

Tech Note 3

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A Unified Case Formula For Platform Loading

A natural concern for the technical designer faced with a platform design problem is how much load the platform will be able to support. This note presents a unified case formula for identifying the maximum permissible load for a simple platform under normal loading conditions. It considers the most common sources of platform failure. Though the final result is a large and intimidating formula, it is nonetheless expressed in a form suitable for rapid numerical evaluation through a spreadsheet, programmable calculator, or other mathematical tool.

A few notes:

- For maximum flexibility, no design factor is applied. The designer will want to make an appropriate choice based on the situation and reduce the rating accordingly.
- An evenly distributed load is assumed. If the load is especially uneven, approaching a point load, this result may not apply.
- These calculations do not take into account off-axis or other complex loading conditions. In particular, it does not take into account any cross-bracing
- The surface of the platform is not taken into account.
- The weight of beams and surface materials should be subtracted from the allowable load.
- A standard $\frac{l}{240}$ deflection criteria is used.
- Standard indoor theatrical conditions are used. When necessary, the most conservative assumptions (e.g. load duration of 10 years) are employed. For less conservative conditions, the load rating can be improved by applying the appropriate adjustment factors to the allowable stress constants.

Definitions. *Let a platform $\mathcal{P} = \langle l, w, h \rangle$ be a simple platform¹ with length l and width w . It is supported by four columns of height h at the four corners.*

¹This rather odd bit of notation expresses the fact that we are defining a simple platform in terms of its length, width, and height. A pure mathematician might point out that by doing so we have defined a *platform space* that is a subset of \mathbb{R}^3 . This is an excellent reason to keep pure mathematicians far away from the theater.

We require the following data about the materials used to construct the platform:

- For the beam in the x direction, the modulus of elasticity E_x , moment of inertia I_x , allowable bending stress $F_{b,x}$, and section modulus S_x .
- For the beam in the y direction, the analogous quantities E_y , I_y , $F_{b,y}$, and S_y .
- For the columns (in the z direction), the cross-sectional column area A , the modulus of elasticity E_z , the allowable compressive stress F_c^* , and the minimum dimension d of the column.

Result. For a platform \mathcal{P} defined as above, we want to find the maximum allowable load per square foot, which we call $P(\mathcal{P})$. We can compute this by taking the smallest of five values:

$$P(\mathcal{P}) = \min \left\{ \begin{array}{l} 0.64 \frac{E_x I_x}{1200 l^3 w} \\ 0.64 \frac{E_y I_y}{1200 w^3 l} \\ \frac{8 F_{b,x} S}{w l^2} \\ \frac{8 F_{b,y} S}{l w^2} \\ \frac{4}{l w} A F_c^* \left(0.625 - 0.1875 \frac{E_z d^2}{h^2 F_c^*} - \sqrt{0.03516 \frac{E_z^2 d^4}{h^4 F_c^{*2}} - 0.1406 \frac{E_z d^2}{h^2 F_c^*} + 0.3906} \right) \end{array} \right.$$

Derivation. We first consider the beams running along the long axis of the platform. These beams can fail due to bending or shear, or they can provide an unsatisfactory level of deflection (the $\frac{l}{240}$ criteria previously mentioned). In practice, under theatrical loading conditions, beam failure due to shear is very rare, and minimizing deflection generally is the limiting factor.

The deflection Δ is given by

$$\Delta = \frac{5pl^4}{384EI}$$

where p is the weight per linear foot of beam length. Since we are interested in maximum weight per square foot, we adjust it by half the width, the area supported by each beam: the maximum allowable weight in square feet, $P = \frac{2p}{w}$. Applying the $\frac{l}{240}$ deflection criteria,

$$\begin{aligned} \frac{l}{240} &\geq \frac{5Pl^4 w}{768EI} \\ P &\leq \frac{768EI}{1200l^3 w} = 0.64 \frac{EI}{1200l^3 w} \end{aligned}$$

This gives us the maximum allowable weight due to deflection in the length axis. By interchanging length and width, we come to the corresponding limit due to deflection in the

other beam:

$$P \leq 0.64 \frac{EI}{1200w^3l}$$

Failure due to bending is also a concern. This imposes the following constraint:

$$\begin{aligned} M_{\max} &\leq F_b S \\ \frac{pl^2}{8} &\leq F_b S \\ \frac{Pwl^2}{8} &\leq F_b S \\ P &\leq \frac{8F_b S}{wl^2} \end{aligned}$$

and in the other beam,

$$P \leq \frac{8F_b S}{lw^2}$$

Next, we consider the columns supporting the platform. We assume the load is distributed evenly among the four columns, i.e. that each column supports a weight of $\frac{Plw}{4}$. We first identify the critical buckling design value

$$F_{cE} = 0.3 \frac{Ed^2}{h^2}$$

and then compute the column stability factor

$$\begin{aligned} C_p &= \frac{1 + \frac{F_{cE}}{F_c^*}}{1.6} - \sqrt{\left[\frac{1 + \frac{F_{cE}}{F_c^*}}{1.6} \right]^2 - \frac{F_{cE}}{F_c^*}} \\ &= 0.625 - 0.625 \frac{F_{cE}}{F_c^*} - \sqrt{0.3906 \frac{F_{cE}^2}{F_c^{*2}} - 0.469 \frac{F_{cE}}{F_c^*} + 0.3906} \\ &= 0.625 - 0.1875 \frac{Ed^2}{h^2 F_c^*} - \sqrt{0.03516 \frac{E^2 d^4}{h^4 F_c^{*2}} - 0.1406 \frac{Ed^2}{h^2 F_c^*} + 0.3906} \end{aligned}$$

which leads to the maximum allowable compressive load

$$P \leq \frac{4}{lw} A F_c^* \left(0.625 - 0.1875 \frac{Ed^2}{h^2 F_c^*} - \sqrt{0.03516 \frac{E^2 d^4}{h^4 F_c^{*2}} - 0.1406 \frac{Ed^2}{h^2 F_c^*} + 0.3906} \right)$$

We take the minimum of these maximum loads, i.e. the most restrictive, and it is our final result for the maximum allowable load. \square

References

- Holden and Sammler, *Structural Design for the Stage*, Focal Press 1999
- Lots and lots of math