

# Homework/Problem Set 2

MAS 961 / API-309

Due April 12 (in class)

Instructor: César A. Hidalgo

75 points + (10 bonus )in total (-10 points for each day after the due date)

## I Link Prediction (15 points)

1) In class we saw a paper on Link Prediction in which David Liben-Nowell and Jon Kleinberg used different metrics to predict the likelihood that a link will be formed in the future (The paper is also available for download as a class reading). Take the sample network of HW1 Q3 and use it to find the 5 links more likely to appear using.

- Graph distance (7 points)
- Common neighbors (8 points)

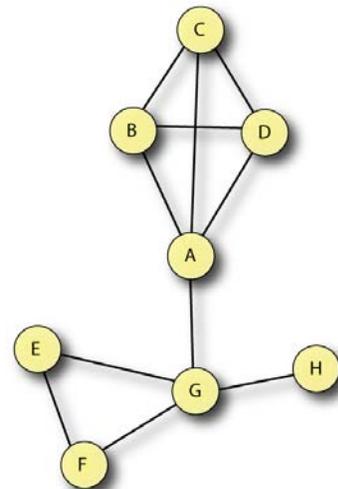
## II Newman and Girvan Modularity (25 points + 10 Bonus Points):

In (Proc Natl Acad Sci U S A. 2006 June 6; 103(23): 8577–8582) Mark Newman defines the modularity of a network divided into two components as (see paper for specification on notation):

$$Q = \frac{1}{4m} \sum_{ij} \left( A_{ij} - \frac{k_i k_j}{2m} \right) S_i S_j,$$

We will now get a better intuition on what this quantity means. Consider the network in the figure below and:

- Find the link with the maximum betweenness centrality (1 point)
- Remove this link, identify two groups and calculate the modularity of this partition (8 points)
- Now, consider the original network from the figure and the groups identified in b. Add a link between nodes E and H and recalculate modularity  $Q$ . Did the modularity  $Q$  go up or down? Why? (8 points)
- Consider the original network from the figure and the groups identified in b. Now add a link between nodes F and A and recalculate modularity  $Q$ . Did  $Q$  go up or down? Why? (8 points)



**BONUS** (10 points) (can be used to make up points from other homework):

Find the eigenvector associated with the maximum eigenvalue of the modularity matrix. Use this to find the vector  $s$  that maximizes modularity.

### III Socially Perceived Averages (35 points):

Social networks matter because they define the reference group to which we measure up to. This makes correlations between the structure of the network and the properties of the individuals that make up the network relevant.

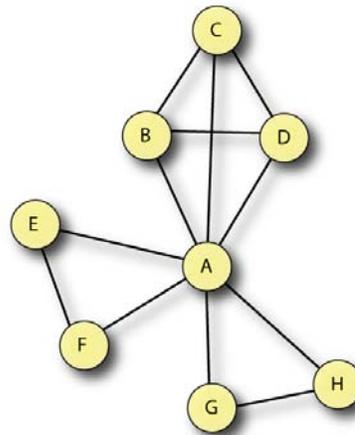
a) Consider that each node  $i$  has a property  $V_i$ , which you can think of as its height, weight, income, etc. Consider also that the network can be described by an adjacency matrix  $A_{ij}$  whose elements are equal to 1 if  $i$  and  $j$  are connected and 0 otherwise.

a.1) Write an expression for the average value of  $V$  for a node's neighbors. We will call this the *local average*. (4 points)

a.2) Write an expression for the *perceived average* as the *mean local average* of all nodes in the network. (4 points)

b) Consider the network in the picture below and the following distribution of  $V$  values

NAME	VALUE (V)
Anna	5
Ben	8
Carl	9
Dennis	9
Earl	1
Frank	2
Gary	4
Harry	2



b.1) Calculate the average value of  $V$  (1 point) (we will call this the Global Average)

b.2) Calculate the local average of  $V$  for each node (8 points)

b.3) Assume a world in which people tend to adjust to the average of their peer group. For whom does it make a difference to consider the global average, versus the local average?

b.4) Calculate the perceived average of the system (2 point)

b.5) Find an allocation of values to nodes such that the perceived average is maximum. (8 points)

b.6) Find an allocation of values to nodes such that the perceived average is minimum (8 points)

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