about the *mass attenuation coefficient*?

3 Applied Questions

1. How thick would a lead apron have to be to shield you from 99.9% of the x-rays from a dental exam, assuming they are generated from a ^{60}Co source?

2. These ^{60}Co gamma rays will generate x-rays in the lead apron. Which interaction mechanism is responsible for this, and what percent of the x-rays will your clothing and skin absorb before they enter your body?

3. For the ^{40}K gamma spectrum shown, answer the following questions:

   (a) Label all visible peaks and major features, their energies, and explain their origins.

   (b) Identify where the locations of any missing peaks should be, their energies, and explain their origins.

   (c) How do you explain the energy difference between the full energy gamma peak and the Compton edge?